Multiuser Detection for Multicarrier DS/CDMA System

*Wan Choi and **Jin Young Kim

*Research & Development Division, Hansol M.com
18F, Hansol M.com Bldg., 1321-11, Seocho-Dong, Seocho-Ku, Seoul, 137-070, Korea
Tel: +82-2-3488-0719, Fax: +82-2-3488-0727
E-mail: choiwan@ms.hansol.co.kr

**IMT-2000 Development Group
Central Research and Development Center
SK Telecom, Korea

Abstract - In this paper, a new multiuser detector combining multicarrier and decorrelating detection schemes is proposed and analyzed in a frequency selective Rayleigh fading channel. The bit error probability is derived and compared with that of the conventional decorrelating detector. From the numerical results, it is shown that the proposed detector achieves better BER performance and lower computational complexity than those of the conventional decorrelating detector.

I. Introduction

A decorrelating detector is a suboptimal multiuser detector that has attracted much attention among several multiuser detector schemes due to its inherent merits. The decorrelating detector provides substantial performance gains over the conventional detector on the various kinds of conditions. It does not need to estimate the received amplitudes and has an error probability independent of the signal energies. It is also known that the decorrelating detector corresponds to the maximum likelihood sequence detector when the energies of all users are not known at the receiver [1].

Since CDMA waveforms have a wide bandwidth, fading characteristics tend to be frequency-selective (this channel characteristic will be more severe in the wideband CDMA in IMT-2000), thus, a rake receiver can be used to enhance performance of the CDMA system. However, it should estimate continuously the relative delay of each path and the gain of each path for maximal-ratio combining (MRC). Even with perfect channel estimation, the near-far problem imposes a fundamental limit on the performance of an existing CDMA receiver. The conventional single-carrier decorrelating detector becomes complicated as the number of users or multi-path increases since the dimension of cross-correlation matrix becomes larger. Dimension of cross-correlation matrix is known to be a critical problem in the implementation of a decorrelating detector [2-4].

Multicarrier DS/CDMA system is regarded as a viable alternative to the classical single carrier DS/CDMA system in that an available bandwidth is decomposed into a set of disjoint equi-bandwidth frequency, and subband of bandwidth is less than the coherence bandwidth of the channel [5-7]. In each subband, the fading has non-selectivity. That is, path diversity is converted to frequency diversity. In [5], Kondo and Milstein examined a multicarrier DS/CDMA system that applied repetition coding to transmit M narrowband, band-limited DS/CDMA waveforms. The multicarrier DS/CDMA system in [5] has several advantages: First, a multicarrier DS/CDMA system is robust to multi-path fading. Second, the system has a narrowband interference suppression effect. And, finally, a good performance can be achieved even with low chip rate, since the entire bandwidth of the system is divided into M equi-width frequency bands, and thus each carrier frequency is modulated by a spreading sequence with a chip duration which is M times as long as that of a single-carrier system. In other words, a multicarrier DS/CDMA system requires a lower speed, parallel type of signal processing, in contrast to a fast, serial-type of signal processing in a single carrier rake receiver. This helps the user to employ a low-power consumption device.

In this paper, a new multiuser detector combining decorrelating detector and multicarrier schemes is proposed and analyzed in a frequency-selective Rayleigh fading channel. The multicarrier transmission scheme considered in our paper is a parallel transmission scheme of narrowband DS waveform in the frequency domain, where the same data sequence multiplied by a spreading sequence modulates multiple carriers in [5]. By combining decorrelating detector and multicarrier schemes, we can reduce complexity of decorrelating detector.

The rest of this paper is organized as follows: In section II, proposed detector and channel models are described. In section III, the BER performance of the proposed detector is analyzed. Numerical results and discussions are presented in section IV. In Section V, some conclusions are drawn.

II. System and Channel Model

We consider K users with BPSK data-modulation transmitting synchronously over their individual multi-path fading channel. This section describes transmitter, channel, and receiver models of the proposed detector.

A. Transmitter Model

A block diagram of a transmitter is shown in Fig. 1(a). The BPSK-modulated signal of the k th user, $b_k(t)$, is spreaded by normalized signature waveform $a_k(t)$. It is assumed that there are N chips per symbol (processing gain = N) and that each user has a different signature waveform. After spreading, the same spread signals modulate M multicarriers, and then are transmitted. The transmitted signal is given by

$$s(t)=\sum_{k=1}^{K}\sqrt{2w_k}\,a_k(t)\,b_k(t)\sum_{i=1}^{M}\cos(2\pi f_i t+\psi_{k,i}) \qquad (1)$$

where w_k is transmit power of k th user, f_i is the i th subband carrier frequency, and $\psi_{k,i}$ is the phase offset of the i th subband carrier for k th user.

