Proceedings of ITC-CSCC 2000, Pusan, Korea

RLS Adaptive IIR Filters Based on Equation Error Methods Considering Additive Noises

Mitsuji Muneyasu, Hidefumi Kamikawa and Takao Hinamoto

Faculty of Engineering, Hiroshima University
1-4-1 Kagamiyama, Higashi-Hiroshima 739-8527 Japan
E-mail: muneyasu@ecl.sys.hiroshima-u.ac.jp

Abstract

In this paper, a new algorithm for adaptive IIR filters based on equation error methods using the RLS algorithm is proposed. In the proposed algorithm, the concept of feedback of the scaled output error proposed by Lin and Unbehauen is employed and the forgetting factor is varied in adaptation process for avoiding the accumulation of the estimation error for additive noise. The proposed algorithm has the good convergence property without the parameter estimation error under the existence of measurement noise.

1 Introduction

Adaptive filters have played a key part in various fields [1], [2]. They are classified into finite impulse response (FIR) and infinite impulse response (IIR) filters from the point of view of the filter model. The adaptive FIR filter has been used widely, because of the feature like stability. However the adaptive IIR filter can achieve a lower order than that of the FIR filter having the same performance.

From the treatment of an error signal, algorithms of the adaptive IIR filters are classified into an output error algorithm and an equation error algorithm [3], [4]. The output error algorithm directly uses the output signal of the filter in the update formula of coefficients. This algorithm should select a small step-size parameter and the associated performance surface may have many local minima. The equation error algorithm uses the desired output instead of the filter output. Its associated performance surface is unimodal and has a good convergence property. However, if a reference signal is corrupted by an additive noise (a measurement noise), the estimation of coefficients by the equation error method has some bias. For the problem like system identification, it becomes a serious defect. To solve this difficulty, several techniques have been proposed [5], [6]. The bias-remedy least mean square error (BRLE) algorithm proposed by Lin and Unbehauen is one of such bias removal algorithms [7]. This algorithm is based on canceling the additive noise in the reference signal by the scaled output error as the estimation of the additive noise.

In Ref.[7], the LMS algorithm is employed as the adaptive algorithm. However the main concept of the BRLE algorithm can be applied to the RLS algorithm. By using the RLS algorithm, we may expect a fast convergence property and a robustness for non-stationary inputs [1]. This paper proposes an algorithm of the RLS adaptive IIR filters based on the BRLE algorithm. In this algorithm, the constant forgetting factor has some problem and its modification is also proposed. By the proposed algorithm, no bias and fast convergence properties are obtained irrespective of the white or colored inputs.

2 Adaptive IIR Filter

An adaptive IIR filter can be expressed by

$$y(n) = \sum_{j=1}^{M-1} a_j(n)y(n-j) + \sum_{i=0}^{N-1} b_i(n)u(n-i) \quad (1)$$

where $u(n)$ is the input signal and $y(n)$ the output signal.

The update algorithm of filter coefficients for adaptive IIR filters is mainly classified into the output error algorithm and the equation error algorithm. Especially, the equation error algorithm has several merits, that is, the associated performance surface is unimodal and an appropriate magnitude of the step-size parameter can be specified. The block diagram of the equation error method is shown in Fig. 1.

The transfer function of the block $A(z)$ which is the inverse filer of the IIR part $1/A(z)$, is given as

$$A(z) = 1 - \sum_{j=1}^{M-1} a_j(n)z^{-j} \quad (2)$$

and the transfer function $B(z)$ is also defined by

$$B(z) = \sum_{i=0}^{N-1} b_i(n)z^{-i} \ . \quad (3)$$

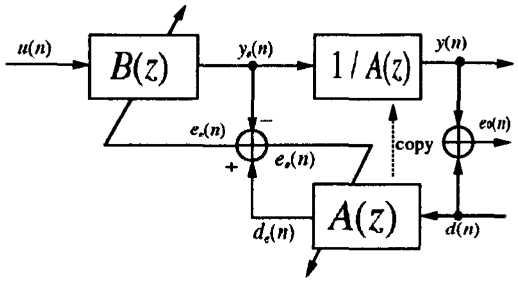

Figure 1: The block diagram of the equation error method.

