Multilayer Embedding Networks Using Immittance Converters, and Its Applications, ITC-CSCC'2000

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Abstract: The method for immittance conversion and its mathematical properties are discussed. Some design methods for complex multilayer embeddeding networks are proposed. Furthermore, as an application of multilayer embedding networks, the equivalent circuit model of a blood vessel is shown with its simulation results.

1. Introduction

The various synthesis techniques have been investigated to simulate $\pm$L, $\pm$C, $-$R, multiplied C, and higher order frequency dependent elements [1][2][3]. One of these techniques is the method of simulating such elements by embedding a passive RC network among immittance converters (IM converters), which is termed "embedding method (EM method)". In the networks designed by the EM method, IM converters play the part for giving the immittance conversion, while RC elements play the part for receiving it. Then we may call these networks as "EM networks" for simplicity.

The conventional EM method has a weak point that EM networks including plural immittance conversion parts cannot be obtained; therefore, this method is not suitable for constituting the complex networks.

In order to overcome this problem, new multiple simulation type EM networks including many different conversion parts have been proposed [4][5]. These new EM networks have a multilayer structure, and each layer can be regarded as the conventional EM network (a single layer network: SEM network); then we may call these new EM networks "multilayer EM networks (MEM networks)." By using the design method of MEM networks, we can obtain various types of EM networks and can simulate complex networks including many different conversion parts.

Focussing our attention on IM converters in MEM networks, we can obtain the essential condition for MEM networks. This condition, called "MEM network condition," tells a mathematical property related to the Transmission Matrices (TM) of converters in MEM networks. Furthermore, we obtain another condition on TM of converters for any closed loop in MEM networks. These conditions on TM can be applied to the design of MEM networks [4].

A fundamental method for designing MEM networks is an equivalent-transformation metehod by consecutive position interchange of IM converters and RC elements (called CPI method) [5].

For the design of a complex MEM network, however, a simple application of the CPI method to the whole network is considered not efficient; therefore, other design methods are investigated in this report for complex MEM networks, in which the MEM networks condition is to be satisfied. A repetitive connection of pre-designed MEM networks at each port makes a new MEM network. As an application of this method, a simple example of equivalent electrical circuit of a blood vessel is shown.

2. Multilayer EM networks

A general figure of MEM network is shown in Fig. 1.

Each $T_{i,j}$ indicates an IM converter whose transmission matrix (TM) is as follows:

$$\vec{T}_{i,j} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} t_A(s) & 0 \\ 0 & t_D(s) \end{pmatrix}, \quad \begin{matrix} i = 1,2,\cdots,m,\cdots,n \\ j = 1,2,\cdots \end{matrix} \qquad (1)$$

where $t_A(s)$ and $t_D(s)$ have the form of K_1/s^n or s^n/K_2 usually; K_1 and K_2 are constant. The subscript i is a branch number and j is a serial number. An arrow of $\vec{T}_{i,j}$ indicates a defined direction of this IM converter, and the following equation is valid.

$$\vec{T}_{i,j} = \overleftarrow{T}_{i,j}^{-1} \qquad (2)$$

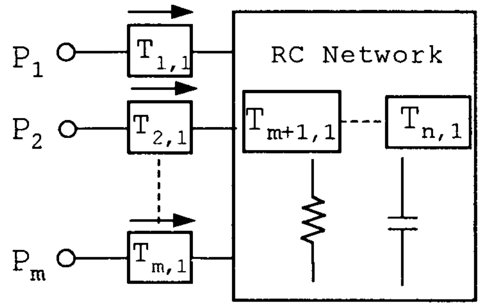

Figure 1. A general figure of multilayer EM network.

In the following, the ground symbols in figures are abbreviated for simplicity.

In Fig. 1, there exists a common return, then RC network parts can be considered as the sets of unit sections (US) which are depicted in Fig. 2(a), (b), and a TM of US is denoted by U as follows: (a) in the case of an ungrounded element

$$U = \begin{pmatrix} 1 & Z \\ 0 & 1 \end{pmatrix} \qquad (3)$$

and (b) in the case of a grounded element

$$U = \begin{pmatrix} 1 & 0 \\ Y & 1 \end{pmatrix} \qquad (4)$$

where Z is an impedance and Y is an admittance.