The entire bandwidth of the system is divided into M equi-width frequency bands, and thus each carrier frequency is modulated by a spreading sequence with a chip duration which is M times as long as that of a single-carrier system.

B. Channel Model

The channel is assumed to be a slow frequency-selective Rayleigh fading channel with delay spread T_m. The chip duration of proposed detector is T_c ($T_c = M\cdot T_c'$) while chip duration of the conventional single-carrier decorrelating detector is $T_c^{'}$.

For the conventional single-carrier decorrelating detector, the frequency selective fading channel for the k th user can be represented as a tapped delay line. The number of resolvable paths in that model is given by [4][5]

$$L = \left\lfloor T_m / T_c^{'} \right\rfloor + 1 \tag{2}$$

where $\lfloor x \rfloor$ is the integer part of x. Then, the complex lowpass equivalent impulse response of the channel of the k th user is given by

$$h_k(t) = \sum_{l=0}^{L-1} \beta_{k,l}\, e^{\mu_{k,l}}\, \delta(t - lT_c^{'}) \tag{3}$$

where $\beta_{k,l}$ and $\mu_{k,l}$ are modeled as zero-mean complex valued stationary Gaussian random variable and uniform random variable over $[0, 2\pi)$, respectively.

For the proposed detector, the channel model can be characterized by the coherence bandwidth, $(\Delta f)_c$, which is given by

$$(\Delta f)_c \approx \frac{1}{T_m} \tag{4}$$

Each subband of the proposed detector has no selectivity if the number of subband multicarriers, M, meets the following condition.

$$\frac{T_m}{T_c} = \frac{T_m}{M \cdot T_c^{'}} \leq 1 \tag{5}$$

In addition, all subbands are subject to independent fading if bandwidth of each subband is larger than the cohrence bandwidth [5], i.e.,

$$(BW)_M = \frac{1}{T_c}(1+\alpha) = \frac{1}{M \cdot T_c^{'}}(1+\alpha) \geq \frac{1}{T_m} \tag{6}$$

where α is roll-off factor and $0 < \alpha \leq 1$.

From (2), (5), and (6), by choosing $M = L$ and $\alpha \geq \frac{T_c^{'}}{T_m}$, we can ensure each subband of a multicarrier system has no selectivity and subject to independent fading.

Then, the complex lowpass equivalent impulse response of the i th sub-carrier channel of k th user is given by

$$h_{k,i}(t) = \alpha_{k,i}\, e^{\phi_{k,i}}\, \delta(t), \qquad i = 1, 2, \cdots, M \tag{7}$$

where $\alpha_{k,i}$ and $\phi_{k,i}$ are an *i.i.d.* Rayleigh random variable with a unit second moment and *i.i.d.* uniform random variable over $[0, 2\pi)$, respectively.

C. Receiver Model

The received signal is given by

$$r(t) = \sum_{k=1}^{K} \sqrt{2w_k}\, a_k(t)\, b_k(t) \sum_{i=1}^{L} \alpha_{k,i} \cos(2\pi f_i t + \theta_{k,i}) + n(t) + n_J(t) \tag{8}$$

where $\theta_{k,i} = \psi_{k,i} + \phi_{k,i}$, $n(t)$ is AWGN with a variance σ^2 and double-sided power spectral density of $N_o/2$, and $n_J(t)$ is partial narrowband interference with a *p.s.d.* (power spectral density) of $S_{n_J}(f)$.

A block diagram of a receiver is shown in Fig. 1 (b). The received signal is coherently demodulated for each carrier and passes through the matched filter banks. The outputs of matched filter banks are decorrelated by decorrelating filters. The decorrelated signals for each user are maximal ratio combined. Then, the combined signals for each user pass through threshold device and data decisions are made.

