Vibration Analysis of Three Span Continuous Reinforced Concrete Bridge with Elastic Intermediate Supports II

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ABSTRACT

A method of calculating the natural frequency corresponding to the modes of vibration of beams and tower structures, with irregular cross sections and with arbitrary boundary conditions was developed and reported by Kim, D. H. in 1974. In this paper, the result of application of this method to the three span continuous reinforced concrete bridge with elastic intermediate supports is presented. Such bridge represents either concrete or sandwich type three span bridge on polymeric supports for passive control or on actuators for active control.

The concrete slab is considered as a special orthotropic plate.

The influence of the modulus of the foundation and D_{22}, D_{12}, D_{66} stiffnesses on the natural frequency is thoroughly studied.

1. INTRODUCTION

The problem of deteriorated highway concrete slab is very serious all over the world. Before making any decision on repair work, reliable non-destructive evaluation is necessary. One of the dependable methods is to evaluate the in-situ stiffness of the slab by means of obtaining the natural frequency. By comparing the in-situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

Recently, use of polymeric bridge supports has become quite popular. Unlike the metal hinges and rollers, these polymers behave like elastic support. The actuators for the active control of the bridge behave, at least partially, as the elastic supports. The reinforced concrete slab can be assumed as a special orthotropic plate, as a close approximation, assuming that the influence of B_{16}, B_{26} D_{16}, and D_{26} stiffnesses are negligible.

2. METHOD OF ANALYSIS

Vibration Analysis

In case of a laminated composite plate with boundary conditions other then Navier or Levy solution types, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical method for engenvalue problems are also very much involved in seeking such a solution.

A simple but exact method of calculating the natural frequency corresponding to the mode of vivration of beam and tower structures with irregular cross-sections and attatched mass/masses was developed and reported by Kim, D. H. in 1974. Recently, this method was extended to two dimensional problems including composite laminates, and has been applied to composite plates with various boundary conditions with/without shear deformation effect and reported at several international conferences including the Eighth Structures Congress(1990) and Fouth materials congress(1996) of American Society of Civil Engineers.

In this paper, the result of application of this method to the subject porblem is presented.

Since the method of analysis used for this paper is given, in detail, in the senior author's book(Kim, 1995), it is not repeated here.

Finite Difference Method

The method used in this paper requires the deflection influence surfaces. Since no reliable analytical method is available for the subject porblem, F. D. M. is applied to the governing equation of the special orthotropic plates.

The number of the pivotal points required in the case of the order of error $\triangle^2$, where $\triangle$ is the mesh size, is five for the central differences of the fourth order single derivative terms. This makes the procedure at the boundaries complicated. In order to solve such problem, the three simultaneous partial differential equations of equilibrium with three dependent variables, w, Mx, and My, are used instead of the ordinary partial differential equation for the bending of the special orthotropic plate.

$$D_1 \frac{\partial^2 Mx}{\partial x^2} - 4D_{66}\frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^2 My}{\partial y^2} = -q(x,y) + kw(x,y) \quad (1)$$

$$M_x = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} \quad (2)$$

$$M_y = -D_{12}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2} \quad (3)$$

If F. D. M. is applied to these equations, the resulting matrix equation is very large in sizes, but the tridiagonal matrix calculation scheme used by Kim, D. H.(1967) is very efficient to solve such equations.

In order to confirm the accuracy of the F.D.M., $[A/B/A]_r$ type laminate with aspect ratio of a/b=1m/1m=1 is considered.

For simplicity, it is assumed that $A=0^0$, $B=90^0$, and r=1. Since one of the few efficient analytical solutions of the special orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, F.D.M. is used to solve this problem and the result is compared with the Navier solution.

Calculation is carried out with different mesh sizes and the maximum errors at the center of the plate are as follows.

10×10 *case* : 0.14%

20×20 *case* : 0.035%

40×40 *case* : 0.009%

The error is less than 1%. This is smaller than the predicted theoretical errors ;

3. Numerical Examination

Structure Under Consideration

The bridge considered is as shown in Figure 1.

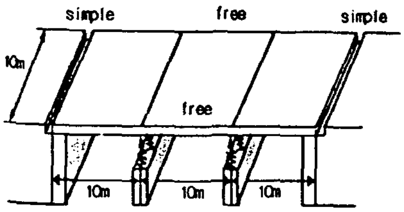


Figure 1. Three span continuous slab bridge

The location of the truck loading is as shown in Figure 2.