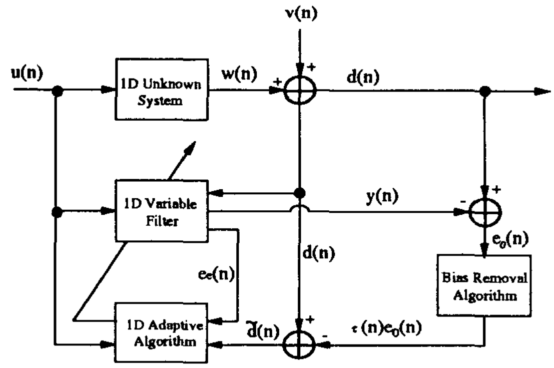

Figure 2: The block diagram of the bias removal algorithm.

The reference signal $d(n)$ is used for the input of the block $A(z)$. Then, the coefficients of the blocks $B(z)$ and $A(z)$ are updated to minimize the error $e_e(n)$ between $d_e(n)$ and $y_e(n)$. The updated coefficients of $A(z)$ is copied to the block $1/A(z)$ for updating the IIR block [1].

The error $e_e(n)$ (called as "equation error") is obtained as

$$\begin{aligned} e_e(n) &= A(z)d(n) - B(z)u(n) \\ &= d(n) - \sum_{j=1}^{M-1} a_j(n)d(n-j) \\ &\quad - \sum_{i=0}^{N-1} b_i(n)u(n-i) \\ &= d_e(n) - y_e(n) \ . \end{aligned} \quad (4)$$

For example, the update formulas by the LMS algorithm are as follows:

$$a_j(n+1) = a_j(n) + 2\mu_a e_e(n)d(n-j) \quad (5)$$

$$b_i(n+1) = b_i(n) + 2\mu_b e_e(n)u(n-i) \quad (6)$$

The equation error method has several merits described in the above. However, the biased estimation of the coefficients of the block $A(z)$ is occurred, if the reference signal is corrupted by the additive noise.

3 Bias Removal Algorithm

To solve the bias problem of the equation error method, one of the bias removal algorithm is proposed by Lin and Unbehauen [7]. This algorithm uses the following scaled output error vector as an estimation of the additive noise:

$$\epsilon(n) = [e_0(n-1), e_0(n-2), \cdots, e_0(n-M+1)]^T \quad (7)$$

$$e_0(n) = d(n) - y(n) \quad (8)$$

This estimation is used for the cancellation of the additive noise.

From this concept, Eq. (5) can be rewritten as

$$\boldsymbol{a}(n+1) = \boldsymbol{a}(n) + 2\mu_a e_e(n)[\boldsymbol{u}_a(n) - \tau(n)\epsilon(n)] \quad (9)$$

where

$$\boldsymbol{u}_a(n) = [d(n-1), d(n-2), \cdots, d(n-M+1)]^T \quad (10)$$

$$\boldsymbol{a}(n) = [a_1(n), a_2(n), \cdots, a_{M-1}(n)]^T \quad (11)$$

and $\tau(n)$ is the following scaling factor:

$$\tau(n) = min\left(1 \ , \ \alpha\frac{||\boldsymbol{u}_a(n)||}{||\epsilon(n)||}\right) \quad \alpha \geq 0 \quad (12)$$

where || || means Euclidean norm.

The value of the scaling factor $\tau(n)$ is inversely proportional to the variance of the error $e_o(n)$. Therefore, in the early stage, the additive noise cannot be reduced because of the large variance of $e_0(n)$. As the variance of $e_0(n)$ decreases, the estimation of the additive noise in $d(n)$ becomes accurate and the value of $\tau(n)$ increases until 1.

The block diagram of this bias removal algorithm is shown in Fig. 2.

4 Equation Error Adaptive IIR RLS Filter Considering Additive Noise

4.1 RLS algorithm considering bias removal

The RLS algorithm considering the bias removal algorithm described in the previous section is summarized as follows. The RLS algorithm for $B(z)$ is omitted, because it is same to the ordinary one.

[RLS algorithm for block $A(z)$ considering bias removal]

Initial conditions $\boldsymbol{P}_a(0) = c_a^{-1}\boldsymbol{I}$, $\hat{a}(0) = 0$

Step.1 Let n be 1.