III. BER Performance

In the performance analysis, the perfect carrier, code, and bit synchronization are assumed. The matched filter output of k th user for the i th subband carrier branch is given by

$$y_{k,i} = \int_0^T r(t)\, a^*(t) \sqrt{2} \cos(2\pi f_i t + \theta_{k,i})\, dt \tag{9}$$

To represent the matched filter output for the i th subband carrier branch by matrix form, we define data bit sequence, signature waveform sequence, signal power, fading matrices as follows:

$$\mathbf{b} = [b_1 \;\; b_2 \;\; \cdots \;\; b_K]^T \tag{10}$$

$$\mathbf{a} = [a_1(t) \;\; a_2(t) \;\; \cdots \;\; a_K(t)]^T \tag{11}$$

$$\mathbf{W} = diag(\sqrt{w_1} \;\; \sqrt{w_2} \;\; \cdots \;\; \sqrt{w_K}) \tag{12}$$

$$\mathbf{C} = \begin{bmatrix} \alpha_{1,i} & 0 & 0 & \cdots \\ 0 & \alpha_{2,i} & 0 & \cdots \\ & & \ddots & \\ \cdots & 0 & 0 & \alpha_{K,i} \end{bmatrix} \tag{13}$$

Then the matched filter output for the i th subband carrier branch is given by

$$\mathbf{y}_i = \mathbf{RWCb} + \mathbf{n}_i \tag{14}$$

where the cross-correlation matrix of normalized signature waveform $\mathbf{R}$ is given by

$$\mathbf{R} = \int_0^T \mathbf{a}^*(\mathbf{t})\mathbf{a}^{\mathbf{T}}(\mathbf{t})\, dt \tag{15}$$

and $\mathbf{n_i}$ is a Gaussian zero-mean K-vector with covariance matrix equal to $\sigma_i^2 \mathbf{R}^*$

The matched filter outputs pass through the decorrelating filters to reduce MAI by matrix inversion. The output of the decorrelating filter for the i th subband carrier branch is given by

$$\mathbf{z}_i = \mathbf{R}^{-1}\mathbf{y}_i = \mathbf{W}\,\mathbf{C}_i\,\mathbf{b} + \mathbf{n}_{zi} \tag{16}$$

where the noise vector $\mathbf{n_{zi}}$ is Gaussian zero-mean with covariance matrix $\sigma_i^2 \mathbf{R}^{-T}$.

Now, the maximal ratio combining (MRC) is used for all the outputs of the decorrelating filter for each carrier branch $\mathbf{z}_i$ ($i = 1, 2, \cdots, M$). Then, the combined signal is obtained by

$$\mathbf{z} = \mathbf{C}_1^* \mathbf{z}_1 + \mathbf{C}_2^* \mathbf{z}_2 + \cdots + \mathbf{C}_M^* \mathbf{z}_M \tag{17.a}$$

$$= \mathbf{C}_1^H\mathbf{C}_1\mathbf{W}\mathbf{b} + \mathbf{C}_1^H\mathbf{n}_{z1} + \mathbf{C}_2^H\mathbf{C}_2\mathbf{W}\mathbf{b} + \mathbf{C}_2^H\mathbf{n}_{z2} + \cdots + \mathbf{C}_M^H\mathbf{C}_M\mathbf{W}\mathbf{b} + \mathbf{C}_M^H\mathbf{n}_{zM} \tag{17.b}$$

Then, the signal-to-noise ratio for the k th user is given by

$$\rho_k = \sum_{i=1}^{M} \frac{w_k|\alpha_{k,i}|^2}{\sigma_i^2(\mathbf{R}^{-1})_{kk}} = \frac{w_k}{(\mathbf{R}^{-1})_{kk}} \sum_{i=1}^{M} \frac{|\alpha_{k,i}|^2}{\sigma_i^2} \equiv \frac{w_k\gamma}{(\mathbf{R}^{-1})_{kk}} \tag{18}$$

where $\gamma = \sum_{i=1}^{M} |\alpha_{k,i}|^2 / \sigma_i^2 = \sum_{i=1}^{M} q_i$.

Since $\alpha_{k,i}$, $i = 1, 2, \cdots, M$, are *i.i.d.* Rayleigh random variables, q_i has an exponential distribution with a *p.d.f.* given by

$$f_q(q_i) = \sigma_i^2 \, e^{-\sigma_i^2 q_i} \tag{19}$$

Then the moment-generating function of γ is given by

$$\Phi_v(s) = \prod_{i=1}^{L} \frac{\sigma_i^2}{s + \sigma_i^2} \tag{20}$$

From (20), the *p.d.f.* of γ is obtained as

$$f_\gamma(\gamma) = F^{-1}\{\Phi_v(s)\} \tag{21}$$

From (18) and (21), the bit error probability of the k th user is obtained as

$$P_k = \int_0^\infty Q\left(\sqrt{\frac{w_k\,\gamma}{(\mathbf{R}^{-1})_{kk}}}\right) f_\gamma(\gamma)\, d\gamma \tag{22}$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}\, dt$.

IV. Numerical Results and Discussions

In this section, we compare the performance and the complexity of the proposed detector with those of the conventional single-carrier decorrelating detector.

