System Capacity Analysis with the Retransmission Limit on ARQ in a Voice/Data DS-CDMA System

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Abstract: In this paper, we investigate the effect of the retransmission limit both the system capacity and the average number of retransmissions in a voice/data DS-CDMA system. Basically, we consider the IS-95 type reverse link of the CDMA system, which supports two kinds of services: a general voice and a packetized data service. ARQ is used for the reliable data transmission. Convolutional code is used for FEC and CRC-CCITT code is used for the error detection in ARQ. The result shows that the number of concurrent data users decreases as we reduce the number of the retransmissions. However, at the same time, we can also reduce the average number of retransmissions. Concludingly, we can select the retransmission limit so as to reduce large amount of retransmissions with small sacrifice in the system capacity.

1.Introduction

In recent years, with the increasing market demands, multimedia communications in both wired and wireless networks have been extensively researched and developed [1]. In a wired communication network, optical fiber based ATM network is developed to serve multimedia information, such as speech, image and data with the high service quality, i.e., the PER (Packet Error Rate) is about $10^{-9} \sim 10^{-12}$ [2]. The demands for the multimedia communication are expected to increase in wireless networks as well.

In the wireless counterpart, third generation wireless system, named IMT (International Mobile Telecommunication)-2000, is expected to support a wide variety of multimedia information utilizing advanced multiple access techniques. One such technique that has recently attracted a great deal of attention is CDMA (Code Division Multiple Access). Since the CDMA technique has many advantages over FDMA (Frequency Division Multiple Access) and TDMA (Time Division Multiple Access), such that the sophisticated frequency planning and time assigning are not needed, it has become a standard of the second-generation digital wireless system, IS-95.

In a CDMA system, many users transmit data simultaneously using the same frequency band and the transmission power of each user produces interference to the other users. Therefore, CDMA system capacity is interference limited, whereas the capacity of FDMA and TDMA system is primarily bandwidth limited. So, in the CDMA system, admission of a new user in the network results in more interference and, consequently, an increase in bit error rate. The CDMA system service quality is dependent not only on the interference caused by other users in the system but also on the wireless channel, which has inferior transmission qualities compared with the wired transmission medium. Hence, for the reliable delivery of various information in wireless networks, adequate error control schemes have to be used according to the information characteristics.

Various error control schemes have been developed to maintain the reliable communication. These include the FEC (Forward Error Control) scheme and ARQ (Automatic Repeat reQuest) scheme and the hybrid scheme. Depending upon the error and delay characteristics of the information, multimedia services can be classified into many categories. In this paper, we consider two service groups. One is the conventional voice service and the other is the packetized data service. It is well known fact that the data services allow the use of more versatile error control mechanism compared with voice because voice is relatively more sensitive to the time delay than data, whereas the data service needs very high transmission qualities.

In order to support the multimedia service and to increase the capacity of the multimedia system, the analysis on the relations between the service parameters and the system capacity has been researched in the DS-CDMA system where the fixed rate voice calls and the variable rate data calls are considered [3]. The effect of adopting various convolutional codes in a voice/data DS-CDMA system was analyzed according to the constraint length of the convolutional codes. Also, in order to increase the capacity region of a voice/data DS-CDMA system, we have examined the influence of adopting an ARQ protocol, where various Hamming codes are used for the error detection in ARQ [4].

In this paper, we analyze the effect of retransmission limits onto the performance of ARQ in terms of the capacity region and the average number of retransmissions. We have restricted our focus on CRC-CCITT for ARQ and 1/3 rate convolutional code for FEC. We have selected CRC-CCITT code for the error detection in ARQ owing to its superior performance on the system capacity [5].

The paper is organized as follows: In Section 2, we show the capacity limit equation to determine the capacity of a voice/data DS-CDMA system. In Section 3, the service quality improvement using ARQ is described. Section 4 is devoted to analyze the effect of retransmission limits. Section 5 presents the system model and analysis results. Finally, Section 6 contains the concluding remarks.

