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# Forward Error Control Coding in Multicarrier DS/CDMA Systems

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**Abstract:** In this paper, forward error control coding in multicarrier direct sequence code division multiple access (DS/CDMA) systems is considered. In order to accommodate a number of coding rates easily and make the encoder and decoder structure simple, we use the rate compatible punctured convolutional (RCPC) code. We obtain data throughputs at several coding rates and choose the coding rate which has the highest data throughput in the SINR sense. To achieve maximum data throughput, a rate adaptive system using channel state information (the SINR estimate) is proposed. The SINR estimate is obtain by the soft decision Viterbi decoding metric. We show that the proposed rate adaptive convolutionally coded multicarrier DS/CDMA system can enhance spectral efficiency and provide frequency diversity.

## 1 Introduction

The radio links for portable and vehicular units can be characterized by time-varying multipath fading, which causes the transmission quality to vary with time. When the transmitter is provided with the channel state information (CSI), the channel can be allowed to be used more efficiently [1], [2]. At each signal to interference and noise ratio (SINR) operating point, we wish to choose the coded modulation scheme which results in the highest throughput under retransmission delay constraints in DS/CDMA environment.

Direct sequence code division multiple access (DS/CDMA) has become a popular multiple access signaling method due to its robustness against fading, anti-interference capability, and multiple-access capacity. The large spreading bandwidth employed typically exceeds the coherence bandwidth of the channel, so that the fading tends to be frequency selective. In a multicarrier system, on the other hand, path diversity is exchanged for frequency diversity. Based on the work of [3], forward error correction is employed in [4] in multicarrier systems, and it is shown that a multicarrier system with forward error correction has better performance than a multicarrier system with repetition coding.

The requirement for different bit error rates implies an adaptive error control coding scheme, allowing it possible to change code rate [1], [5]. We wish to change the code rate and hence the correction power of the code according to source and channel needs.

In this paper, forward error control coding in multicarrier DS/CDMA systems is considered. The rate adaptive multicarrier DS-CDMA system model is presented in Section 2, system analysis is given in Section 3, and simulation results are shown in Section 4. Some concluding remark is given in Section 5.

## 2 System Model

### 2.1 Transmitter

The transmitter for the rate adaptive convolutionally coded orthogonal multicarrier (CC/OM) DS/CDMA system considered in this paper is shown in Fig. 1. For user $k$, the information bits $\{b_k^i\}$, each with duration $T_b$, are encoded by the RCPC encoder of rate $r$. The relationship between $T_b$ and the duration $T_s$ of a coded binary symbol can be written as $T_s = rMT_b$, where $M$ is the number of subchannels. The $M$ coded binary symbols are allocated to $M$ subchannels to get frequency diversity. They are interleaved to get time diversity as well as frequency diversity, and are spread by each user's pseudo noise (PN) signature waveform $c_k(t)$ with chip duration $T_c = T_s/N$, where $N$ is the processing gain of the DS waveforms modulated by subcarriers. Note that for CC/OM systems, we have $M = \frac{2B_T}{B_S} - 1$, where $B_T$ and $B_S$ are the total and subchannel bandwidths, respectively. Since we fix the subchannel bandwidth $B_S$ (or equivalently, the symbol duration $T_s$) in this paper, $T_b$ varies according to $T_s = rMT_b$ when the code rate $r$ changes.

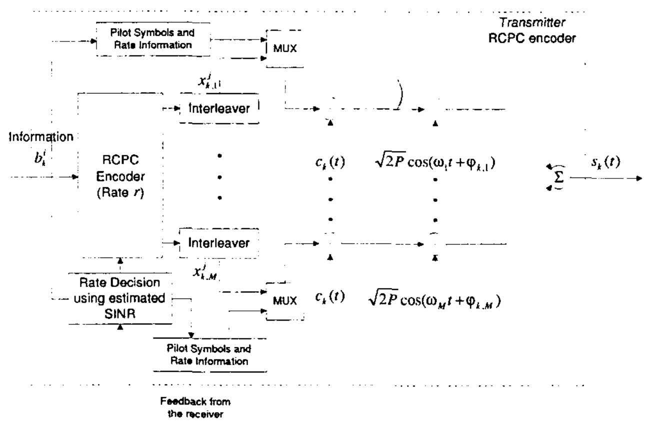


Figure 1: The transmitter model for user $k$ in the adaptive rate CC/OM DS/CDMA system

