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Multi-Level and Multi-Objective Optimization of Framed Structures Using Automatic Differentiation

조효남* 민대홍** 이광민*** 김환기***
Cho, Hyo-Nam Min, Dae-Hong Lee, Kwang-Min Kim, Hoan-Kee

ABSTRACT

An improved multi-level (IML) optimization algorithm using automatic differentiation (AD) for multi-objective optimum design of framed structures is proposed in this paper. In order to optimize the steel frames under seismic load, two main objective functions need to be considered for minimizing the structural weight and maximizing the strain energy. For the efficiency of the proposed algorithm, multi-level optimization techniques using decomposition method that separately utilizes both system-level and element-level optimizations and an artificial constraint deletion technique are incorporated in the algorithm. And also to save the numerical efforts, an efficient reanalysis technique through approximated structural responses such as moments, frequencies, and strain energy with respect to intermediate variables is proposed in the paper. Sensitivity analysis of dynamic structural response is executed by AD that is a powerful technique for computing complex or implicit derivatives accurately and efficiently with minimal human effort. The efficiency and robustness of the IML algorithm, compared with a plain multi-level (PML) algorithm, is successfully demonstrated in the numerical examples.

1. Introduction

In the practical optimization problems, more than one important objective is usually required to be optimized, such as, minimum weight or cost, maximum stiffness, minimum displacement at specific structural points, maximum natural frequency of free vibration, maximum structural strain energy. Recent trends when designing steel framed structures have shown more than two objectives. For example, cost saving by decreasing structural weight may be one objective, and performance maximization by increasing structural strain energy to endure earthquakes or minimizing deflection may be the other. An optimum design model for more than two objective functions requires a multi-objective optimization method, which has recently been developed to adjust many conflicting designs to structural optimizations. Most of the studies were concerned mainly with the theory, but applications to engineering design can hardly be found. Not until recently did a few studies on multi-objective optimization appear in structural optimization [6, 14]. Only recently, studies on multi-objective optimizations using multi-level optimization [10, 11, 16] started to improve their efficiency because too much of computing times are usually required with multi-objective optimization. As one of the most recent study, Gang Li et al. (1999) has proposed a plain multi-level (PML) algorithm for multi-objective optimization for eight-story, one-bay shear steel frame, which has the objective functions of weight and structural strain energy [5]. The PML algorithm is the latest development for multi-level and multi-objective optimization with dynamic property. However, the algorithm shows robust results only when an initial value is set near an optimum value. The algorithm also did not perform the reanalysis of dynamic properties. Thus the direct application of mathematical programming without approximation reanalysis is very costly, mainly because a great number of structural analyses is required. In the paper, an improved multi-level (IML) algorithm for multi-objective optimization is proposed to overcome these drawbacks in the PML algorithm [5]. In the IML algorithm, an artificial constraint deletion is introduced to increase its robustness. In addition, in order to increase the computing efficiency, optimization of system level that requires a great number of dynamic analyses is achieved by approximating all the structural responses such as bending moments, frequencies, and strain energy under seismic load. Structural responses are also approximated using the intermediate variables that represent characteristic of each structural response. Sensitivity analysis of structural dynamic response with respect to the intermediate variables also is executed by automatic differentiation (AD) that is a technique for computing complex or implicit

* 한양대학교 토목환경공학과 교수

** 한양대학교 토목환경공학과 박사과정

*** 한양대학교 토목환경공학과 석사과정

derivatives accurately and efficiently with minimal human effort. The new optimization algorithm is applied to a multi story steel framed structure which is the same numerical example as that in the Gang Li's study and it is demonstrated that the IML algorithm has better robustness and efficiency than the PML algorithm [5].

2. General Formulations

For steel framed structures, the multi-objective and multi-level optimization problem can be stated in the following form as in the reference [5].

The system-level optimization:

$$\begin{aligned} &\text{Find } X \text{ such that minimize } \{-U(X), W(X)\} \\ &\text{subject to } G_j(X) \leq 0 \quad j = 1, 2, \dots, n_c, \quad X^L \leq X \leq X^U \end{aligned} \quad (1)$$

where X is the design variables at system-level, which are the moment of inertia; $U(X)$ is the structural strain energy part of multi-objective function; $W(X)$ is the structural steel weight part of multi-objective function; $G_j(X)$ is j-th constraint; n_c is the number of constraint; and X^L and X^U are the lower and upper bounds of the system design variables, respectively.