2.Capacity Bound of a Voice/Data DS-CDMA System

We consider a typical multimedia DS-CDMA system that supports one group of voice service and one group of data service. In the reverse link, the received signal-to-interference plus noise power ratio (*SINR*) of each voice and data user can be represented as [6]

$$SINR_k = \frac{S_k}{I_{home} + I_{other} + \eta_0} \tag{1}$$

where S_k is the received signal power of the desired k-th user, I_{home} is the interference caused in the home cell, I_{other} is the interference caused in the other cells and η_0 is the background noise.

The base station is assumed to receive equal signal power levels from each mobile and those are constant during the call access time. We further model that other cell interference level is constant and that is a scaled version of the home cell interference. If the background noise is negligible, the received signal power of voice and data users has the following relation [7].

$$\begin{aligned} S_v &= \Lambda \cdot S_d \\ &= \left(G_d (E_b / N_0)_d^{-1} + (1+f)\right) \\ &\quad \cdot \left(G_v (E_b / N_0)_v^{-1} + \alpha(1+f)\right)^{-1} \cdot S_d \end{aligned} \tag{2}$$

And the system capacity is bounded by

$$\alpha N_v + \Lambda^{-1} N_d \le \left(\frac{G_d (E_b / N_0)_{d,req}^{-1}}{1+f} + 1 \right) \cdot \Lambda^{-1} \tag{3}$$

where N_v and N_d are the number of concurrent voice and data users, respectively. In Table 1, all parameters in Eq.(2) and Eq.(3) are explained.

In Eq.(3), we can find some parameters that play an important role to determine the system capacity region, such as, voice activity factor, other cell interference factor and the required E_b/N_0 which satisfies the service quality. As an example, 1/2 rate and 1/3 rate convolutional codes with constraint length 9 are used to reduce the required Eb/No in the forward link and reverse link, respectively, in the present IS-95 based CDMA system.

In Section 3, we discuss the effect of adopting additional error control mechanism to the data service. Generally, data transmissions are delay-insensitive and error-sensitive. From this characteristic, we adopt an ARQ protocol for the data users to reduce the required signal power while BER (Bit Error Rate) is maintained above the required service level. Here, reducing the required signal power can be directly translated into the suppression of the interference level, and finally the overall system capacity is increased.

3.Undetected Error Rate in ARQ

Table 1. Definition of Parameters

Symbol	Description
S_v	Received Power of a Voice User
S_d	Received Power of a Data User
G_v	Processing Gain of a Voice User
G_d	Processing Gain of a Data User
$(E_b/N_0)_v$	Received (E_b/N_0) of a Voice User
$(E_b/N_0)_d$	Received (E_b/N_0) of a Data User
α	Voice Activity Factor
F	Other-Cell Interference Ratio

When a coded packet is received, the receiver tests the received packet to detect errors. For each received packet, the result of error testing is divided into three cases as in Eq.(4).

$$P_c + P_e + P_d = 1 \tag{4}$$

where P_c is the probability that the received packet has no error, P_e is the probability that the received packet has errors that are undetectable and P_d is the probability that the received packet has errors that are detectable. When the receiver detects errors, the retransmission request is automatically sent to the transmitter, and this procedure is repeated until either the packet is correctly received or the number of retransmissions reaches the specified limit.

Generally, for any (n,k) linear block code, when k is smaller than 25, we can get the weight distribution of the code directly from the computer generation of all codewords. If we know the weight distribution of a code, we can easily find out the undetected WER (Word Error Rate), $P_e(E)$, of the codes as follows [8]:

$$P_e = \sum_{i=1}^{n} A_i \left(\frac{p}{q-1} \right)^i (1-p)^{n-i} \tag{5}$$

where A_i is the weight distribution of the code, q is equal to 2 if binary code is used, and p is the symbol error rate of the received codeword.

