# Multi-Resolution Modeling Technique Using Mesh Segmentation

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Abstract: This paper presents an algorithm for simplification of 3D triangular mesh data, based on mesh segmentation. The proposed algorithm is first attempt to segment the entire mesh into several parts using the orientation of triangles. Then simplification algorithm is applied to each segment that has similar geometric property. The proposed two step multi-resolution modeling scheme would yield better performance then conventional algorithms like edge collapse technique, since the segmentation step can give global information on the input shape. The experimental results show that the proposed algorithm is performed efficiently.

#### 1. Introduction

In Computer Vision and Computer Graphics, polygonal models are often used for efficient representation of the individual objects that are acquired by laser scanners and so on. Simplex polygons, i.e., triangles, are used primarily because they are easy and efficient to render. Although throughput of graphics systems has increased considerably over the years, the size and complexity of such mesh models have been growing even faster. Thus some problems still remain in storing, transmission and real-time rendering. To solve the problem that is brought by rendering all triangles, the technique that changes the resolution of the object according to importance in the image is widely used. This technique is called multiresolution modeling technique, which is also known as LOD (Levels-Of-Detail) Problem. In LOD representation for polygonal models, if viewpoint is far, low-resolution object is rendered, or else high-resolution object is rendered. To represent 3D object efficiently.

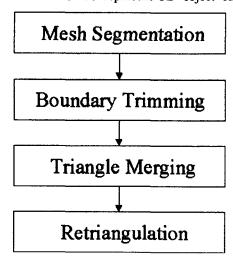


Fig. 1. Block diagram

applying of LOD technique is very important and the research for this is also needed.

This paper proposes a new technique that decreases complexity of the mesh model. First, the entire mesh model is segmented into several parts. Then, in each segment, simplification is performed respectively. Each segment consists of the linked faces that have similar orientation on the object surface. Thus it can be fitted to planar patch. The vertices to be removed are selected by the distance between this plane and each vertex. The block diagram of the proposed algorithm is shown in Fig. 1.

The remainder of the paper proceeds as follows: The next section reviews some aspects of the literature that are related to the proposed algorithm. Section 3 explains how mesh model is segmented and simplified. Experimental results are shown in Section 4. Final concluding remarks are given in Section 5.

## 2. Related Work

Multi-resolution modeling technique of 3D triangular mesh data is the simplification technique that decreases the data from the highest resolution to the desired resolution. Simplification is often achieved by performing a series of local operations. Such operation tries to simplify the polygonal model to minimize an energy function. Note that the type of such operation is selected based on the primitive, i.e., vertex, edge or face. Simplification algorithm generally chooses one of these operation types and applies it repeatedly to its input surface, until desirable complexity is achieved.

Turk [1] and Schroeder et al. [2] apply the vertex remove approach as part of their simplification algorithms. In each stage, the vertex that minimizes the error metric is chosen and removed. Holes caused by vertex removal, are retriangulated using a planar projection approach. This process is shown in Fig. 2. Soucy and Laurendeau [3] also apply the approach based on the vertex remove operation. They fill the hole by using an unconstrained Delaunay triangulation algorithm. The main disadvantage of this algorithm is that the order of computation is very large. Since they evaluate all the errors from removed vertices, the evaluation will take

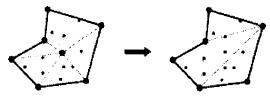


Fig. 2. Vertex remove operation

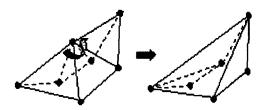


Fig. 3. Choo's algorithm

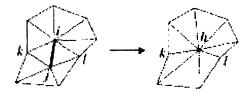


Fig. 4. Edge collapse operation

longer as the process goes on. Choo et al. [4] proposed the algorithm using an edge criterion for removing a vertex. In this procedure, when the interior angle between the edges connected to a vertex is large enough to be approximated as a straight line, the vertex is removed. This process is shown in Fig. 3.

Hoppe [5], [6] proposed an edge collapse technique. In this process, the two vertices of an edge are merged into a single vertex. The triangles that contain both of the vertices degenerated into 1-dimensional edges and are removed from the mesh. This typically reduces the mesh complexity by 2 triangles. In order to choose the edge collapsed in that stage, Hoppe uses energy function consists of the number of vertices, volumetric error, and the length of the edge. Fig. 4 shows this procedure.

Hamann [7], Gieng et al. [8] use the face collapse operation in their simplification systems. In this operation, all three vertices of a triangular face are merged into a single vertex as in Fig. 5. This operation is similar to the edge collapse operation, except that it is more coarse-grained. The coarser granularity of this operation may allow the simplification process to proceed more quickly, at the expense of the fine-grained local control of the edge collapse operation.