The transmitted signal $s_k(t)$ of user $k$ can be written as

$$s_k(t) = \sqrt{2P} \sum_{j=-\infty}^{\infty} \sum_{m=1}^{M} x_{k,m}^j c_k(t - jT_s)\cos(\omega_m t + \varphi_{k,m}), \quad (1)$$

where $P$ is the transmitted power per subcarrier, $x_{k,m}^j$ is the $k$-th user's $j$-th coded symbol in the $m$-th subchannel after interleaving, $\omega_m = 2\pi f_m$ is the carrier angular frequency of the $m$-th subchannel, and the random phases $\{\varphi_{k,m}\}$ of the subcarriers are independent and identically distributed (i.i.d.) uniform random variables on $[0, 2\pi)$. In (1), $c_k(t) = \sum_{n=0}^{N-1} c_{k,n} p(t - nT_c)$ is the signature waveform, where$\{c_{k,n}\}$ are the spreading chips that take values $\pm 1$, and $p(t)$ is a rectangular pulse whose value is 1 on $[0, T_c)$ and 0 outside.

The channel is assumed to be frequency selective Rayleigh fading and not to vary during one symbol duration. However, the subchannels are assumed to be nonselective by choosing the number of subcarriers appropriately as [3] $MT_c \geq T$, where $T$ is the maximum delay spread of the channel. Then the complex lowpass impulse response of the subchannels of user $k$ can be modeled as $h_{k,m}(t) = \alpha_{k,m} e^{j\beta_{k,m}} \delta(t)$, where $\alpha_{k,m}$ is the fading amplitude and $\beta_{k,m}$ is the random phase of the $m$-th subchannel, $m = 1, 2, \ldots, M$. The phases $\{\beta_{k,m}\}$ are i.i.d. uniform random variables on $[0, 2\pi)$. In general, the fading amplitudes $\{\alpha_{k,m}\}$ are correlated, but we can assume that they are i.i.d. Rayleigh random variables, once the coded symbols are properly interleaved in the time domain.

We assume frame by frame transmission. Each frame of duration $T_f$ consists of a header of duration $T_p$ and data symbols of duration $T_d$. The header contains pilot symbols and information on the rate and channel state.

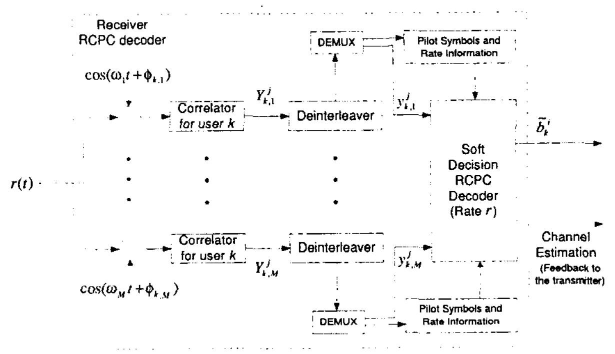


Figure 2: The receiver model for user $k$ in the adaptive rate CC/OM DS/CDMA system

## 2.2 Receiver

The receiver for the adaptive rate CC/OM DS/CDMA system in this paper is shown in Fig. 2. Let us assume that there are $K$ users in a cell and power control is employed. Then the received signal at the receiver can be written as

$$r(t) = \sqrt{2P} \sum_{j=-\infty}^{\infty} \sum_{k=1}^{K} \sum_{m=1}^{M} \alpha_{k,m} x_{k,m}^j c_k(t - \tau_k - jT_s) \cdot \cos(\omega_m t + \phi_{k,m}) + n(t), \quad (2)$$

where the propagation delays $\{\tau_k\}$ are i.i.d. uniform random variables on $[0, T_b)$, $\{\phi_{k,m} = (\varphi_{k,m} + \beta_{k,m} - \omega_m \tau_k) \bmod 2\pi\}$ are i.i.d. uniform random variables on $[0, 2\pi)$, and $n(t)$ is the additive white Gaussian noise (AWGN) with mean zero and variance $N_o/2$.

The received signal is coherently demodulated by the subcarrier and then correlated by each user's signature waveform. Let the first user be the desired user, $\tau_1 = 0$, and $j = 0$ without loss of generality. Then the correlator output of the $q$-th subcarrier of the desired user is given by

$$Y_q = \int_0^{T_s} r(t) c_1(t) \cos(\omega_q t + \phi_{1,q}) dt, \quad (3)$$

where we dropped the subscript 1 used to denote the user and the subscript 0 used to denote the correlator output in $Y_{1,q}^0$ for convenience. After the correlator output $Y_q$ is deinterleaved, the output goes through a soft decision RCPC decoder using Viterbi algorithm.

At the decoder, we compute the decoding metric to estimate the SINR, which will be used in the decision of the code rate for the next frame. In this paper, we use the SINR estimate feedback because, if some error occurs during the feedback information transmission in practical systems, the error of the code rate would affect more adversely than the error of the estimated SINR. We assume perfect feedback and no error in the decoding of the header for analysis purpose.