The element-level optimization:

$$\begin{aligned} &\text{Find } x \text{ such that minimize } W_i(x) \\ &\text{subject to } g_j(X_i^*, x) = 0; \quad g_k(x) \leq 0 \quad k = 1, 2, \dots, n_i; \quad x^L \leq x \leq x^U \end{aligned} \quad (2)$$

where x is the design variables at element-level, which are the member cross-sectional dimensions; $W_i(x)$ is the objective function, which is the weight of i-th member; $g_k(x)$ is k-th constraint; n_i is the number of the constraints; x^L and x^U are the lower and upper bounds of the element variables, respectively; and $g_j(X_i^*, x)$ is the additional relative equality constraint that connects design variables of the two levels and thus simplifies the coupling between them.

2.1 Design variables

In general, the design variables of a steel frame at system-level $\{X\}$ and element-level $\{x\}$ are taken as the moments of inertia of each member and element cross-sectional dimensions respectively, which may be given in vector forms as

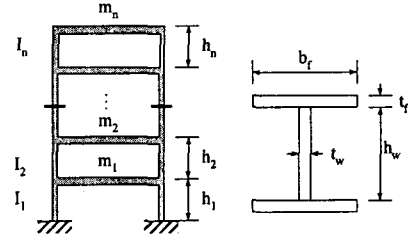


Fig. 1. A steel framed structure with I Section

$$X = (X_1, X_2, \dots, X_n) = (I_1, I_2, \dots, I_n) \quad (3)$$

$$x = (x_1, x_2, x_3, x_4) = (h_w, b_f, t_f, t_w) \quad (4)$$

Noting that the design variables are moments of inertia of frame members, the relationships between cross sectional area A , section modulus S and moment of inertia I must be given in a multi-level optimization. The following relations can be obtained.

$$A(I) = \left(\frac{A_0}{\sqrt{I_0}} \right) \sqrt{I}, \quad S(I) = \left(\frac{I}{I_0} \right)^{0.75} S_0 \quad (5a), (5b)$$

where A_0 and I_0 are the initial values of the cross-sectional area and moment of inertia of an element at the beginning of the system-level optimization in a single iteration cycle.

2.2 Objective function

Both maximum structural strain energy and minimum structural weight are simultaneously considered as the objective function in the system-level optimization as in the reference [5]. The multi-objective function often has a linearly weighting form by employing the weighting method for system-level optimization as

$$F(I) = w_1 \left(\frac{W(I)}{W_0} \right) + w_2 \left(-\frac{U(I)}{U_0} \right); \quad (w_1, w_2 > 0, \text{ and } w_1 + w_2 = 1) \quad (6)$$

where $U(I)$ and $W(I)$ are structural strain energy and weight, respectively; U_0 and W_0 are initial values at the beginning of each iteration; and w_1 and w_2 are weight factors to be determined by considering engineering problems in practice.

For a shear steel frame under earthquake load, the structural strain energy can be derived as

$$U(I) = \sum_{j=1}^m \sum_{i=1}^n \frac{1}{2} Q_{ij} \delta_{ij} \quad (7)$$

where Q_{ij} and δ_{ij} are shear force and story drift of the i -th element with respect to the j -th mode of vibration, respectively; m is the total number of modes of vibration; and n is number of elements.

Based on Eq. (5a), the structural weight can be written as

$$W(I) = \sum_{i=1}^n \rho A_i h_i = \sum_{i=1}^n \rho h_i \frac{A_0}{\sqrt{I_0}} \sqrt{I} \quad (8)$$

where ρ is material mass density; and A_i and h_i are cross-sectional area and height of the i -th element, respectively.

Besides, note that the objective function in the element-level becomes the weight of the element:

$$W_i(I) = \rho A_i h_i = \rho h_i (h_w t_w + 2bt_f) \quad (9)$$

2.3 Constraints

The basic and design constraints required for optimum (seismic) design of steel frame according to the AISC-LRFD and AISC seismic provisions may be summarized as in Table 1.

