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Numerical Solution of the Radiation Problem by the B-Spline Higher Order Kelvin Panel Method for a Half-Immersed Cylinder in Wave and Current

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ABSTRACT: The improved Green integral equation of overdetermined type applied to the radiation problem for an oscillating cylinder in the presence of weak current is presented. A two-dimensional Green function for the weak current is also presented. The present numerical solution of the improved Green integral equation by the B-spline higher order Kelvin panel method is shown to be free of irregular frequencies which are present in the usual Green integral equation.

1. Introduction

When the current is present, the wave field is modified and it is difficult to formulate the potential problem for the radiation wave due to an oscillating body. In this study, it is assumed that the wave is transferred by the current without deformation. Under the assumption, the boundary value problem for the radiation potential in the presence of the uniform horizontal current is equivalent to the so called, forward speed radiation problem for a ship advancing in waves. In this paper, the two-dimensional boundary value problem for the radiation potential with weak current is solved by making use of the improved Green integral equation of overdetermined type(Hong and Lee 1999) and using the two-dimensional Kelvin-type Green function.

2. Linearized Boundary-Value Problem

A body is oscillating in the free surface of deep water under gravity and in the presence a horizontal current with uniform speed U. The magnitude U^2 is assumed to be of $O(\varepsilon)$ where ε, being as small as the wave slope, is the measure of smallness. Let oxy be a Cartesian co-ordinate system attached to the mean position of the cylindrical body, with y vertically upward, x in the negative direction of the current velocity and o in the mean waterplane W. The body performs simple harmonic oscillations of small amplitude about its mean position with circular frequency ω.

With the usual assumptions of the incompressible, inviscid fluid and irrotational flow without capillarity, the fluid velocity $\vec{v}$ can be given by the gradient of a velocity potential Φ which satisfies the Laplace equation $\nabla^2\Phi = 0$, in the fluid region. Here, the potential at P in the fluid region can be decomposed as follows:

$$\Phi(P,\ t) = \Phi_S(P) - Ux + Re\{\Psi(P)e^{-i\omega t}\} \tag{1}$$

where Φ_S denotes a steady potential due to the presence of the body in the current, Ψ a complex-valued unsteady potential. The magnitude of both Φ_S and Ψ is of $O(\varepsilon)$.

The free surface boundary condition on the mean free surface $y = 0$ is as follows:

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)p = 0 \quad on \quad y = 0 \tag{2}$$

where p denotes the pressure :

$$p = -\rho\left(\frac{\partial \Phi}{\partial t} + \frac{v^2}{2}\right) \tag{3}$$

Substituting (1) and (3) into (2), the following free surface boundary conditions for Φ_S and Ψ can be found respectively:

$$\left[U^2\frac{\partial^2}{\partial x^2} + g\frac{\partial}{\partial y}\right]\Phi_S = 0 \quad \text{on} \quad y = 0 \tag{4}$$

$$\left[(-\omega^2 + 2i\omega U\frac{\partial}{\partial x} + g\frac{\partial}{\partial y}\right]\Psi = 0 \quad on \quad y = 0 \tag{5}$$

Under the assumption of small amplitude oscillation, the displacement vector $\vec{A}(M)$ of a point M on the wetted surface S of the body at its mean position can be expressed as follows:

$$\vec{A}(M) = Re\{\vec{\alpha}(M)e^{-i\omega t}\}, \qquad M \in S \tag{6}$$

$$\vec{\alpha}(M) = \sum_{k=1}^{2} a_k\vec{e}_k + a_3\vec{e}_3 \times \overrightarrow{OM}, \qquad M \in S \tag{7}$$

where $a_k(k=1,2,3)$ denotes complex valued amplitude of sway, heave, roll respectively and O the center of rotation of the body.

Applying the impermeability condition on S, the following body boundary condition can be found:

$$\begin{aligned}(\vec{n} + a_3\vec{e}_3 \times \vec{n}) \cdot \nabla(\Phi_S - Ux + \Psi) \\ = (\vec{n} + a_3\vec{e}_3 \times \vec{n}) \cdot (-i\omega\vec{\alpha})\end{aligned} \tag{8}$$

where $\vec{n}$ denotes a unit normal to S directed into the fluid region, at its mean position and $(\vec{n} + a_3\vec{e}_3 \times \vec{n})$ the Taylor expansion of the normal at its instantaneous position.

Neglecting second-order quantities, the following linearized body boundary condition for Φ_S and Ψ can be found respectively:

$$\frac{\partial \Phi_S}{\partial n} = Un_1 \quad on \quad S \tag{9}$$

$$\frac{\partial \Psi}{\partial n} = -i\omega\vec{\alpha} \cdot \vec{n} + Ua_3n_2 \quad on \quad S \tag{10}$$

With these linearized boundary conditions on S and on $y=0$, the unsteady potential and the steady potential problems can be solved independently and the latter will be dropped from the present study.