These simplification algorithms applying local operations repeatedly, are processed in the direction of local minimum. Thus, the solution is also local optimum and these approaches cannot guarantee that the solution is the global optimum. This limitation is caused by the local operation itself.

# 3. Multi-Resolution Modeling

In order to increase the convergence to global optimum, in our approach we employ the mesh segmentation

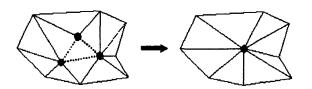


Fig. 5. Face collapse operation

technique. The mesh segmentation means partitioning a 3D surface mesh into meaningful, connected patches. In mesh segmentation, the edges that contain important topological information become the boundary of each segment. Thus, keeping this boundary from being corrupted, leads to global optimum more.

## 3.1 Mesh Segmentation

In this paper, the mesh segmentation technique is used as a kind of preprocessing of the multi-resolution modeling technique. The proposed mesh segmentation technique uses the normal vector of the surface. The basic idea of the mesh segmentation is to merge the linked faces that have similar normal direction into the same segment. This process uses the angle between the normal vector of each face. When the angle is less than the given threshold, the faces are merged. Note that mesh segmentation can be classified into two categories:

- Min-# problem : For given angle threshold  $\delta$ , minimize the number of segmented patches.
- Min-δ problem: For given the number of segmented patches, to minimize the angle threshold

Let T and S denote a triangular face and the segment, respectively. In order to determine whether the triangle  $T_i$  that is adjacent to the k-th segment  $S_k = \{T_0, T_1, \dots, T_n\}$  but not contained yet, would be merged into  $S_k$ , the angle between the triangle  $T_i \notin S_k$  and  $T_i \in S_k$  is tested.  $T_i$  is merged into  $S_k$  when the following decision rule is satisfied.

$$\max_{i} \left\{ ang(T_{j}, T_{i}) \right\} \leq \delta_{Seg} \tag{1}$$

where  $\delta_{Seg}$  is the angle threshold and the function  $ang(T_j, T_i)$  means the angle between the triangle  $T_j$  and  $T_i$ .

After the segmentation, the entire mesh data is segmented into patches that have similar geometric property, and the result is used to the proposed multi-resolution modeling technique.

## 3.2 Boundary Trimming

Since the faces that have similar normal direction are merged into the same segment during mesh segmentation the boundary of each segment contains the perceptually important geometric information of the entire mesh model. Although this boundary is important to keep the topology of the entire mesh model, every vertex composed of the boundary does not affect the topology. Since some of the boundary vertices may not affect the topology, the boundary trimming is performed to detect and remove them.

The boundary vertices that less corrupt the topology can be detected by investigating the angle at the boundary vertex. The term, the angle of the boundary vertex, means the angle between the adjacent boundary edges connected to the boundary vertex. When the following criterion is satisfied, the vertex is contained to candidate set.

$$|180^{\circ} - the \text{ angle of the boundary vertex}| \le \delta_B$$
 (2)

where  $\delta_B$  is the angle threshold.

Although this procedure is similar to the Choo's

algorithm [4], in our approach all vertices contained to candidate set are not removed. If the vertex is common to three or more segments, although the angle of the boundary vertex in one segment is close to 180 degrees, this may not in other segment. Thus, when the boundary vertex is common to only two segments and the angle of the boundary vertex is close to 180 degrees, the boundary vertex can be removed.

# 3.3 Triangle Merging

In mesh segmentation, the orientation on the mesh surface is used to segment the mesh model. Thus, each segment consists of the faces that have similar normal property, and can be fitted to planar patch. In our approach, the best fitting plane to each segment is calculated by using Least-Square method. Let the best fitting plane to k-th segment be  $z = a_k x + b_k y + d_k$ . Then, the square error distance e is given by

$$e_k = \sum_i \frac{\left(a_k x_i + b_k y_i - z_i + d_k\right)^2}{a_k^2 + b_k^2 + 1}$$
(3)

where  $(x_i, y_i, z_i)$  is the vertex in the same segment. When the partial derivatives on  $a_k$   $b_k$   $d_k$  equals zero, respectively, the error  $e_k$  is minimized, yielding following equations

$$a_{k} \sum_{i} x_{i}^{2} +b_{k} \sum_{i} x_{i} y_{i} +d_{k} \sum_{i} x_{i} = \sum_{i} x_{i} z_{i}$$
(4)
$$a_{k} \sum_{i} x_{i} y_{i} +b_{k} \sum_{i} y_{i}^{2} +d_{k} \sum_{i} y_{i} = \sum_{i} y_{i} z_{i}$$
(5)
$$a_{k} \sum_{i} x_{i} +b_{k} \sum_{i} y_{i} +d_{k} n_{k} = \sum_{i} z_{i}$$
(6)

where  $n_k$  is the number of the vertices that contained in the k-th segment.