Table 1. Constraints for Strength and Local Buckling

Level	Design Constraints		Remarks
System	Strength	For $\frac{P_u}{\phi_c P_n} \geq 0.2$, $\frac{P_u}{\phi_c P_n} + \frac{8M_u}{9\phi_b M_n} \leq 1.0$ For $\frac{P_u}{\phi_c P_n} < 0.2$, $\frac{P_u}{2\phi_c P_n} + \frac{M_u}{\phi_b M_n} \leq 1.0$	P_u and P_n are required and nominal axial compressive or tensile strength, respectively; M_u and M_n are required and nominal flexural strength, respectively; and ϕ_c and ϕ_b are resistance factors for the axial and flexure.
Element	Local Buckling	For the flange, $\frac{t_w/2}{t_f} \leq \frac{52}{\sqrt{F_y}}$ For the web, If $\frac{P_u}{\phi_b P_y} \leq 0.125$, $\frac{h_w}{t_w} \leq \frac{520}{\sqrt{F_y}} \left(1 - \frac{1.54P_u}{\phi_b P_y}\right)$ If $\frac{P_u}{\phi_b P_y} > 0.125$, $\frac{h_w}{t_w} \leq \frac{191}{\sqrt{F_y}} \left(2.33 - \frac{P_u}{\phi_b P_y}\right) \geq \frac{253}{\sqrt{F_y}}$	F_y is yield strength of member and flange (ksi); and P_y is nominal yield strength.

Frequency constraints

The frequency constraints are used to make the natural frequencies avoid a range called restricted zone of frequency, which can be given in the following form [5]:

$$\omega_j \notin [\omega_a, \omega_b] \quad j=1, \dots, m \quad (10)$$

where ω_j is the j -th natural frequency; and ω_a and ω_b are the lower and upper bound of the restricted zone, respectively. Three alternative cases can be considered assuming that $\omega_1 \leq \omega_2 \leq \dots \leq \omega_m$:

In case all the structural natural frequencies are less than the restricted zone, the frequency constraint Eq. (10) can be changed into the following form:

$$\omega_m \leq \omega_a \quad (11a)$$

1) In case all the structural natural frequencies are greater than the restricted zone, the frequency constraint Eq. (10) can be changed into the following form:

$$\omega_b \leq \omega_1 \quad (11b)$$

2) In case when a couple of adjacent frequencies called concerned frequencies, ω_i and ω_{i+1} , which can cover the entire restricted zone, the frequency constraint can be written as

$$\omega_i \leq \omega_a \quad \omega_{i+1} \geq \omega_b \quad (11c)$$

Additional relative constraint

In order to assure the consistency between system and element design variables, the coupling between the system-level and element-level is treated by an additional relative constraint in the element-level optimization. The appropriate additional relative constraint is very important in a multi-level optimization problem because it affects both the convergence of the optimization procedure and the final optimal solutions. The equality constraint of moment of inertia of an element is introduced during the element-level optimization process to treat the coupling effectively. The additional relative constraint can be written as:

$$I_i(h_w, b_f, t_f, t_w) = I_i^* \quad (12)$$

where I_i^* is the optimum moment of inertia for the i -th element at system-level.

3. Multi-Level Optimization Algorithm

3.1 Plain Multi-Level Algorithm

In the reference [5], a plain multi-level (PML) optimization algorithm shown in Fig. 2 proposed for a multi-objective optimization for steel framed structures. However, it is found that the following problems are incurred when the algorithm is applied to numerical examples:

- Only in case an initial value is not far from a near-optimum value in the algorithm, robust results could be obtained. If an initial value is feasible but far from the optimum, unreliable results may be produced.
- The dynamic analysis is performed to evaluate objective function and constraint in each iterative step without sensitivity analysis. Dynamic reanalysis in each iterative step will be more time-consuming in a large structure, and therefore, the computational efficiency can not be expected when it is applied to large structures.

