A Note on Synthesis of a Complex Coefficient BPF Based on a Real Coefficient BPF

Kazuhiro Shouno*, and Yukio Ishibashi**

*University of Tsukuba Doctoral Program in Engineering
**University of Tsukuba Institute of Electronics and Information Sciences
305-8573 1-1-1 Tennoudai Tsukuba Ibaraki Japan
University of Tsukuba Institute of Information Sciences and Electronics Ishibashi Lab.
Phone & Fax: +81-298-53-5565 Email: baja@fillab.is.tsukuba.ac.jp

Abstract — A complex coefficient filter obtained by directly exchanging several reactance elements included in a real coefficient filter for imaginary valued resistors is described. By using the proposed method, four varieties of complex coefficient filters are obtained. The stability problem is described. Finally, the frequency responses of the proposed filters are shown.

1. Introduction

Recently, many techniques concerned with complex coefficient filters (complex filters) have been proposed not only in the field of digital circuit but also in the field of the analog circuit. Their frequency characteristics are unsymmetrical with respect to the axis of D.C. It is well known that this property is important for the applications to the orthogonal communication system and so on.

There have been proposed many methods for designing the complex filter in the analog field. One is the frequency shifting (FS) [1], which is the simplest method. This method is carried out by shifting the poles and zeros of the real transfer function in the direction of the frequency axis on the s-plane. The others are the ELHT [2] and the ELLT [3]. These methods have the advantage that we can immediately design a complex BPF which satisfies the required specifications. All the above methods belong to the frequency transformation. The imaginary resistors arise after the above transformations.

In this paper, we discuss complex filters obtained by directly exchanging several reactance elements included in the real coefficient BPF (real BPF) for the imaginary resistors. The proposed method gives four varieties of complex filters. Two of them are the same as the complex filters obtained by using the conventional method. The stability of the proposed circuit is described. Finally, their frequency responses are shown.

2. Proposed Method

The imaginary resistor is a resistor whose element value is imaginary [1]. Its impedance $Z_{JR}(j\omega)$ is defined by

$$Z_{JR}(j\omega) = jR \tag{1}$$

where j is the imaginary unit and R is a real constant. Equation (1) shows that the imaginary resistor doesn't make power dissipation and that the imaginary resistor has the inductive property when $R > 0$ and the capacitive property when $R < 0$. Thus, the imaginary resistor can be regarded as a kind of reactance element.

In this paper, we discuss a complex filter obtained by exchanging several reactance elements included in the real filter for the imaginary resistor whose element value is given by the following formulae.

A) The case of exchanging an inductor L for the imaginary resistor jR_L

$$jR_L = j\omega_0 L \tag{2}$$

B) The case of exchanging a capacitor C for the imaginary resistor jR_C

$$\begin{aligned} jR_C &= 1/j\omega_0 C \\ &= -j/\omega_0 C \end{aligned} \tag{3}$$

where ω_0 is a real constant. Equations (2) and (3) predict that both of the real BPF and the complex BPF have the similar frequency responses near ω_0. In this paper, the complex filter obtained by applying the above replacement to the real BPF whose center frequency and bandwidth are given by ω_0 and B_W, respectively, is described.

Figure 1(a) shows the conventional LPF-BPF transformation. When we apply the above replacement to each one of 2 reactance elements included in the shunt arms and the series arms of the real BPF, we have four varieties of complex coefficient filters shown in Fig.1(b).

(1) Case 1 (FS)

The admittance $Y_{shunt}(j\omega)$ of the shunt arm of the real BPF in Fig.1(a) are given by

$$Y_{shunt}(j\omega) = \frac{j\omega C}{B_W} + \frac{\omega_0^2 C}{j\omega B_W} \tag{4}$$

The second term of the right side of Eq.(4) indicates the admittance of the inductor whose element value is $B_W/\omega_0^2 C$. Exchanging this inductor for the imaginary resistor by using Eq.(2) gives the imaginary resistor whose element value is $jB_W/\omega_0 C$. The resultant admittance $Y_{1shunt}(j\omega)$ leads to

$$Y_{1shunt}(j\omega) = \frac{j\omega C}{B_W} - j\frac{\omega_0 C}{B_W} \tag{5}$$

On the other hand, the impedance $Z_{series}(j\omega)$ of the series arm of the real BPF leads to

$$Z_{series}(j\omega) = \frac{j\omega L}{B_W} + \frac{\omega_0^2 L}{j\omega B_W} \tag{6}$$

