점탄성 모델을 도입한 사출/압축 성형공정의 준 3차원 수치해석

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Quasi-Three-Dimensional Viscoelastic Simulation of Injection/Compression Molding Process

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Introduction

In an injection molded optical products such as CD, DVD and various lenses, residual stresses and birefringence are of great importance because of dimensional accuracy and optical preformance, respectively. In such optical products, injection/compression molding process have many advantages over the injection molding process since injection/compression molding process can reduce the birefringence compared with injection molding process. Kwon et al.[3] have studied the birefringence distribution in injection/compression molded center-gated disk based on Leonov model. In the present study, the numerical analysis system to predict the residual stresses and birefringence in the injection/compression molded products of general 3-Dim thin part geometry such as lens was developed based on the same physical modeling in Kwon et al.[3]. We verified the developed numerical system by comparing the numerical results with those by Kwon et al. in the center-gated disk and applied the system to convex lens to predict the birefringence in injection/compression process.

Physical Modeling

For the creeping flow in the injection molding filling, compression and packing stages of three dimensional thin part geomety, the Hele-Shaw approximation is valid and the governing equations[1] such as mass, momentum and energy conservations can be reduced to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0 \tag{1}$$

$$-\frac{\partial p}{\partial x}+\frac{\partial \tau_{xz}}{\partial z}=0,-\frac{\partial p}{\partial y}+\frac{\partial \tau_{yz}}{\partial z} = 0,-\frac{\partial p}{\partial z}+\frac{\partial \tau_{zz}}{\partial z}=0 \tag{2}$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\tau_{xz}\frac{\partial u}{\partial z}+\tau_{yz}\frac{\partial v}{\partial z} \tag{3}$$

To take into account the flow-induced residual stresses and birefringence, Leonov model[2] is employed as a nonlinear viscoelastic fluid constituive equation as follows.

$$\boldsymbol{\tau} = 2\eta_0 s\boldsymbol{d} + \sum_{k=1}^{N} \frac{\eta_k}{\theta_k}\boldsymbol{c}_k \tag{4}$$

$\boldsymbol{\tau}$ is a stress tensor except the isotropic pressure term, $\boldsymbol{d}$ is a rate of deformation tensor, $\boldsymbol{c}_k$ is the k-th mode Finger strain tensor representing the elastic deformation of polymeric material. Meanwhile, stress-optical law was applied to predict the birefringence, namely,

$$n_i - n_j = C(\sigma_i - \sigma_j) \tag{5}$$

where C is the stress-optical coefficient, n_i the refractive index and σ_i the principle stress.

Numerical Modeling

Applying the constitutive equation to momentum equations(2) and continutiy equation(1) one can obtain the following equation for σ_{zz}.

$$\frac{\partial}{\partial x}\left(\mathcal{S}\,\frac{\partial \sigma_{zz}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mathcal{S}\,\frac{\partial \sigma_{zz}}{\partial y}\right) + \frac{\partial F^x}{\partial x} + \frac{\partial F^y}{\partial y} = -\dot{b} \tag{6}$$

In equation(6) $\mathcal{S}$, F^x and F^y are defined as follows.

$$\begin{aligned}
\mathcal{S} &= \frac{1}{s}\int_0^b \frac{z^2}{\eta_0}dz \\
F^x &= bu_s - \frac{A}{s}\int_0^b \frac{z}{\eta_0}dz + \frac{1}{s}\sum_{k=1}^{N}\frac{\eta_k}{\theta_k}\left[\int_0^b \frac{z\, c_{xz,k}}{\eta_0}dz - \int_0^b \frac{z}{\eta_0}\int_0^z \frac{\partial c_{zz,k}}{\partial x}d\tilde{z}dz\right] \\
F^y &= bv_s - \frac{B}{s}\int_0^b \frac{z}{\eta_0}dz + \frac{1}{s}\sum_{k=1}^{N}\frac{\eta_k}{\theta_k}\left[\int_0^b \frac{z\, c_{yz,k}}{\eta_0}dz - \int_0^b \frac{z}{\eta_0}\int_0^z \frac{\partial c_{zz,k}}{\partial y}d\tilde{z}dz\right]
\end{aligned}$$

where b is the current thickness of cavity, u_s, v_s is the slip velocity by geometric condition[4] and $A = \tau_{xz}(0)$, $B = \tau_{yz}(0)$ is each shear stress of x, y direction on the lower cavity surface. Also the energy equation can be restated as follows.

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \eta_0 s\dot{\gamma}^2 + \sum_{k=1}^{N}\frac{\eta_k}{\theta_k}\left(C_{xz}\frac{\partial u}{\partial z} + C_{yz}\frac{\partial v}{\partial z}\right) \tag{7}$$

where $\dot{\gamma} = \sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}$.

Based on the numerical modeling described above, we have developed a numerical analysis ststem using FEM for general 3-Dim thin part geometry.

In addition, by including the packing stage which has been already developed in the previous study[5] injection/compression molding process was simulated to predict the flow-induced residual stresses and birefringence.