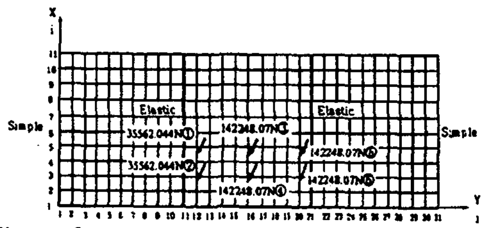


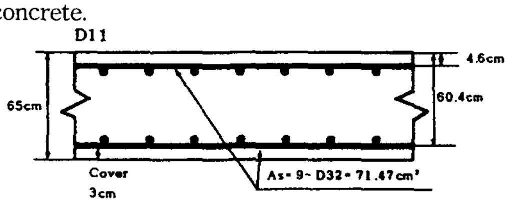
Figure 2. Location of truck loading

Figure 3 shows the cross section of the slab with unit width.

$\sigma_{ck}=210kg/cm^2=20.5942926MPa$ and

$E_c=15000\sqrt{\sigma_{ck}}=21.317118060GPa.$

Poissons ratio ν_{12} = ν_{21} = 0.18 for concrete.



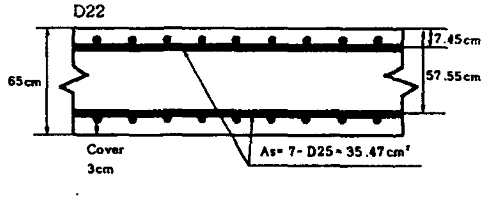


Figure 3. Cross section of the slab with unit width

Three different concepts are adopted for obtaining the stiffnesses, D_{ij}. For all cases, the effect of the bending extension coupling stiffness, B_{ij}, is assumed as negligible.

Case 1. Balanced design using the transformed area for steel in calculating the moment of inertia of the cross-section.

Case 2.

With $E_c=15000\sqrt{\sigma_{ck}}=21.317118060GPa$ and $E_s=199.92GPa$, and with concrete $Q_{11}=E_c/(1-\nu_{12}^2)$ and steel $Q_{11}=E_s$, the typical formulas for D_{ij} are used.

Case 3. Using the cracked section concept by the maximum moment, the moment of inertia of the cross section is obtained to calculate D_{ij}.

Table 1. shows the flexural stiffnesses of three cases. For all cases, the uncracked section is used to obtain D_{66} and the concrete dead weigth is,

$2.5t/m^3\times0.65m$

$=15925$ *Pa.*

Table 1. Flexural stiffnesses of three cases. (N · m)

Case / stiffness	Case1	Case2	Case3
D_{11}	351761502.8	323428383.7	323416426.7
D_{22}	155665708.1	151828300.8	151827047.8
D_{12}	90690632.4	90690632.4	90690632.4
D_{66}	206573097.2	206573097.2	206573097.2

Numerical Result

The deflections at the wheel load points for three cases, when the molulus of foundation, $k=14504\times10^{6}N/m^{2}$, are given in Table 2.

Table 2. The deflections at wheel loading points for three-cases (Unit : m)

Case / load point	Case1	Case2	Case3
1	0.2786E-03	0.2955E-03	0.2955E-03
2	0.2314E-03	0.2458E-03	0.2458E-03
3	0.2132E-02	0.2300E-02	0.2300E-02
4	0.1901E-02	0.2054E-02	0.2054E-02
5	0.3900E-03	0.4155E-03	0.4155E-03
6	0.3288E-03	0.3504E-03	0.3504E-03

Table 3 shows the natural frequencies of three-cases, under the same value of $k=14504\times10^{6}N/m^{2}$.

Table 3. The natural frequencies for three-cases (Unit : rad/sec)

Case	Natural Frequency (rad/sec)
Case 1	12.92903
Case 2	12.33828
Case 3	12.33805

The influence of the modulus of foundation, k, is studied by changing k values from $14504\times10^{3}N/m^{2}$ to $14504\times10^{7}N/m^{2}$. Table 4 shows the natural frequencies for Case 2, under changing values of k.

Table 4. The natural frequency for Case 2-1 (Unit : rad/sec)

$k(N/m^{2})$	Natural Frequencies (rad/sec)
14504×10^{3}	0.8068337E+01
14504×10^{4}	0.1223924E+02
14504×10^{5}	0.1232987E+02
14504×10^{6}	0.1233830E+02
14504×10^{7}	0.1233943E+02

4. Conclusion

This paper illustrates better and rational method of analysis of the reinforced concrete bridge by the use of the special orthotropic plate theory. The bridge is three span continuous structure with elastic intermediate supports.

Vibration analysis of such structure is complicated but the simple and accurate method of vibration analysis developed by the senior author is used to solve such problem with relative ease. The effects of the stiffnesses and the moduli of foundation are thoroughly studied.

5. References

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