3.2 Improved Multi-Level Algorithm

An Improved multi-level (IML) algorithm shown in Fig. 3 is proposed for an efficient multi-objective optimization, which shows better performance compared to the PML algorithm [5]. In the IML algorithm, an artificial constraint deletion technique and reanalysis using approximation structural responses such as moments, natural frequencies, and total strain energy with respect to intermediate design variables are introduced to carry out optimum design efficiently for fast convergence. The artificial constraint deletion for frequency constraint is introduced to improve the efficiency and the robustness of the optimization algorithm. After optimization is carried out for all constraints except a frequency constraint, the feasibility for frequency constraint is checked. If feasibility is satisfied with frequency constraint, it is assumed that the optimization process arrived at a near optimum design. Otherwise, an optimization with frequency constraints is executed. In order to find an efficient optimization technique for steel framed structures that involve nonlinear constraints, the efficiency and robustness of various techniques available in the ADS are comparatively examined with the optimization of the example structure of this study. Since the ALMM (Augmented Lagrange Multiplier Method) provides most reasonable and effective solutions in most cases, it is used as the basic optimization algorithm for the system and element optimization problem in the paper.

4. Approximation Reanalysis Using Automatic Differentiation

4.1. Reanalysis of Structural Responses

The quality of approximation reanalysis is very important in the system optimization of structures. A great amount of work has been carried out to enhance the solution of structural optimization problems. In most case it is better to approximate the functions with respect to some intermediate variables that represent characteristic of each structural response, when the design variables are the actual section properties. For instance, these variables can be considered as the vector of moments of inertia. The quality of approximation is significantly higher as all the functions under consideration contain the values of the

intermediate variables. The reason for choosing these intermediate variables is that the stiffness and mass matrices of the steel frame structure contain the values of moments of inertia of elements. Thus the quality of approximation will be enhanced with these variables. In this study, the optimization is achieved by approximating all the structural responses.

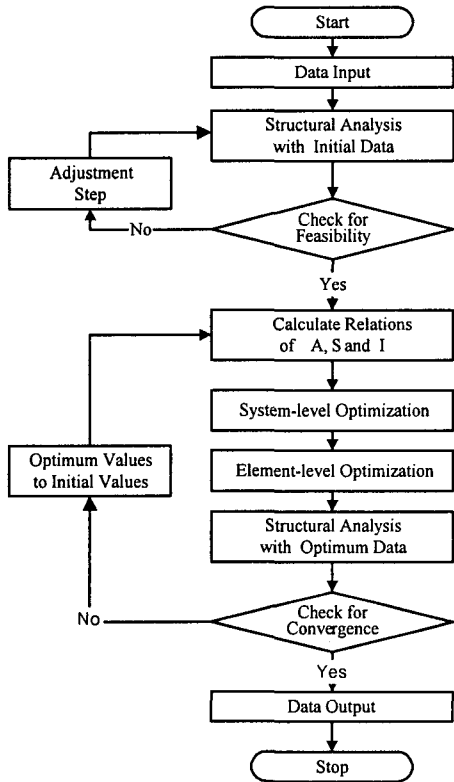


Fig. 2. The PML Algorithm (Li, et al.)

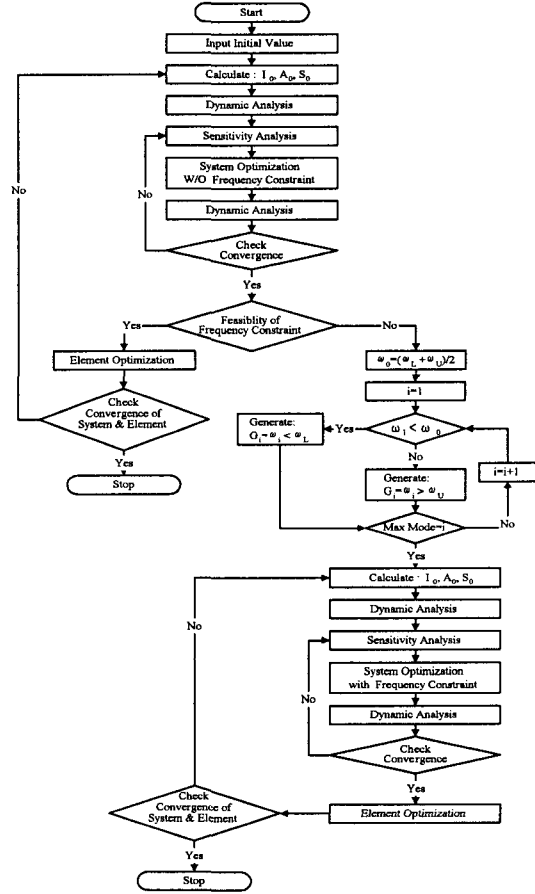


Fig. 3. The IML Algorithm

1) Equivalent static lateral force

The UBC-91 stipulates a seismic provision that the structure should be designed for a total base shear force given by the following formula:

$$V = \frac{ZIC}{R_w} W \quad (13)$$

in which Z is the seismic zone coefficient; I is an importance factor; R_w is a numerical coefficient related to the type of construction; W is dead load used to calculate seismic loads; and C is the dynamic factor that may be given as

$$C = \frac{1.25S}{T^{2/3}} \quad (14)$$

in which S is site coefficient; T is fundamental period of vibration.

2) Approximation reanalysis problem

The simplest local approximation is the linear approximations to structural responses for objective functions and constraints, which are based on the value of the function and its derivatives at one design point. Let the vector of structural responses in each member be denoted by \mathbf{R} . If \mathbf{X} represents the vector of design variables with n unknowns, then \mathbf{R} can be approximated as

$$\mathbf{R}(\mathbf{X}) = \mathbf{R}(\mathbf{X}_0) + \nabla \mathbf{R}(\mathbf{X}_0) \delta \mathbf{X} \quad (15)$$

in which $\delta \mathbf{X} = \mathbf{X} - \mathbf{X}_0$; \mathbf{X}_0 is the starting design point about which the Taylor series expansion is created; and the symbol ∇ represents the gradients of the function. This linear approximation is inaccurate even for design points \mathbf{X} , close to \mathbf{X}_0 . Retaining additional terms in the Taylor series expansion can increase accuracy. This, however, requires the costly calculation of higher-order derivatives. A more attractive alternative is to find intermediate variables that would make the approximated function behave more linearly. In general, the intermediate variable \mathbf{Y} can be defined in terms of design variable \mathbf{X} in the following form;

$$\mathbf{Y} = \mathbf{X}^\beta \quad (16)$$

where \mathbf{Y} is the function of the design variables called intermediate variables. Then, linear approximation, $\mathbf{R}(\mathbf{X})$, in terms of the intermediate variables, may be given as

$$\mathbf{R}(\mathbf{X}) = \mathbf{R}(\mathbf{X}_0) + \nabla \mathbf{R}(\mathbf{Y}_0) \delta \mathbf{Y} \quad (17)$$

where $\mathbf{Y}_0 = \mathbf{Y}(\mathbf{X}_0)$ and $\delta \mathbf{Y} = \mathbf{Y} - \mathbf{Y}_0$. First all the structural responses for constraints and objective function, such as bending moments, natural frequencies, and strain energy are approximated in terms of the intermediate variables that represent characteristic of each structural response. Thus the strain energy and bending moments due to the equivalent static lateral force under seismic load are, respectively, approximated using each intermediate variables $\mathbf{Y} = \mathbf{X}^{1/3}$ and $\mathbf{Y} = \mathbf{X}^{1/3}$. The natural frequencies based on the structural stiffness and mass are also approximated using intermediate variables $\mathbf{Y} = \mathbf{X}^{1/2}$.

4.2. Automatic Differentiation

To evaluate structural responses with respect to intermediate design variables, design sensitivity analysis using derivatives involved, which is usually formidable task in the system optimization of large scale structures. However, the conventional techniques for computing the derivatives, such as hand-coding, finite difference approximations, and symbolic differentiation, are hard to apply to real scale structures because of the large amount of computational time and the accumulation of computing errors. To overcome the problems, an automatic differentiation (AD) technique is introduced in this paper for accurately and efficiently computing derivatives with minimal human effort[1]. AD techniques rely on the fact that every function, no matter how complicated, is executed on a computer as a (potentially very long) sequence of elementary operations such as additions, multiplications, and elementary functions (sine, cosine, etc.). By applying the chain rule of differential calculus over and over again to the composition of those elementary operations, one can compute the derivative information exactly (up to machine precision) and in a completely mechanical fashion [7].