When k is bigger than 25, it is almost impossible to find the weight distribution of the code by the computer simulation. In this case, we use the MacWilliams identity to find out the weight distribution of its dual code, instead of finding out that of the original code [9]. If we get the weight distribution of the dual code, we can also determine the undetected WER of the original code. In the case of the shortened binary cyclic redundancy code, the weight distribution of its dual code can be calculated by [10]. If we know the weight distribution of the dual code of any binary CRC code, the undetected WER of the received signal is determined as follows [8]:

$$P_e = q^{-r} \sum_{i=0}^{n} B_i \left(1 - \frac{qp}{q-1} \right)^i - (1-p)^n \tag{6}$$

where B_i is the weight distribution of the dual code and r is the number of parity bits, which is equal to n-k.

4.Effect of Retransmission Limits

Large number of retransmissions causes longer delay for the successful transmission. In a practical system, which provides data services with ARQ, the number of retransmissions is generally limited to reduce the long service delay. When the retransmission is limited into a certain number, while the required service quality is maintained, we should allocate more power for the data transmission, so that we can finish the transmission successfully before the pre- specified retransmission limit.

For M retransmissions, Eq.(4) becomes

$$P_c \sum_{i=0}^{M} P_d^i + P_e \sum_{i=0}^{M} P_d^i + P_d^{M+1} = 1 \quad (7)$$

where the first term represents the probability that a packet is correctly received after M retransmissions, the second term represents the probability that a packet has undetectable errors after M retransmissions and the last term represents the probability that a packet has detectable errors after M retransmissions. If M is limited into a certain number, the WER, $P_w(E)$, consists of the second term and last term.

If we know the WER of a code, the BER is given by its upper bound and lower bound as follows [11]:

$$\frac{1}{k} P_w(E) \le P_b \le P_w(E) \quad (8)$$

where $P_b(E)$ is the BER of the information signal after finishing the retransmission. When the word error is translated into only one information bit error, that is the lower bound of BER. When the word error is translated into all information bit errors, this is the upper bound of BER. In this paper, we make use of the worst case, that means we use the upper bound for the BER of the received signal.

When the number of retransmissions is limited to M, the average number of retransmissions can be calculated as follows:

$$N_{ave} = \sum_{i=0}^{M} i P_d^i \quad (9)$$

where N_{ave} is the average number of retransmissions.

5.Analysis Model and Results

In order to analyze the effect of retransmission limits in a voice/data DS-CDMA system, we use the system parameters in Table 2, which are similar to the typical 2nd generation cellular mobile system, what is called, the IS-95 DS-CDMA system. We use 1/3 rate convolutional code with constraint length 9 for FEC. It is assumed that the return channel for the retransmission request is error free.

Fig. 1 and Fig. 2 show the system capacity region and maximum data user capacity, respectively, when the various lengths of CRC-CCITT are used for the error detection in ARQ and the retransmission is infinitely allowed. In Fig.1, the system capacity region increases when we use ARQ for data services. However, the rate of increase in the capacity region becomes smaller as we increase the code length. In addition, the system capacity is bounded by a linear line, which means that when the data user is maximized the total system capacity is maximized. From Fig.2, we can see that the system capacity is maximized when m is equal to 5, this means the (256,240) CRC-CCITT code is used for the error detection in ARQ. If m is greater than 8, the system capacity is rather decreased because of the poor error detection capability in spite of the processing reduction.

Fig. 3 and Fig. 4 present the effect of retransmission limits in terms of the maximum concurrent data users and the average number of retransmissions, respectively. In

Table 2. Analysis Model

Description	Value
Required BER of Voice Users	10^{-3}
Required BER of Data Users	10^{-9}
Transmission Rate of Voice Users	9600bps
Transmission Rate of Data Users	9600bps
Spreading Bandwidth	1.2288MHz
Voice Activity Factor	3/8
Other-Cell Interference Factor	0.55

Fig.3, the system capacity is reduced when we limit the number of retransmissions because each user uses more power to transmit the information successfully within the limit. We can see that the number of maximum data users is exponentially increased as the retransmission limits becomes larger. In this figure, the horizontal line represents the number of maximum data users of the infinite retransmission system. The system capacity is almost equal to that of the infinite retransmission system when the retransmission is limited to 10. In Fig.4, it is shown that the average number of retransmissions is non-linearly increased. As the retransmission limits becomes larger, the average number of retransmissions converges to that of the infinite retransmission system. However, the system capacity converges faster than the average retransmission. This means that we can reduce the larger amount of average retransmission while the system capacity is minimally sacrificed.