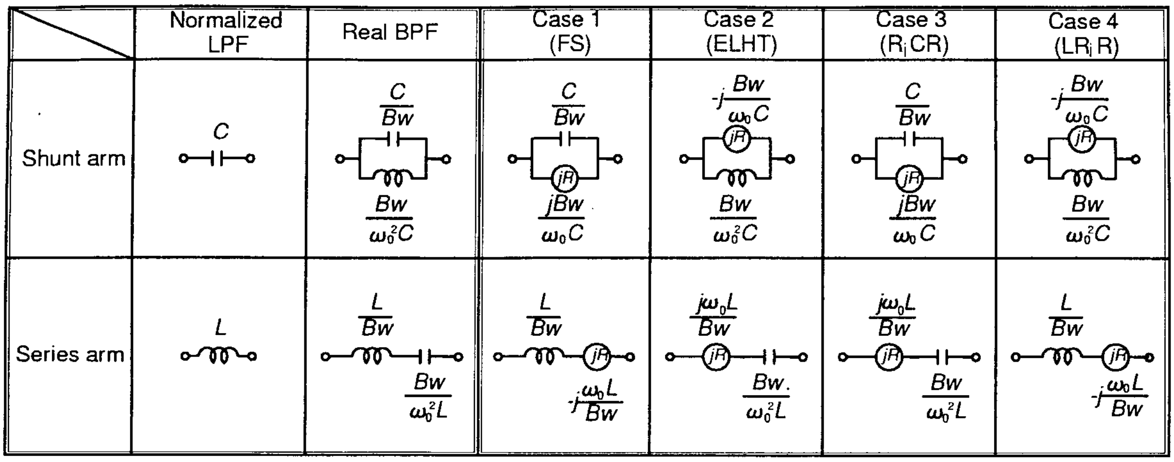

Figure 1: (a)Conventional LPF-BPF transformation, (b)Proposed transformations.

The second term of the right side of Eq.(6) indicates the impedance of the capacitor whose element value is $B_W/\omega_0^2 L$. Exchanging this capacitor for the imaginary resistor by using Eq.(3) gives the imaginary resistor whose element value is $-j\omega_0 L/B_W$. The resultant series impedance $Z_{1series}(j\omega)$ leads to

$$Z_{1series}(j\omega) = \frac{j\omega L}{B_W} - j\frac{\omega_0 L}{B_W} \tag{7}$$

Let us consider the transformation indicated by Eq.(5). In Fig.1(a), the admittance $Y_{LPF}(jx)$ of the shunt arm of the normalized real LPF leads to

$$Y_{LPF}(jx) = jxC \tag{8}$$

Setting $Y_{LPF}(jx) = Y_{1shunt}(j\omega)$ gives

$$x = \frac{1}{B_W}(\omega - \omega_0) \tag{9}$$

Similarly, when we consider the transformation indicated by Eq.(7), Equation (9) is obtained. Although the reactance elements are replaced with the imaginary resistor without taking the frequency transformation into account, both of the shunt and the series arms have the same frequency transformation. Figure 2 shows the frequency transformation indicated by Eq.(9). This figure shows that this transformation is equivalent to the conventional FS [1]. The resulting frequency response becomes a complex bandpass characteristics whose arithmetic center frequency and bandwidth are given by ω_0 and $2B_W$, respectively.

(2) Case 2 (ELHT)

The first term of the right side of Eq.(4) indicates the admittance of the capacitor whose element value is C/B_W. Exchanging this capacitor for the imaginary resistor by using Eq.(3) gives the imaginary resistor whose element value is $-jB_W/\omega_0 C$. The resultant admittance

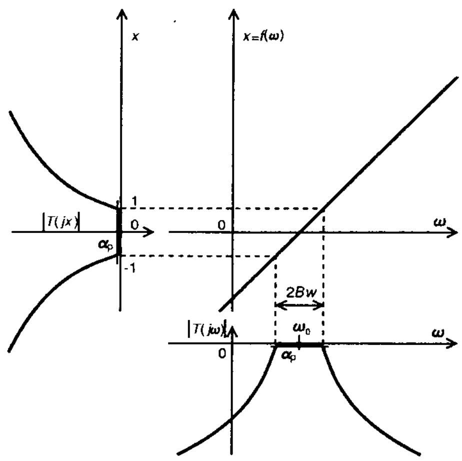

Figure 2: Frequency transformation in case 1.