Traditionally, two approaches to AD, so-called forward and reverse modes, have been developed. A mode of differentiation, where the derivatives are maintained with respect to the independent variables, is called the forward mode of AD. The best known alternative to the forward mode is the reverse mode, which maintains derivatives of intermediate values with respect to the final results and is a discrete analog of the adjoint. The associativity of the chain rule of differential calculus actually allows many ways to compute derivatives, which may greatly differ in cost. These are discussed in the proceedings edited by Griewank and Corliss (1991) and Berz et al. (1996)[1, 8]. Various AD tools are available. Available tools include ADIFOR [2,

3], ODYSSEE [12], and ADOL-F [15] for FORTRAN programs; and ADOL-C [9] and ADIC [4] for C programs. This study employed the ADIFOR tool [2, 3]. ADIFOR differentiates programs written in FORTRAN77.

5. Numerical Example

In this section, an eight-story, one-bay shear steel frame with I-sections as shown in Fig. 1 which is almost the same example used in the reference [5], is optimized by an optimum design program developed for the multi-objective and multi-level optimization based on the theory and formulations discussed above. The basic data for design are summarized in Table 2.

Table 2. Basic Data for Design

Item	Data
Building story and story height	$N = 8, h_i = 4 \text{ m}$
Concentrated mass and mass density of steel	$m_i = 4000 \text{ kg}, \rho = 7850 \text{ kg/m}^3$
Elastic modulus and yield strength	$E = 200,000 \text{ Mpa}, F_y = 248 \text{ Mpa (36 ksi)}$
Restricted zone of frequency	$[\omega_a, \omega_b] = [15, 20]$
Bounds of system variables (10^{-4}m^4)	$x^U = (7.0, 7.0, 5.0, 5.0, 4.0, 4.0, 3.0, 3.0)$ and $x^L = (1.5, 1.5, 1.0, 1.0, 0.7, 0.7, 0.5, 0.3)$
Bounds of element variables (10^{-2}m)	$x^U = (80, 50, 3.0, 1.6)$ and $x^L = (25, 15, 1.0, 0.5)$

Table 3. Initial Values

Initial value	Story	1	2	3	4	5	6	7	8
Case 1	h_w (cm)	40.0	40.0	40.0	40.0	30.0	30.0	30.0	30.0
	b_f (cm)	20.0	20.0	20.0	20.0	12.0	12.0	12.0	12.0
	t_f (cm)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	t_w (cm)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Case 2	h_w (cm)	43.0	38.0	32.0	26.0	23.0	19.0	19.0	18.0
	b_f (cm)	19.0	18.0	16.0	13.0	8.0	6.0	5.0	6.0
	t_f (cm)	1.2	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	t_w (cm)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

To investigate the robustness and the efficiency of the two algorithms, each optimization is individually performed with two different sets of initial values of Table 3. The new IML and the PML algorithms are applied to the above design example. The results of the application using the two algorithms are comparatively shown in Table 4.

The frequencies at each mode obtained from the dynamic analysis as a part of the optimum design algorithm using 2 cases of initial values are shown in Table 3. Fig. 4 illustrates the difference between the frequencies by modes obtained from optimum design using IML algorithm and those using the PML algorithm. As previously described in the formulation procedure, the proper type of restricted zone for frequency constraints should be set up in an appropriate way depending on the cases. In this case, the restricted frequency zone ranges from 15 to 20 rad/sec. The PML algorithm generally fixes the type of restricted zone by referring to the frequencies at the modes obtained from the initial dynamic analysis. The frequencies computed by the PML algorithm are 10.01, 25.87, and 41.10 at the modes, respectively, from the 1st to the 3rd with the first case of initial value. Then, the frequency at 2nd mode is forced to be larger than 20 while the frequency at the 1st mode is forced to be smaller than 15, which would become a frequency constraint condition of the optimum design process. With the 2nd case of initial value, the frequencies by the PML are 6.00, 15.23, and 23.37 at from the 1st to the 3rd mode. Then, the frequency at the 2nd mode, 15.23, is forced to be smaller than 15 while the frequency at the 3rd mode is forced to be larger than 20, which would become a frequency constraint condition of the optimum design process. However, in the case of the IML algorithm the optimization is carried out for all constraints except a frequency constraint, and then the feasibility for frequency constraint is checked. If feasibility is not satisfied with frequency constraint, type of restricted frequency zone is set up at the current point. Therefore, regardless of initial values, it gives definitely consistent type of restricted frequency zone.