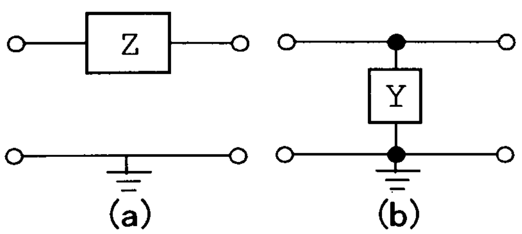

Figure 2. Unit sections of RC network parts.

As mentioned above, EM networks consist of the parts for giving the immittance conversion and the parts for receiving it, and they are IM converters and RC elements respectively. As one of the features of EM networks, the fundamental characteristics of the IM converter parts are unrelated to the kinds of RC elements and their values. In order to develop the concept of EM method, we only pay attention to the IM converter parts, and assume that TM of every US is replaced by the unit matrix (UM). In the following, we express the TM between port P_i and P_j as $T_{M[i,j]}$, and the UM as $\mathbf{1}$. In Fig. 1, it is the condition for forming the EM networks that the following relation is valid.

$$T_{TM[i,j]} = T_{TM[j,i]} = \mathbf{1}, \quad i \neq j \qquad (5)$$

where $T_{TM[i,j]}$ is the product of all IM converters in an arbitrarily selected route between P_i and P_j; namely,

$$T_{TM[i,j]} = \vec{T}_{i,1}\vec{T}_{i,2}\cdots\vec{T}_{k,q}\vec{T}_{k,q+1}\cdots\vec{T}_{j,2}^{-1}\vec{T}_{j,1}^{-1} \qquad (6)$$

where each $\vec{T}_{i,j}$ can be chosen arbitrarily but one of these $\vec{T}_{i,j}$'s must be decided to satisfy equation (5).

We may divide the right side of equation (6) in two parts at an arbitrary point, for example:

$$T_{TM[i,j]} = (\vec{T}_{i,1}\vec{T}_{i,2}\cdots\vec{T}_{k,q})(\vec{T}_{k,q+1}\cdots\vec{T}_{j,2}^{-1}\vec{T}_{j,1}^{-1}) = \vec{T}_x\vec{T}_y \qquad (7)$$

From equation (5) and (7), the following relation is derived.

$$\vec{T}_x\vec{T}_y = \vec{T}_x\vec{T}_x^{-1} = \vec{T}_y^{-1}\vec{T}_y = \mathbf{1} \qquad (8)$$

First of all, we insert a US with a RC elements between $\vec{T}_x$ and $\vec{T}_y(=\vec{T}_x^{-1})$, then the US is converted in the same manner as of a single layer EM network.

Similarly, it is easily shown that the product of all IM converters in any closed loop in MEM networks is UM, and we call this relation "closed loop theorem."

Applying TM to US, an ungrounded impedance Z for example, is converted into $(t_{xA}(s)/t_{xD}(s))Z$ where $t_{xA}(s)$ is the A element of $\vec{T}_x(=\vec{T}_y^{-1})$, and $t_{xD}(s)$ is the D element. The conversion method mentioned above can be applied to any place in the network shown in Fig. 1; therefore, by arranging proper IM converters and US's, we can design the complex networks including many different conversion parts.

3. Design techniques by CPI method

By using the consecutive position interchange (CPI) method [5], RC elements are converted to appropriate elements by an IM converter moved from one side to the other side of the elements and the conversion is identical to that given by the EM method. We are going to show the existence of an appropriate US which satisfies the following:

$$\vec{T}U = U'\vec{T} \qquad (9)$$

By multiplying both sides of the above equation by $\vec{T}^{-1}$ from the right-hand side, we obtain

$$U' = \vec{T}U\vec{T}^{-1} = \vec{T}U\overleftarrow{T} \qquad (10)$$

This equation shows that the CPI method gives a conversion equivalent to the EM method. By moving an IM converter by the CPI method, RC elements are converted to other elements.

If a network has branches, we must move an IM converter into all branches at a node. If branches are connected to a branch at a node, same IM converters in these branches are united into one same IM converter at the node and the IM converter moves into the branch. This method can be also applied to MEM networks naturally.

As a simple example of CPI method, Fig.3 shows a conversion of L into R, where $R = (K/s) \cdot (sL) = K \cdot L$. Thus the resistor between two converters K/s and s/K realizes the simulated inductance. In this figure, each value of converters represents the D-element of TM, while the A-element is supposed to be 1.

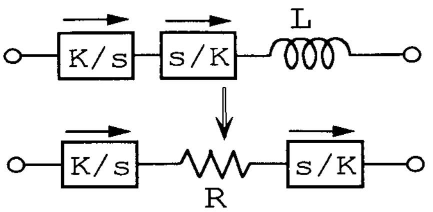

Figure 3. An example of immittance conversion.