# 3 Rate Adaptation

## 3.1 Channel quality estimation

We can express the correlator output (3) as

$$Y_q = S_q + U_q + A_q + N_q, \quad (4)$$

where $S_q = \sqrt{\frac{P}{2}} T_s \alpha_{1,q} x_{1,q}^0$ is the desired signal, $U_q$ is the interference from other users in the same subchannel, $A_q$ is the interference from adjacent subchannels, and $N_q$ is the correlator output of the AWGN [4]. The variance of $U_q$ is

$$\begin{aligned} E\{U_q^2\} &= \frac{P}{2T_s} \sum_{k=2}^{K} E\{\alpha_{k,q}^2 \cos^2(\Delta\phi_q^{k,1})\} \\ &\quad \cdot \int_0^{T_s} \left(R_{k,1}^2(\tau) + \tilde{R}_{k,1}^2(\tau)\right) d\tau \\ &= \frac{PT_s^2}{12N^3} \sum_{k=2}^{K} r_{k,1}, \end{aligned} \quad (5)$$

where we assume $E\{\alpha_{k,q}^2\} = 1$ and $r_{k,h}$ is the average interference parameter defined in [4], [6]. Similarly, the variance of $A_q$ is

$$E\{A_q^2\} = \frac{PT_s^2}{4\pi^2 N^3} \sum_{k=2}^{K} [\mu_{k,1}(0) - \mu_{k,1}(1)] \sum_{\substack{m=1 \\ m \neq q}}^{M} \frac{1}{(m-q)^2}, \quad (6)$$

where $\mu_{k,h}(n)$ is defined in [4]. The variance of the correlator output corresponding to the AWGN is $E\{N_q^2\} = \frac{N_o T_s}{4}$. Since scaling the correlator output with a constant does not affect the performance, we can normalize the correlator output as follows [4]:

$$y_q \triangleq \sqrt{\frac{2}{P}\frac{Y_q}{T_s}} = \alpha_q x_q^0 + I_q, \qquad (7)$$

where $I_q = \sqrt{\frac{2}{P}\frac{1}{T_s}}(U_q + A_q + N_q)$. For convenience, we omitted the subscripts which represent the user. Note that the mean of $I_q$ is zero.

In this paper, we use random sequences as the spreading sequences so that $I_q$ can be assumed to be a Gaussian random variable from the central limit theorem when $K$ and $N$ are large enough. Then

$$\sigma_{I_q}^2 = \frac{K-1}{3N} + \frac{K-1}{2\pi^2 N}\sum_{\substack{m=1\\ m\neq q}}^{M}\frac{1}{(m-q)^2} + \frac{N_o}{2E_s} \qquad (8)$$

[4], [6]. Note that the bit energy is $E_b = \frac{E_s}{r^2 M}$ from $T_s = rMT_b$.

Let $\mathbf{y} =< y_1^1, \cdots, y_M^1, \cdots, y_1^W, \cdots, y_M^W >$ be the received codeword during one frame, where $y_q^j$ is the $q$-th correlator output at time index $j$ after interleaving and $W = 1/T_d$ is the number of coded symbols in a frame. In practice, perfect CSI is not available, but partial CSI (estimates of the fading amplitudes) can be reliably estimated to achieve good error performance by using pilot symbols in a frame. Thus, we consider the decoding metric $m(\mathbf{y}, \mathbf{x}; \hat{\alpha}) = \sum_{w=1}^{W}\sum_{q=1}^{M}|y_q^w - \hat{\alpha}_q x_q^w|^2$, where $\hat{}$ denotes estimate and $\mathbf{x}=< x_1^1, \cdots, x_M^1, \cdots, x_1^W, \cdots, x_M^W >$ is the path codeword, since we may assume that the differences among the noise variances of the subchannels are not large [4].

Let $\{\hat{x}_q^w\}$ be the estimated values of $\{x_q^w\}$ at the decoder. Then, if the available CSI is ideal, i.e., if $\hat{\alpha}_q = \alpha_q$, we have $\sum_{w=1}^{W}\sum_{q=1}^{M}|y_q^w - \hat{\alpha}_q\hat{x}_q^w|^2 = \sum_{w=1}^{W}\sum_{q=1}^{M}|\alpha_q(x_q^w - \hat{x}_q^w) + I_q|^2$. The term $\alpha_q(x_q^w - \hat{x}_q^w)$ is equal to zero if the decoder does not make any error. Typically, errors may occur when $\alpha_q$ is small: in such a case, however, $\alpha_q(x_q^w - \hat{x}_q^w)$ is also small and may be neglected [2]. We thus have $E\left\{\sum_{w=1}^{W}\sum_{q=1}^{M}|y_q^w - \hat{\alpha}_q\hat{x}_q^w|^2\right\} \approx WM \cdot E\{|I_q|^2\}$, and a useful estimate of the SINR can be obtained from $\widehat{\text{SINR}} = 10\log_{10}\frac{WM}{m(\mathbf{y},\hat{\mathbf{x}};\hat{\alpha})}$.