To ensure the same energy per symbol (E_b) for the both detectors, E_b / M should be transmitted over each multicarrier subband. Then the average received energies of a proposed detector and the conventional single-carrier decorrelating detector are given by $E\left[\frac{E_b}{M}\sum_{i=1}^{M}\alpha_{k,i}^2\right]$ and $E\left[E_b\sum_{l=1}^{L-1}\beta_{k,l}^2\right]$, respectively. And, to ensure the same average received energy for both system and unit energy constraint on the fading, it is required that

$$E\left[\frac{E_b}{M}\sum_{i=1}^{M}\alpha_{k,i}^2\right] = E\left[E_b\sum_{l=1}^{L-1}\beta_{k,l}^2\right] = E_b \tag{23}$$

For an exponential multi-path intensity profile (MIP), we have

$$\sigma'^2_{k,l} = \sigma'^2_{k,1}\, e^{-\frac{l-1}{L'}} \tag{24}$$

where $\sigma'_{k,l} = E[\beta_{k,l}^2]$ and the parameter L' represents the rate of power decay on successive paths. From (23) and the exponential MIP, for an arbitrarily taking $L' = L$, we obtain

$$\sigma'^2_{k,1} = \frac{1 - e^{-1/L}}{1 - e^{-1}} \tag{25}$$

A. Comparison of Complexity

A dimension of cross-correlation matrix is critical factor in determining the complexity of a decorrelating detector. The dimensions of cross-correlation matrix of the conventional single-carrier decorrelating detector and the proposed detector are $KL \times KL$ and $K \times K$, respectively. In the conventional decorrelating detector, $(KL)^2$ multiplications and $(KL)^2$ additions are performed during $KL \times KL$ matrix inversion process. While, in the proposed detector, $K \times K$ matrix inversions are performed M ($= L$) times. So, $M \times K^2$ multiplications and $M \times K^2$ additions are performed during matrix inversion process. Thus the computational complexity of the proposed detector is $1/M$ of that for the decorrelating detector. As the number of resolvable multi-path increases, the computational complexity of the proposed detector is smaller than that of the conventional decorrelating detector.

B. Comparison of BER Performance

For the numerical examples, the number of users $K = 8$, the processing gain of the proposed detector $N = 63$, and the processing gain of conventional single-carrier decorrelating detector $N' = 255$ were assumed.

In Fig. 2, the bit error probability of the proposed detector and the conventional single-carrier decorrelating detector are compared in a frequency-selective Rayleigh fading channel. The typical values of the multi-path delay spread suggest the multi-path diversity reception with two branches (two-ray model) in suburban and four to five branches in an urban environment [8]. The proposed detector has a performance gain compared with the conventional one for the same BER. The gain is induced by the reduction of the dimension for cross-correlation matrix. An effect of noise enhancement in decorrelaing procedure depends on the dimension of cross correlation matrix. So, the proposed detector has a performance improvement compared with the conventional one.

V. Conclusions

The new multiuser detection system combining decorrelating detection and multicarrier transmission was proposed and analyzed in terms of BER performance in a frequency-selective Rayleigh fading channel. It was shown that the proposed detector achieves lower bit error probability and lower computational complexity than those of decorrelating detector. In terms of both complexity and performance, the proposed detector can be considered as a promising multiuser detection scheme. The future directions of this research can include the application of the proposed detector to the multi-rate CDMA system. The proposed detector can be applied to the receiver design of a wideband CDMA system.

References

[1] R. Lupas and S. Verdu, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 35, pp. 123-136, Jan. 1989.

[2] Z. Zvonar and D. Brady, "Multiuser detection in single-path fading channels," *IEEE Trans. Commun.*, vol. 42, no. 2/3/4, pp. 1729-1739, Feb./Mar./Apr. 1994.

[3] W. Choi, Y. Cho, and J. H. Lee, "Performance of frequency domain multiuser detector for synchronous CDMA

communication over Rician fading channels," *in Proc. of the IEEE VTC '98*, pp. 1725-1729, Ottawa, Canada, May 1998.

[4] Z. Zvonar and D. Brady, "Suboptimal multiuser detector for frequency-selective Rayleigh fading synchronous CDMA channels," *IEEE Trans. Commun.*, vol. 43, no. 2/3/4, pp. 154-157, Feb./Mar./Apr. 1995.