4. Design methods for complicated MEM networks

In this chapter we investigate some design methods to obtain complicated MEM networks. From equation (5), the MEM network condition must be satisfied for any pair of ports in designed MEM networks; therefore, we should always verify the MEM network condition for the networks obtained by these methods. Especially, in the case of connecting non-MEM networks, an appropriate IM converter is to be inserted at ports or nodes in networks. In the following, conditions for design methods are given together with new concepts concerning TM's of IM converters.

4.1 MEM and MEM connection

The CPI method can be applied to obtain complicated MEM networks in principle. However, it is not necessary to apply the CPI method to the overall design of given networks. If the complicated networks can be divided into some simple parts, we can obtain the whole MEM networks by consecutive connections of the parts that are converted by CPI method. In biological system, for example, we can find many instances of equivalent circuit models that include iterative arrays of simple partial circuits.

Since a connection of any two MEM networks also forms a MEM network, a converter to be inserted is necessary in order to maintain the MEM networks condition.

4.2 MEM and Closed Loop connection

From the closed loop theorem mentioned above, another method for designing MEM networks is given. Cutting off a single closed MEM network at an arbitrary node, a new network with two ports is obtained. Obviously, it is shown from the closed loop theorem that this new network satisfies the MEM network condition for these two ports.

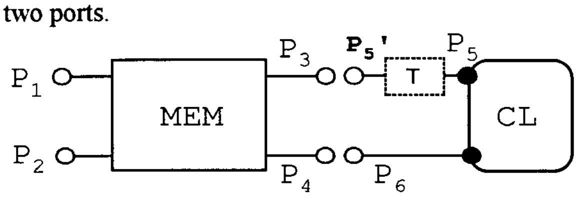

Figure 4. Connection of MEM and CL with inserted IM converter.

On the other hand, in the case of adding terminals to a closed loop, a new condition to satisfy the MEM networks condition is required for those terminals. In Fig.4, the product of TM's of converts in closed loop is UM, but the product of converters between arbitrarily selected nodes, P_5 and P_6, for instance, is not always UM. Since it is easily shown from the MEM network condition and the close loop theorem that the product of TM's of IM converters between arbitrarily selected nodes in a MEM network is uniquely given, we can easily determine the TM of IM converter to be inserted. The TM in this case is called the relative positional matrix for nodes or ports in the MEM network.

In Fig.4, the MEM network condition $T_{M[5,6]} = \mathbf{1}$ is invalid for new ports P_5 and P_6, selected from nodes in a close loop CL. But, by inserting an IM converter with $T = T_{M[5,6]}^{-1}$ at port P_5, the MEM network condition is valid for new ports, $P_5{}'$ and P_6.

4.3 Conversion section and its equivalent TM from ports

The TM intrinsic to a conversion section (section TM shortly) in a MEM network is characterized by the product of IM converters on an arbitrarily chosen route from any port to one of the nodes of the conversion section. From the property of relative positional matrix, it is easily shown that a section TM is also given uniquely.

This section TM represents the equivalent single TM from any port to a conversion section. The section TM is used to obtain the conditions for inserted converters to modify a conversion section.

The section, whose section TM equals to UM, tells that this section (U_i in Fig.5) has no conversion for any port in MEM network, and we can easily replace this section with any MEM network (MEM_2 in Fig.5) without inserting any IM converters to maintain the MEM network condition.

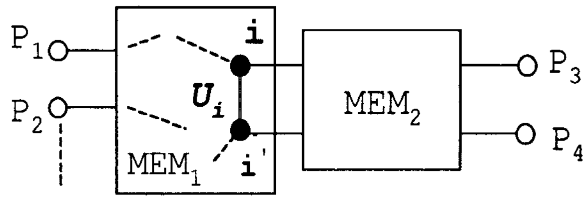

Figure 5. Section TM and insertion of MEM.

5. Application to equivalent circuit model of a blood vessel

In this section, we take an example of a blood vessel model as an electrical circuit, and design its MEM circuit. As shown in Fig.6, this model includes iterative arrays of simple partial circuits (ladder part.) It is apparently easy to design this ladder part by consecutive application of CPI method, section by section. Thus the MEM networks of Fig.6 is obtained by moving the IM convert from node 'a' as a starting point, and converting L into R.