Table 4 Results of optimum design of IML and PML algorithm ($w_1=0.5$, $w_2=0.5$)

Algorithm	Building story	Moment of inertia(cm ⁴)	h _w (cm)	b _f (cm)	T _f (cm)	T _w (cm)	Weight (kg)	Energy (N-m)	Number of analysis
PML algorithm Initial values 1	1	33053.00	44.18	19.53	1.24	1.11	1322.26	1618.35	1176
	2	19921.99	38.65	18.31	1.06	0.95			
	3	12097.77	32.92	16.88	1.01	0.75			
	4	7066.21	27.17	15.51	1.00	0.51			
	5	3723.33	24.42	8.37	1.00	0.81			
	6	2654.06	22.33	7.14	1.00	0.74			
	7	4748.31	25.89	9.59	1.00	0.85			
	8	3135.88	23.28	7.79	1.00	0.77			
PML algorithm Initial values 2	1	30603.28	43.35	18.64	1.25	1.09	1156.86	1559.31	892
	2	17975.76	37.73	17.77	1.03	0.93			
	3	10930.58	31.53	17.31	1.01	0.67			
	4	6276.43	29.05	10.50	1.00	0.74			
	5	3203.52	23.25	8.03	1.01	0.77			
	6	1511.48	19.07	5.62	1.00	0.63			
	7	1297.66	18.36	5.19	1.00	0.61			
	8	1476.51	18.74	5.83	1.00	0.61			
IML algorithm Initial values 1	1	30627.88	43.34	18.99	1.23	1.09	1155.95	1558.42	8
	2	18235.08	38.16	17.60	1.02	0.96			
	3	10926.49	31.40	17.29	1.01	0.67			
	4	6272.87	29.03	10.50	1.00	0.74			
	5	3232.81	23.46	7.93	1.00	0.78			
	6	1521.96	18.90	5.89	1.00	0.61			
	7	1297.13	18.35	5.19	1.00	0.61			
	8	1476.08	18.72	5.83	1.00	0.61			
IML algorithm Initial values 2	1	30665.55	43.40	19.04	1.23	1.09	1157.37	1560.34	7
	2	18257.51	38.20	17.62	1.02	0.96			
	3	10939.93	31.44	17.31	1.01	0.68			
	4	6280.59	29.06	10.51	1.00	0.74			
	5	3236.79	23.48	7.94	1.00	0.78			
	6	1523.83	18.93	5.89	1.00	0.61			
	7	1298.72	18.38	5.19	1.00	0.61			
	8	1477.90	18.75	5.84	1.00	0.61			