### 3.2 Rate adaptation

We now propose a threshold based adaptation scheme which adaptively changes the coding rate using $\widehat{\text{SINR}} = 10\log_{10}\frac{WM}{m(\mathbf{y},\hat{\mathbf{x}};\hat{\alpha})}$. Let $\theta_0 = -\infty, \theta_1, \theta_2, \cdots, \theta_Q = \infty$ be the SINR threshold values which are chosen such that between $\theta_{j-1}$ and $\theta_j$ the channel coding rate $R_j$ has the highest throughput. Here, $Q$ is the number of possible code rates. Then, the transmitter mode (rate) adaptation scheme can be defined as follows:

Choose $R_j$ if $\theta_{j-1} \leq \widehat{\text{SINR}} < \theta_j, j = 1, \cdots, Q$.

In the proposed method, we choose the adaptation interval $T_a$ so that $T_a$ is long enough to allow the transmission of at least one frame and short enough to react quickly to the possible change of the SINR. The transmitter can then adapt its data rate every $\lceil \frac{T_a}{T_f} \rceil T_f$.

## 4 Simulation Results

In this section, we first investigate the effects of coding rates on the throughput, and then investigate the performance of the adaptive rate CC/OM DS/CDMA system via computer simulation using Monte Carlo methods. In the simulation, we use RCPC codes with rate 1/4 convolutional codes of constraint length $L_c = 5$ and 9 as the parent codes [1], [5]: as the error control capability varies when the constraint length change, we use two values of $L_c$. The number $M$ of subcarriers is 4 and 9. The processing gain $N$ is 192 and 96, for $M = 4$ and $M = 9$, respectively, when the total bandwidth $B_T = \frac{N(M+1)}{T_s}$ is fixed. We assume each frame contains 144 symbols with $T_f = 10$ms and $T_p = 0$ (that is, we assume perfect feedback to simplify the simulations).

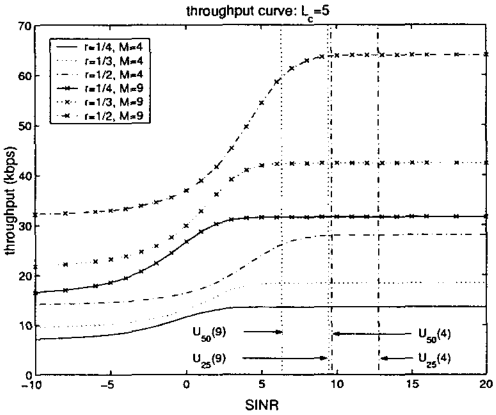


Figure 3: The throughput curves when $L_c = 5$. (Note: $U_k(m)$ is the maximum possible SINR when $K = k$ and $M = m$ with $U_1(m) = \infty$.)

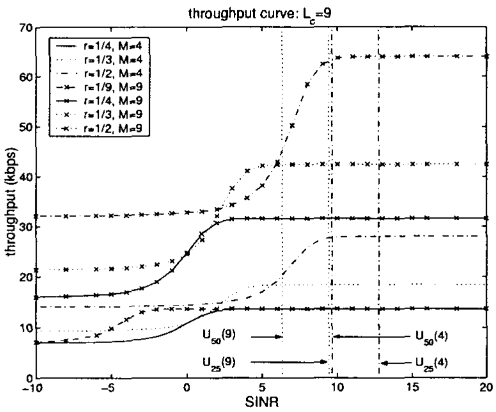


Figure 4: The throughput curves when $L_c = 9$. (Note: $U_k(m)$ is the maximum possible SINR when $K = k$ and $M = m$ with $U_1(m) = \infty$.)