Results and Disscussion

The numerical analysis ststem developed in the present study was applied to the center-gated disk. Fig. 1 shows the geometry and mesh data of center-gated disk with the diameter of 10.16cm and thickness of 2.0mm. Processing conditions are as follows : flow rate is 23.8cm^3/sec, melt temperature 220^oC and mold temperature 40^oC. Fig. 2 shows the predicted distribution of birefringence by the injection process without a packing stage. And shown in Fig. 3 are the distribution of birefringence by the injection/compression process which have the following processing conditons : polymer melt is injected into the cavity until the melt front reaches the radius of 3.5cm with an initial cavity thickness 4.25mm and a flow rate of 23.8cm^3/sec and the subsequent compression stage started with a closing velocity of 1.0cm/sec. As shown in Figs. 2 and 3, the birefringence by the injection/compression process is greatly reduced as compared with injection molding process. In this respect, the injection/compression process is more suitable for manufacturing the center-gated disk. The numerical results of birefringence at the several radial locations based on quasi 3-Dim analysis(Fig. 3) have been compared with the corresponding ones, Fig. 4, obtained from 2-Dim analysis for the same processiong condition. Fig. 3 and 4 show that quasi-three dimensional analysis results coincide well with those by 2-Dim analsysis, which validates the quasi-three dimensional analysis program.

The quasi-3D numerical analysis was then performed for a lens product which has variable thickness and non-shperical surface as shown in Fig. 5. Fig. 6 shows the gap-wise distribution of birefringence at the several locations in the injection/compression process including a packing stage. Inner peaks showed up by the additional flow during packing stage in the injection/compression process. Also birefringence at the thickest center point of C is smallest because of small deformation due to small velocity gradient and fast relaxation due to high temperature in the region. However, at the points of A and E which have smaller thickness birefringence becomes higher since the higher birefringence developed by the fast flow during cavity filling and packing stages and is frozen by the fast cooling due to small thickness.

References

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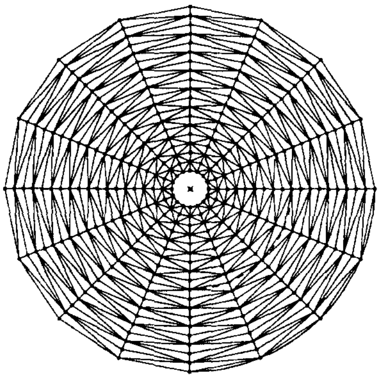

Fig. 1. Disk mesh

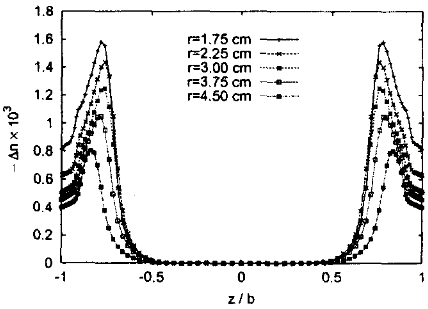

Fig. 2. Birefringence (Injection only)

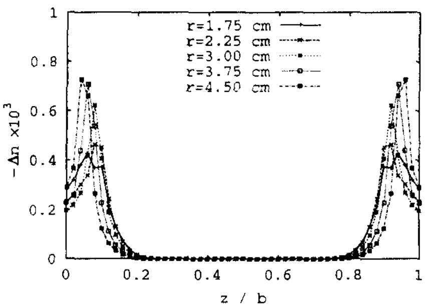

Fig. 3. Injection/compression (2.5D)

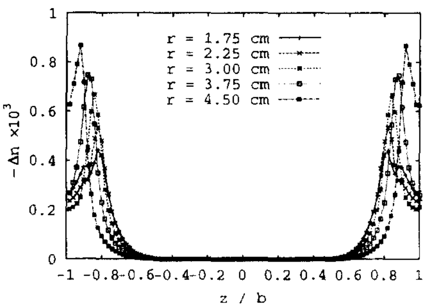

Fig. 4. Injection/compression (2D)

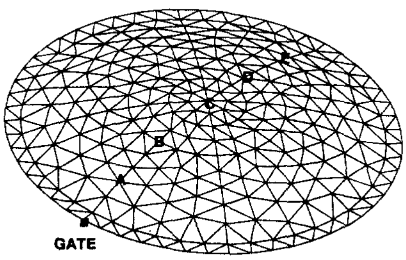

Fig. 5. Lens mesh

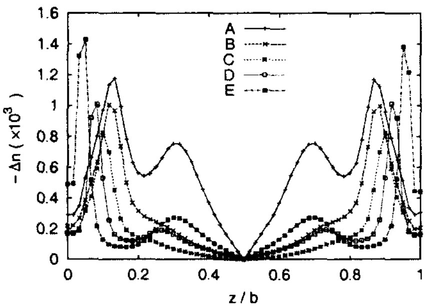

Fig. 6. Birefringence (Incluing packing)