6.Concluding Remarks

In this paper, we present an analytical approach to the effect of retransmission limits on the system capacity and the average number of retransmissions in a voice/data DS-CDMA system which adopts an ARQ protocol. Using the ARQ protocol, we can reduce the required Eb/No value of data users so that the overall interference level decreases. Considering the retransmission limits, the system capacity decreases while the average number of retransmissions is also reduced. The system capacity converges faster than the average retransmission to that of the infinite retransmission system. Therefore, we can find the optimum retransmission limit where the average retransmission is greatly reduced while the system capacity is minimally reduced. There results can also be used for determining the system capacity and average retransmission under the specific limits on the retransmission.

As a further consideration, we may think more realistic system model. Imperfect power control in CDMA system can be considered with the practical channel modeling. In addition, we may consider the practical limitation of the ARQ protocol such that the return channel is not error free. Also, using the results about the average number of retransmission, we can analyze the system throughput and delay performance.

Reference

[1] R. Schneiderman, *Wireless Personal Communications*, IEEE press, 1994.

[2] H. Armbruster, "The Flexibility of ATM: Supporting Future Multimedia and Mobile Communications," *IEEE*

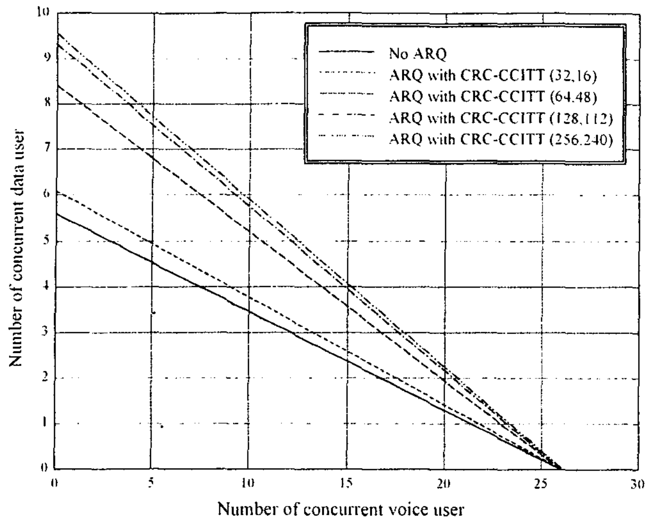

Figure 1. Capacity Region for Various Lengths of CRC-CCITT Codes in ARQ

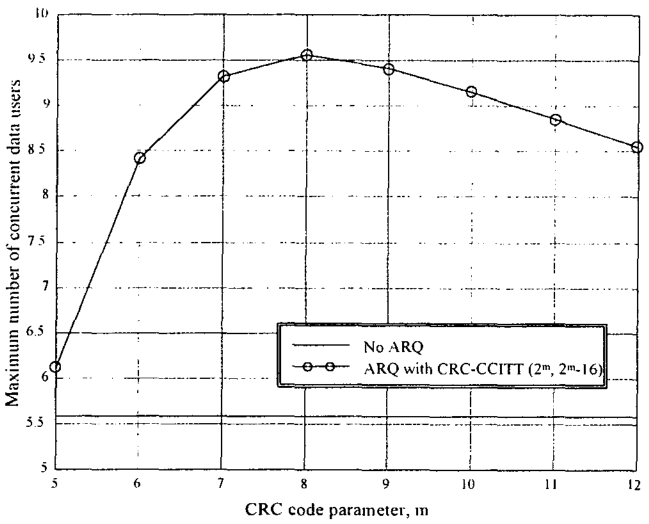

Figure 2. Maximum Number of Concurrent Data Users for Various Lengths of CRC-CCITT Codes in ARQ where $n=2^m$ and $k=2^m-1$ with $r=16$.