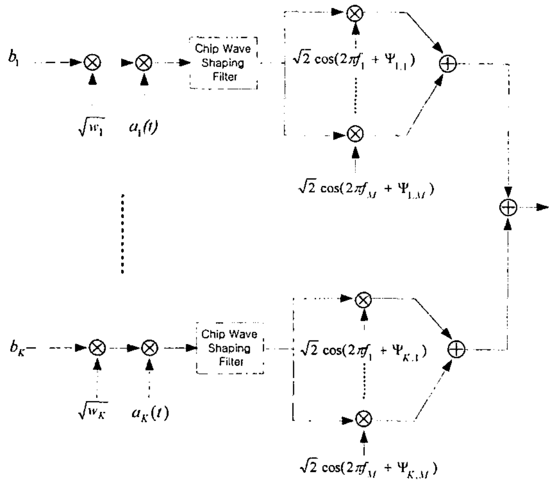
[5] S. Kondo and L. B. Milstein, "Performance of multicarrier DS CDMA systems," *IEEE Trans. Commun.*, vol. 44, no. 2, pp. 238-246, Feb. 1996.

[6] S. Kondo and L. B. Milstein, "Multicarrier DS CDMA systems in the presence of partial band interference," *in Proc. of the IEEE MILCOM '94*, Fort Monmouth, NJ, U.S.A., pp. 588-592, Oct. 1994.

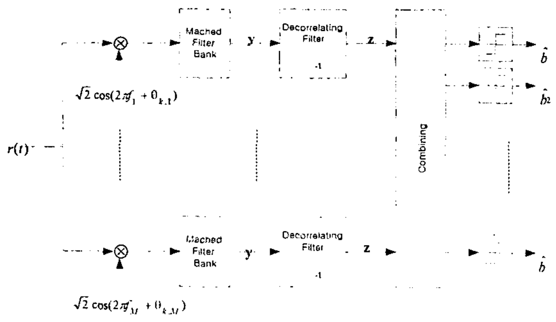
[7] E. A. Sourour and M. Nakagawa, "Performance of orthogonal multicarrier CDMA in a multi-path fading channel," *IEEE Trans. Commun.*, vol. 44, no. 3, pp. 356-367, Mar. 1996.

[8] W. Lee, "Overview of cellular CDMA," *IEEE Trans. Veh. Technol.*, vol. 40, no. 2, pp. 291-302, May 1991.

[9] W. Choi and J. Y. Kim, "Performance of a multiuser detector with multicarrier transmission for a DS/CDMA system," *To be submitted to Kluwer Academic Journal of Wireless Personal Communications.*

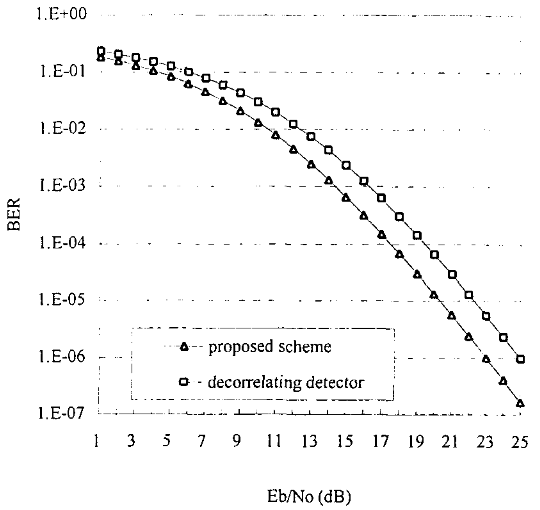


(a) Transmitter

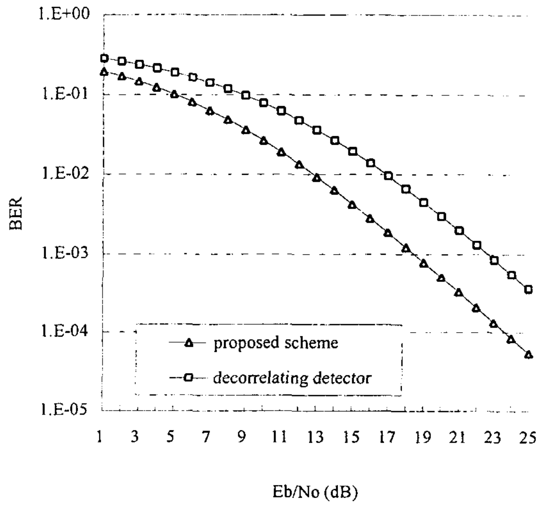


(b) Receiver

Fig. 1. Block diagrams of the proposed multiuser detector (a) Transmitter, (b) Receiver



(a) $L=M=4$



(b) $L=M=2$

Fig. 2. BER of the proposed system and the decorrelating detector without NBI (a) $L=M=4$ (b) $L=M=2$