In Figs. 3 and 4, we obtain data throughput curves for $L_c = 5$ and 9, and $M = 4$ and 9. When the SINR is very low, the bit error rate is almost the same irrespective of the coding rate: thus, the throughput will be higher with higher coding rate than with lower coding rate. When the SINR is very high, the bit error rate is higher for higher coding rate. However, as we can send more information bit with the higher coding rate, we obtain higher throughput with higher coding rate than with lower coding rate at very high SINR also. When $L_c = 9$, we can see that there are 4 crossing points when $M = 9$ (and when $M = 4$ also) in Fig. 4. To maximize the data throughput, crossing points around 2.5 and 5.5 (dB) are more important than the other two points. Therefore, we can set these two points as the thresholds and construct a rate adaptive system as described in Section 3.2. The rate $1/9$ code in Fig. 4 was obtained from the rate-compatible convolutional code [1] of a parent code with rate $1/4$.

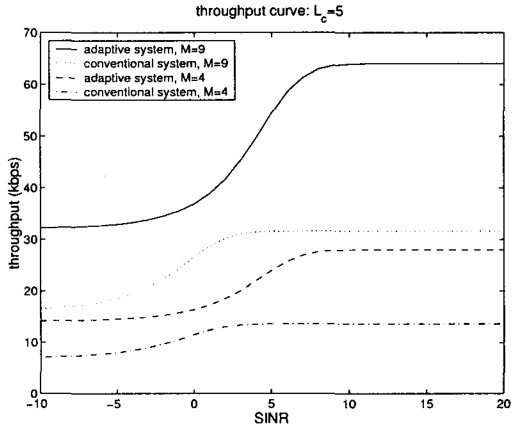


Figure 5: The adaptive throughput curves and the conventional throughput curves when $L_c = 5$

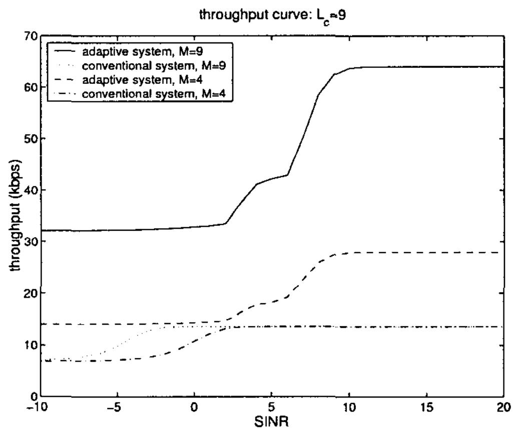


Figure 6: The adaptive throughput curves and the conventional throughput curves when $L_c = 9$.

We implement our rate adaptive CC/OM DS/CDMA system based on the above observations. When $L_c = 5$, it is clear from Fig. 3 that fixing the coding rate to $1/2$ allows us to get the highest throughput: the result is Fig. 5. When $L_c = 9$, using the thresholds $\theta_1 = 2.5$dB and $\theta_2 = 5.5$dB, we get Fig. 6. In Figs. 5 and 6, the throughput of the conventional system is also shown. It is clear that we can get higher throughput with the adaptive system.

## 5 Conclusion

In this paper, forward error control coding in multicarrier DS/CDMA systems was considered. In order to accommodate a number of coding rates easily and make the encoder and decoder structure simple, we used the rate compatible punctured convolutional (RCPC) code. We first investigated the effects of coding rates on the throughput. Data throughputs were obtained at several coding rates by varying the number of subcarrier and constraint length. Then we chose the coding rate which had the highest data throughput in the SINR sense. To achieve maximum data throughput, we proposed a rate adaptive system based on the channel state information (the SINR estimate). The SINR estimate was obtained by the soft decision Viterbi decoding metric. Finally, we showed that the proposed rate adaptive convolutionally coded multicarrier DS/CDMA system could enhance spectral efficiency and provide frequency diversity.

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## References

[1] P. Frenger, P. Orten, T. Ottosson, and A. Svensson, "Rate-compatible convolutional codes for multirate DS-CDMA systems," *IEEE Trans. Commun.*, vol. 47, pp. 828-836, June 1999.

[2] K. Balachandran, S. R. Kadaba, and S. Nanda, "Channel quality estimation and rate adaptation for cellular mobile radio," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1244-1256, July 1999.

[3] S. Kondo and L. B. Milstein, "Performance of multicarrier DS CDMA systems," *IEEE Trans. Commun.*, vol. 44, pp. 238-246, February 1996.

[4] Y. H. Kim, I. Song, H. G. Kim, and J. Lee, "Design and performance analysis of the convolutionally coded orthogonal multicarrier DS/CDMA systems," *IEEE Trans. Vehic.Techn.*, U.S.A. (accepted for publication).

[5] J. Hagenauer, "Rate-compatible punctured convolutional codes (RCPC Codes) and their applications," *IEEE Trans. Commun.*, vol. 36, pp. 389-400, April 1988.

[6] M. B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communication - Part I: System analysis," *IEEE Trans. Commun.*, vol. 25, pp. 795-799, August 1977.