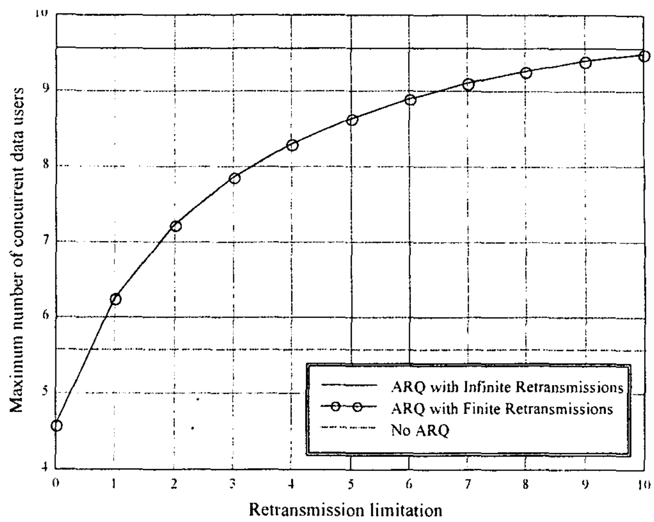

Figure 3. Maximum Number of Concurrent Data Users for the Retransmission Limit

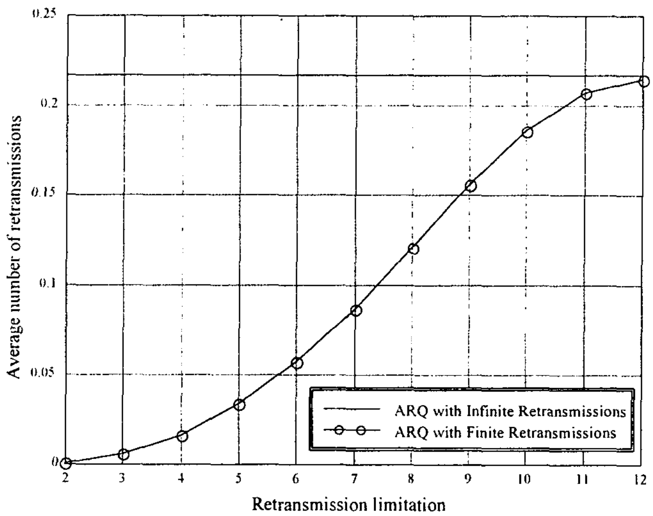

Figure 4. Average Number of Retransmission for the Retransmission Limit

Commun. Mag., vol.33, no.2, pp.76-84, Feb. 1995.

[3] K. Kim, J. An, Y. Choi, G. Ko and J. Yang, "Analysis on the Capacity Plane and RRAC Algorithm of Multimedia DS-CDMA System," *Proc. of 3rd Mobile Communication Workshop*, pp.8-13, 1997.

[4] C. Lee and K. Kim, "Capacity Enhancement Using an ARQ Scheme in a Voice/Data DS-CDMA System," *IEE Electronics Letters*, Vol.34, No.4, pp.327-329, Feb. 1998.

[5] C. Lee, H. Lee and K. Kim, "System Performance Analysis with Error-Controlled Data in a Voice/Data DS-CDMA System," *Proc. of VTC'99-Fall*, Vol.1, pp.343-347, 1999.

[6] A. Viterbi, *CDMA : Principles of Spread Spectrum Communication*, Addison-Wesley, 1995.

[7] C. Lee, G. Go, J. Yang, A. Ahmad and K. Kim, "Capacity Enhancement Using FEC and ARQ Schemes in a Multimedia DS-CDMA System," *Proc. of ITC-CSCC'97, pp.849-852*, Jul. 1997.

[8] J. Wolf and A. Michelson, "On the Probability of Undetected Error for Linear Block Codes," *Trans. on Commun.*, Vol. Com-30, No.2, pp.317-324, Feb 1982.

[9] W. Peterson and E. Weldon, *Error-Correcting Codes*, MIT Press, 1972.

[10] J. Wolf and R. Blakeney, "An Exact Evaluation of the Probability of Undetected Error for Certain Shortened Binary CRC Codes," *Proc. of MILCOM'88*, pp.287-292, 1988.

[11] S. Wicker, *Error Control Systems for Digital Communications and Storage*, Prentice-Hall, 1995.