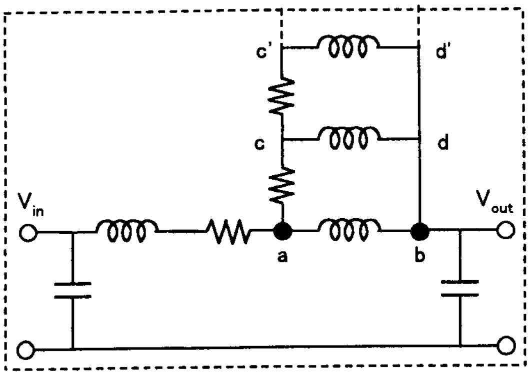

Figure 6. Electrical circuit model of a blood vessel.

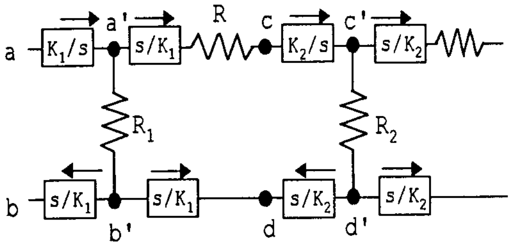

Figure 7. Converted Ladder part by CPI method.

In Fig.7 the converted ladder part in Fig.6 is shown.

The equivalent circuit model of a blood vessel is supposed to work under a rather low frequency; therefore this model consists of large inductances and capacitances. It is useful to investigate the real behavior of this model experimentally, but it is sometimes difficult to prepare electrical parts with the value of large inductance or capacitance. IM converters facilitate to simulate those elements independently of their types and values. In Fig.7, simulated inductances are realized by resistors (See Fig.3). However, this circuit is not always the best design. More simplified circuit shown in Fig.8 can be obtained by moving the converters near nodes into conversion sections.

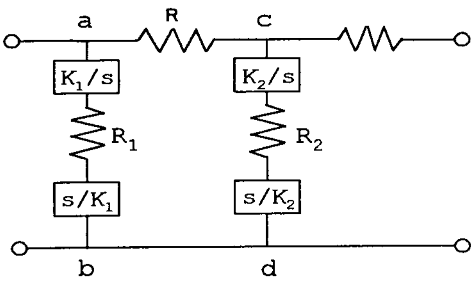

Figure 8. A simplified circuit of Fig.8.

The simulation results are shown in Fig.9 and Fig.10 for a single impulse and its output response. The effect of inserting ladder part, which suppresses the oscillation of output response, is easily understood from Fig.9. In this simulated circuit, Antoniou's circuit is used for IM converter. The conversion by Antoniou's circuit is determined only by the D element of TM, because the value of the A element is equal to 1.

MEM networks with simulated large C or L are considered useful to investigate the behavior of a model in a low frequency range.

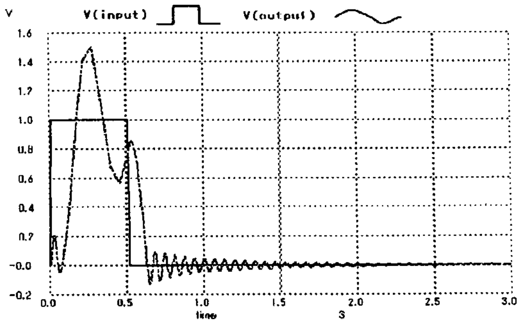

Figure 9. Response for a single impulse with ladder part.

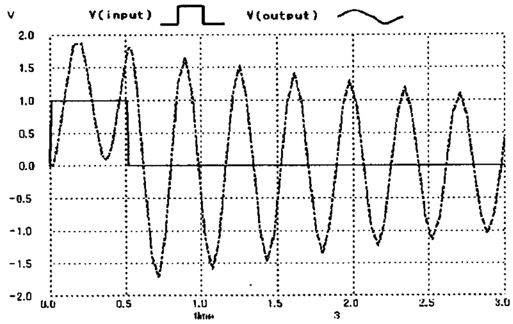

Figure 10. Response for a single impulse without ladder part.

6.Conclusion

Fundamental methods for immittance conversion and related mathematical properties are discussed.

For complicated MEM networks, some design methods by using pre-designed MEM networks are proposed. In connection of pre-designed MEM networks, appropriate converters are inserted to preserve the MEM network condition.

The section TM characterizes the conversion section viewed from any port; therefore, it can be utilized to obtain the conditions for connecting new MEM networks

An application of MEM network is shown as a circuit model of blood vessel together with its simulation results.

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