$Y_{2shunt}(j\omega)$ leads to

$$Y_{2shunt}(j\omega) = \frac{j\omega_0 C}{B_W} + \frac{\omega_0^2 C}{j\omega B_W} \tag{10}$$

On the other hand, the first term of the right side of Eq.(6) indicates the impedance of the inductor whose element value is L/B_W. Exchanging this inductor for the imaginary resistor by using Eq.(2) gives the imaginary resistor whose element value is $j\omega_0 L/B_W$. The resultant shunt impedance $Z_{2series}(j\omega)$ leads to

$$Z_{2series}(j\omega) = \frac{j\omega_0 L}{B_W} + \frac{\omega_0^2 L}{j\omega B_W} \tag{11}$$

In the same fashion as the case 1, both of the shunt arm and the series arm come to have the same frequency transformation. The relationship between x and ω leads to

$$x = -\frac{\omega_0^2}{\omega B_W} + \frac{\omega_0}{B_W} \tag{12}$$

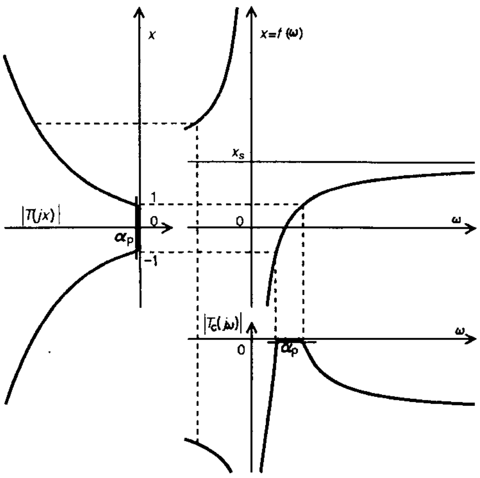

Figure 3: Frequency transformation in case 2.

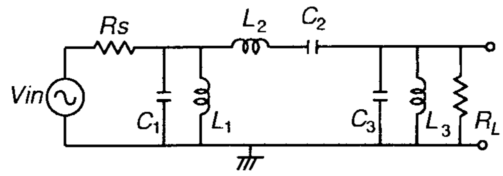

Figure 4: Real BPF.

Table 1: Element values.

Element	Value	Element	Value
R_S	1	R_L	1
C_1, C_3	1.0118	jR_{C1}, jR_{C3}	$-0.098834j$
L_1, L_3	0.0098834	jR_{L1}, jR_{L3}	$0.098834j$
C_2	0.020119	jR_{C_2}	$-4.9705j$
L_2	0.49705	jR_{L_2}	$4.9705j$

Figure 3 shows the frequency transformation indicated by Eq.(9). Now, the frequency transformation proposed in the ELHT[2] is given by

$$x = -\frac{\omega_C}{\omega - \omega_S} - x_S \tag{13}$$

Because substituting $\omega_S = 0$, $\omega_C = \omega_0^2/B_W$ and $x_S = -\omega_0/B_W$ into Eq.(13) gives Eq.(12), this case is regarded as the special case of the ELHT. The passband edges of the resultant filter are given by $\omega_0^2/(\omega_0 + B_W)$ and $\omega_0^2/(\omega_0 - B_W)$, respectively.

(3) Case 3 (RiCR configuration)

When the complex filter is synthesized by using the shunt arm given by Eq.(5) and the series arm given by Eq.(11), their frequency transformations differ from each other. Thus, its frequency response is unfortunately distorted. However, this filter is a very attractive configuration from the viewpoint of its active realization because it includes no inductors. In this paper, we call this filter an RiCR filter. The narrower the passband width B_W, the smaller the distortion of the transfer response is.

(4) Case 4 (LRiR configuration)

We synthesize the complex filter using the shunt arm given by Eq.(10) and the series arm given by Eq.(7). In the same fashion as the case 3, its frequency response is unfortunately distorted. Because this filter includes no capacitors, we call this filter an LRiR filter.

3. Stability

The imaginary resistor can be replaced with an ideal transformer [4]. By using this replacement, the 2-port complex filter is converted into the 4-port network without the imaginary resistors. The input and the output signals are decomposed into their real and imaginary components. Because both of the 4-port network and the 2-port network are equivalent to each other, we have the solution to the stability of the proposed circuits by examining the stability of the 4-port network. All the proposed complex filters consist of the imaginary resistors and the positive valued passive elements. The 4-port network converted by using the above method includes the positive valued passive elements and the ideal transformers only. This circuit is obviously stable. Thus, it is the indirect proof, but it is concluded that all the proposed circuits are stable.

4. Examples of Frequency Responses

In order to show the example of the frequency response of the proposed filters, we use the real BPF which satisfies the following specifications.

Third-order Chebyshev BPF	
Passband ripple	1dB
Geometric center frequency ω_0	10rad/s
Passband width B_W	2rad/s

Figure 4 shows a BPF which satisfies the above specifications. The proposed complex filters are shown in Fig.5(a)-(d), respectively. Table 1 shows their element values. Figure 6 shows their frequency responses. This figure shows that all the proposed filters have the bandpass characteristics. As predicted in Sect.2, the RiCR and the LRiR filters don't have the equiripple property. Figure 7 shows the frequency response of the RiCR and the LRiR filters whose passband width $B_W = 0.5$rad/s. From this figure, it is found that as the passband is narrower, the frequency response of the passband ripple has better equiripple property.