Step.2 Calculation of $\tau(n)$:

$$\tau(n) = min\left(1, \alpha\frac{||\boldsymbol{u}_a(n)||}{||\epsilon(n)||}\right)$$

Step.3 Removal of additive noises:

$$\tilde{\boldsymbol{u}}_a(n) = \boldsymbol{u}_a(n) - \tau(n)\epsilon(n)$$

Step.4 Calculation of gain vectors:

$$\boldsymbol{k}_a(n) = \frac{\boldsymbol{P}_a(n-1)\tilde{\boldsymbol{u}}_a(n)}{\lambda_a + \tilde{\boldsymbol{u}}_a^T(n)\boldsymbol{P}_a(n-1)\tilde{\boldsymbol{u}}_a(n)}$$

Step.5 Calculation of a priori estimate errors:

$$\eta_a(n) = y_e(n) - d(n) + \boldsymbol{a}^T(n-1)\boldsymbol{u}_a(n)$$

Step.6 Renovation of the estimate value of the coefficient vectors:

$$\boldsymbol{a}(n) = \boldsymbol{a}(n-1) + \boldsymbol{k}_a(n)\eta_a(n)$$

Step.7 Renovation of the correlation matrix:

$$\boldsymbol{P}_a(n) = \frac{1}{\lambda_a}\{\boldsymbol{P}_a(n-1) - \boldsymbol{k}_a(n)\tilde{\boldsymbol{u}}_a^T(n)\boldsymbol{P}_a(n-1)\}$$

Step.8 Let n be $n+1$ and go to Step. 2.

4.2 Problem of the RLS algorithm considering bias removal

Compared to the LMS algorithm, the RLS algorithm requires more amount of computation, but it gives more fast convergence property, because the RLS algorithm can use all the past input information to calculate the tap correction term. Therefore it is considered that the RLS algorithm accumulates all the past input in its parameters.

This fact causes a problem for the bias removal RLS algorithm in the previous section. In early stage of this algorithm, the estimation of the additive noise is inaccurate, this error of the estimation remains in all adaptive process from the nature of the RLS algorithm. As a result, the estimation of tap coefficients also becomes incorrect.

To solve this problem, the use of a variable forgetting factor $\lambda(n)$ is considered. The value of $\lambda(n)$ sets small one in early stage of adaptation and it gradually increases for progress of adaptation, because the estimation error of the additive noise is large in early stage and it gradually becomes small.

The schedule of increasing $\lambda(n)$ is empirically decided. For example, the case which used for the simulation in the latter section is shown in Fig. 3. In this case, the initial value of $\lambda(n)$ is 0.2 and it adds 0.1 at each 100 iteration until it becomes 1.

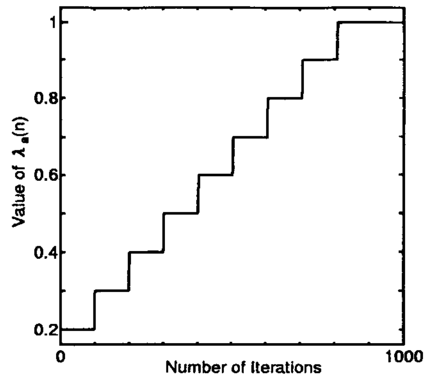

Figure 3: An example of the scheduling of $\lambda_a(n)$

5 Simulation

To verify the effectiveness of the proposed method, it is applied to system identification. The following system is used for an unknown system to be identified:

$$H(z) = \frac{1 + 2.0z^{-1} + 3.0z^{-2} + 4.0z^{-3} + 7.0z^{-4}}{1 - 0.3z^{-1} - 0.48z^{-2} + 0.084z^{-3} + 0.072^{-4}} \qquad (13)$$

The following mean squared parameter error (MPSE) is also used for the performance measure:

$$MSPE(i) = 10log\frac{mspe(i)}{mspe(0)} \qquad (14)$$

$$mspe(i) = \frac{\sum_{j=200\times i}^{200\times i+199}||\boldsymbol{a}(j) - \boldsymbol{a}_{true}||^2}{200} \qquad (15)$$

where $\boldsymbol{a}_{true}$ is the coefficients vector of the unknown system.

The white Gaussian noise whose mean is 0 and variance 1, is selected as the input signal $u(n)$ and the additive noise $v(n)$. The step-size parameters in the LMS algorithm are chosen as $\mu_a = 0.0001$ and $\mu_b = 0.0002$ and this selection gives the best convergence property. The parameters in the RLS algorithm are chosen as $c_a = e^7$, $c_b = 1.0$, $\lambda_b = 1.0$ and λ_a specified in Fig.3. For each algorithm, $\alpha = 1.0$ and the initial values of coefficients are 0. The mean and variance of the reference signal corrupted by the additive noise are -0.003035 and 266.485976. The convergence properties of $A(z)$ and $B(z)$ are shown in Figs. 4 and 5, respectively and each estimation value of coefficients after 30000 iteration is also shown in Tables 1 and 2, respectively.