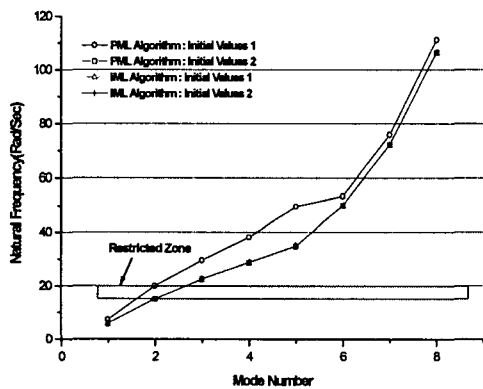


Fig. 4 Frequency at each mode

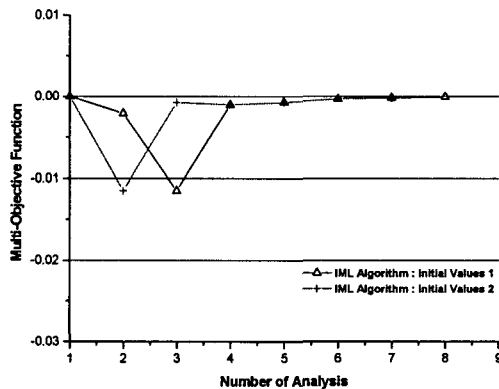


Fig. 5 Convergence of multi-objective function

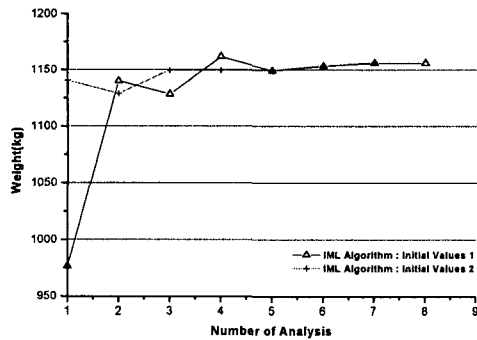


Fig. 6 Convergence of weight

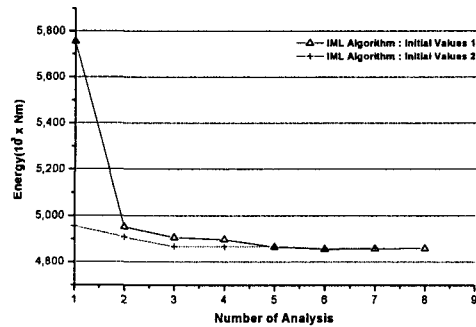


Fig. 7 Convergence of strain energy

From the convergence of the optimum solution in Fig.5, Fig. 6 and Fig.7, the essential features of the proposed algorithm are comparatively discussed with emphasis on robustness of the algorithms. For each optimization, the convergence history of multi-objective function is presented in Fig. 5. The results from optimum design using the IML algorithm with both cases of initial values commonly show the convergence to zero(0) value of the objective function. The weight factors, w_1 and w_2 , in the multi-objective function of the optimum design are 0.5 and 0.5, respectively and the weight and strain energy functions in the multi-objective function are normalized. When focused on the weight and the strain energy, the results from the two algorithms with the 1st case of initial values are completely different, which is mainly due to the difference in the type of restricted zones of the two algorithms. If the initial values are far away from the optimum, then the type of frequency constraint in the algorithm is not reasonably selected and thus a real optimum solution cannot be obtained. However, the IML algorithm by using artificial constraint deletion can always produce reasonable type of frequency constraint. Therefore, since reasonable optimum results can be obtained from the IML algorithm regardless of initial values, it may be argued that the IML is more robust than the PML.

An efficient algorithm could be defined as the one that reaches the convergence within a reasonable number of analysis which is also important factor to prove superiority of the algorithm. In order to demonstrate the efficiency of the IML algorithm, the number of analysis of the algorithm are compared with those of the PML algorithm. Table 4 shows the IML algorithm needs less number of analysis than the PML algorithm does. The IML algorithm using structural dynamic responses such as bending moments, frequencies, and strain energy under seismic load in term of intermediate variable implements only one analysis at each step of iteration. The PML algorithm requires much more numbers of structural analyses that has a great effect on computational cost in order to evaluate the objective function and constraint at each step of iteration. It may be noted that the IML algorithm (7~8) could drastically reduce the number of reanalysis, compared with the PML algorithm (892~1176). Moreover, noting that the more a large scale structure is to be optimized, the more efficient optimization algorithm is required, thus, it may be stated that the IML algorithm is expected to be a lot more efficient in the optimization of these large-scale structures, compared with the available algorithm.

5. Conclusion

In the paper, An IML optimization algorithm using automatic differentiation(AD) for multi-objective optimum design of framed structures is proposed. To optimize steel frames under seismic load, structural dynamic responses usually require bending moments, natural frequencies, and the structural strain energy for constraint and objective function. In order to increase the computing efficiency, optimization is achieved by approximating all the structural responses. using the intermediate variables that represent characteristic of each structural response. Sensitivity analysis of structural dynamic response in terms of intermediate variables is executed by an AD technique for accurately and efficiently computing derivatives with minimal human effort. In the IML algorithm, an artificial constraint deletion technique is also introduced to increase robustness and efficiency of the algorithm by overcoming the drawback of the frequency constraints in the PML algorithm.

In order to demonstrate robustness and efficiency of the IML algorithm, an eight-story, one-bay steel frame is used as the numerical example, and the optimum results of the proposed algorithm are compared with those of the PML algorithm. Based on the results of the application example, it may be concluded that the IML algorithm is considerably more efficient and robust than the PML method. Therefore, it may be argued that the IML algorithm proposed in this study may be successfully applied to large-scale structural optimization problems with robustness and efficiency.

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