5. Conclusions

By directly exchanging the reactance elements included in the real BPF for the imaginary resistors, we obtained four varieties of tcomplex BPFs. Two of the proposed filters are the same as the filter designed by using the conventional FS and ELHT. The others become

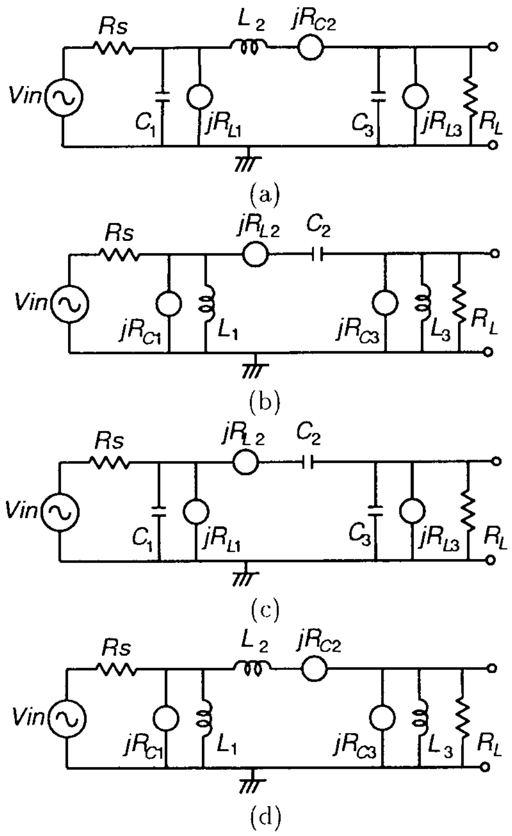

Figure 5: Proposed filter circuits, (a)Case 1, (b)Case 2, (c)Case 3, (d)Case 4.

attractive filters which include no inductors or no capacitors. The stability of the 2-port complex filter is described. Finally, the examples of the frequency responses of the proposed filters are shown.

Further investigation is required to establish the design method of the RiCR and LRiR filter which have the equiripple characteristics.

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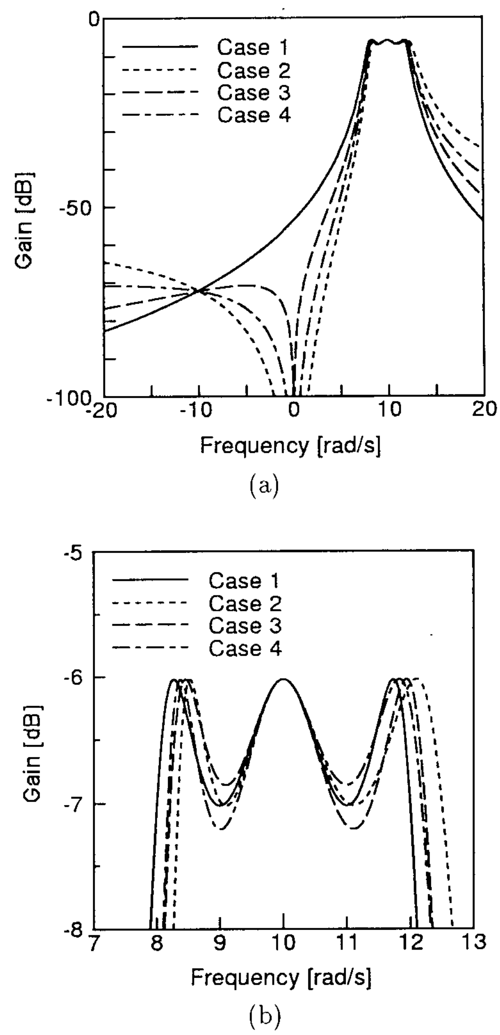

Figure 6: Frequency response (B_W=2rad/s), (a)Overall, (b)Enlarged scale figure near passband.

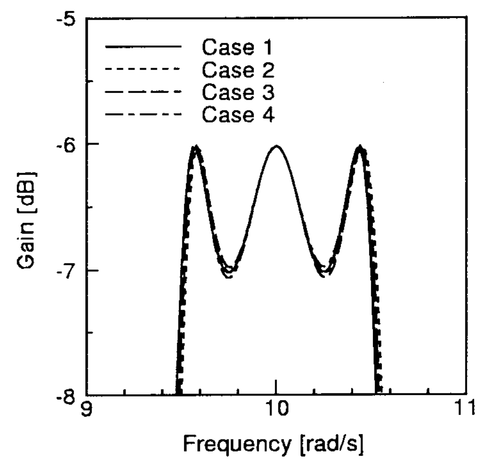

Figure 7: Enlarged scale figure near passband (B_W=0.5rad/s).