From the above results, it is found that the proposed method gives a good convergence property.

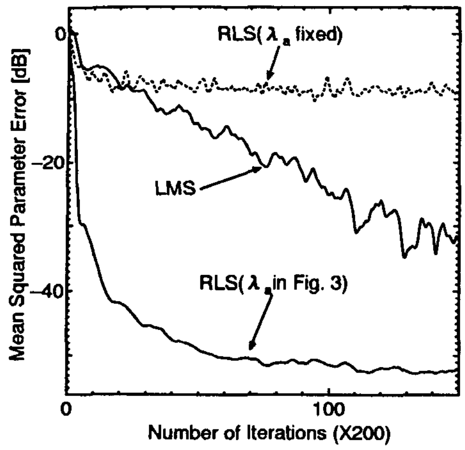

Figure 4: Convergence property of $A(z)$

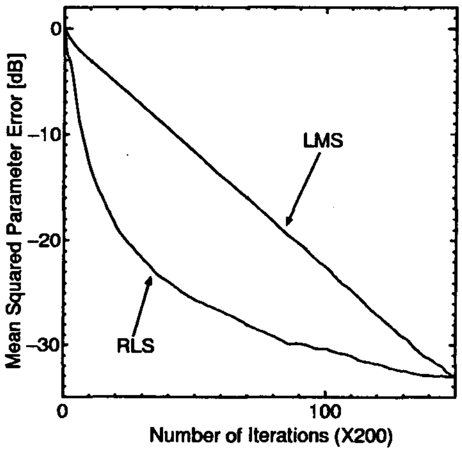

Figure 5: Convergence property of $B(z)$

6 Conclusion

In this paper, we have proposed an equation error adaptive IIR filter based on RLS algorithm considering the bias removal. Then, the problem of this algorithm has also been pointed out and an improvement method has been shown. The simulation result has indicated the effectiveness of the proposed algorithm.

The theoretical analysis of the proposed method, the reduction of the amount of computation and the scheduling algorithm of λ_a are left for the future works.

Table 1: Estimation value of coefficients of $A(z)$

	a_1	a_2	a_3	a_4
True value	0.3	0.48	-0.084	-0.072
LMS	0.3139	0.4751	-0.0821	-0.0733
Proposed	0.3034	0.4801	-0.0855	-0.0720

Table 2: Estimation value of coefficients of $B(z)$

	b_0	b_1	b_2
True value	1.0	2.0	3.0
LMS	1.0014	1.9703	2.9728
Proposed	1.0056	1.9807	3.0305

	b_3	b_4
True value	4.0	7.0
LMS	3.8994	6.8394
Proposed	4.0139	6.9175

References

[1] S. Haykin, "Adaptive Filter Theory," Prentice-Hall, Englewood Cliffts, New Jersey, 1986.

[2] B. Widrow and S. D. Stearns, "Adaptive Signal Processing," Prentice-Hall, Englewood Cliffts, New Jersey, 1985.

[3] C.R. Johnson, Jr., "Adaptive IIR filtering: Current results and open issues," IEEE Trans. Inform. Theory, Vol.IT-30, No.2, pp. 237-250, Mar. 1984.

[4] J.J. Shynk, "Adaptive IIR filtering," IEEE ASSP Mag., pp. 4-21, April 1989.

[5] P. Stoica and T. Söderstrom, "The Steiglitz-McBride identification algorithm revisited - convergence analysis and accuracy aspects," IEEE Trans. Automat. Contr., Vol.AC-26, No.3, pp. 712-717, June. 1981.

[6] H. Fan and W.K. Jenkins, "A new adaptive IIR filter," IEEE Trans. Circuits Syst.,, Vol.CAS-33, No.10, pp. 939-947, Oct. 1986.

[7] J.-N. Lin and R. Unbehauen, "Bias-remedy least mean square equation error algorithm for IIR parameter recursive estimation," IEEE Trans. Signal Process., Vol.40, No.1, pp. 62-69, Jan. 1992.