After the best fitting plane is found, simple statistical processing is required to select the vertex to be removed. The mean m and standard deviation  $\sigma$  of square error distance e are obtained by

$$m_{k} = \frac{\sum_{i} q_{i}}{n_{k}}, \quad \sigma_{k} = \frac{\sum_{i} q_{i}^{2}}{n} - m_{k}^{2}$$
where,  $q_{i} = \sum_{i} \frac{(ax_{i} + by_{i} - z_{i} + d)^{2}}{a^{2} + b^{2} + 1}$  (7)

The vertices far from the plane can be removed if they are not boundary vertex, because they less affect the topology of the entire mesh model. The vertices are removed when following restriction is satisfied.

$$q_i \ge m_k + \alpha \sigma_k \tag{8}$$

where  $\alpha$  is the control parameter.

# 3.4 Retriangulation

Through the boundary trimming and triangle merging, the vertices that satisfy equation (8) are removed. The holes caused by removing of the vertices are filled by using the Delaunay triangulation [9].

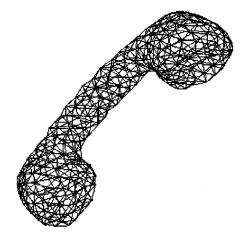


Fig. 6. Original phone model

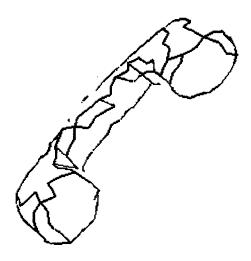


Fig. 7. Segmented phone model

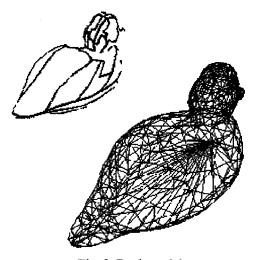


Fig. 8. Duck model

## 4. Experimental Results

To evaluate the performance of the proposed algorithm experiments are carried out on the phone model shown in Fig. 6. It consists of 363 vertices and 722 triangles. Fig. 7 shows the result of the mesh segmentation when  $\delta_{Seg}$  is set to 80 degrees. In Fig. 7, only boundary of the segment

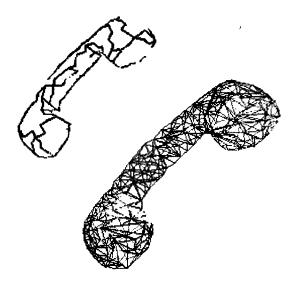


Fig. 9. Phone model ( $\delta_B = 20$ )

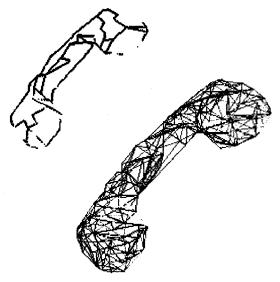


Fig. 10. Phone model ( $\delta_B = 60$ )

is rendered. In this case, phone model segmented into 34 patches. Since the original phone model has rough surface, the angle threshold is large comparatively. Fig. 8 shows the duck model that consists of 427 vertices and 850 triangles. It is partitioned to 54 segments when  $\delta_{Seg}$  is 50 degrees. Note that  $\delta_{Seg}$  is chosen experimentally.

Fig. 9 and Fig. 10 are the phone model after the vertices are removed when  $\delta_B$  is 20 degrees and 60 degrees respectively. The number of the vertices that removed is 190 and 270 respectively. According as the angle threshold is increased, vertices are efficiently removed. In both cases,  $\delta_{Seg}$  is set to 80 degrees and  $\alpha$  is set to -1.5 respectively. Especially in case that  $\delta_B$  is 60 degrees, about 75% of the data is removed. However Fig. 10 shows that the topology of the phone model is still maintained well.

## 5. Conclusion and Future Work

We have described the multi-resolution modeling technique using mesh segmentation. The results show that the topology of the LOD polygon models is keeping well and the proposed algorithm is performed efficiently. Further research should be done on the mesh segmentation, because the performance of the multi-resolution modeling technique using mesh segmentation depends on the mesh segmentation result.

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