The unsteady potential is known as the radiation potential Ψ_R which can be decomposed as follows:

$$\Psi_R = -i\omega\sum_{k=1}^{3} a_k\Psi_k - Ua_3\Psi_2 \tag{11}$$

Then the body boundary conditions for $\Psi_k(k=1,2,3)$ are

$$\frac{\partial \Psi_k}{\partial n} = n_k \quad on \quad S, \qquad k = 1,2 \tag{12}$$

$$\frac{\partial \Psi_3}{\partial n_0} = (\vec{e}_3 \times \overrightarrow{OM}) \cdot \vec{n} \quad on \quad S \tag{13}$$

The potentials $\Psi_k(k=1,2,3)$ must also satisfy the free surface boundary condition (5) and the radiation condition at infinity.

3. Improved Green Integral Equation

Since we have assumed that U^2 is of $O(\varepsilon)$, the two-dimensional Kelvin-type Green function(Haskind 1954) can be simplified as follows:

$$\begin{aligned}G(z_P, z_M) = \frac{1}{2\pi}Re\left\{\log\left(\frac{z_P - z_M}{z_P - \bar{z}_M}\right)\right. \\ \left. + I_1 + I_2\right\} + i\frac{1}{2\pi}\ Im\{I_1 - I_2\}\end{aligned} \tag{14}$$

with

$$I_1 = \frac{-1}{1+2\gamma}e^{\zeta_1}[Em_1(\zeta_1) + 2i\pi] \tag{15}$$

$$I_2 = \frac{-1}{1-2\gamma}e^{\zeta_2}Em_1(\zeta_2) \tag{16}$$

$$\zeta_j = -ik_j(z_P - \bar{z}_M), \quad j = 1,2 \tag{17}$$

$$k_j = \frac{\omega^2}{g[1-(-1)^j 2\gamma]}, \quad j = 1,2 \tag{18}$$

where $z = x + iy$ denotes the complex plane, g the gravitational acceleration and z_P, z_M the source and field points respectively. $\gamma = u\omega/g$ is known as the Brard number. $Em_1(\zeta)$ is the modified complex exponential integral defined as follows;

$$Em_1(\zeta) = \begin{cases} E_1(\zeta) & \text{for} \quad Im(\zeta) > 0 \\ E_1(\zeta) - 2i\pi & \text{for} \quad Im(\zeta) < 0 \end{cases}$$

Discretization of the body surface in (26) into a set of curvilinear segments s, $(s=0,1,2,,,N^S-1)$, will then yield (Lee and Kerwin 1999):

$$\alpha(P)\ \sum_j \Psi_j^v N_j + \sum_{s=0}^{N^S-1}\sum_{j=0}^{N^v-1} \Psi_j^v \int_S N_j \frac{\partial G}{\partial n}\, dl$$
$$- 2i\nu[(\Psi^v G)_C - (\Psi^v G)_D] \qquad (28)$$
$$= \sum_{s=0}^{N^S-1} \int_S \frac{\partial \Psi}{\partial n} G dl\ ,\ P \in S \cup W$$

The number of unknown potential vertices N^v is greater than the number of the curvilinear segments or panels since $N^v = N^S + p$ according to the properties of the B-spline basis functions. Since the equation (26) is overdetermined, we can place any number of control points N^P on $S \cup W$ which is greater than or equal to N^v. This linear system will be solved by the usual Gauss elimination when $N^P = N^v$ and by a least square approach when $N^P > N^v$.

5. Numerical Results and Discussion

The hydrodynamic pressure forces due to the unsteady potential can be obtained as

$$\vec{F} = -\int_S p\, \vec{n}\, dl \qquad (29)$$

Substituting (3) and (8) into (29) and introducing non-dimensional added-mass and wave-damping coefficients M_{ik} and D_{ik}, we have the following expression for F_i :

$$F_i = -\rho A \sum_{k=1}^{3} [M_{ik}\, \ddot{a}_k + \omega D_{ik}\, \dot{a}_k],\ i=1,2 \qquad (22)$$

where A denotes the sectional area of the cylinder.
The hydrodynamic coefficients of a half immersed circular cylinder are computed for various Froude numbers, $F_n = u/\sqrt{gD}$, based on the diameter of the circle $D=1$. They are all plotted as functions of $K = \omega\sqrt{D/g}$.
After extensive numerical tests for the convergence, 30 higher order panels with 60 control points in S are employed as well as 38 control points in W to show the present numerical values for two different Froude numbers: $F_n = 0.05$ and $F_n = 0.099$.

We have found numerically that the irregular frequencies exist in the solution of the Green integral equation (22) denoted by GIE while the solution of the improved Green integral equation (26) denoted by IGIE, is free of the irregular frequencies as shown Figure (1).

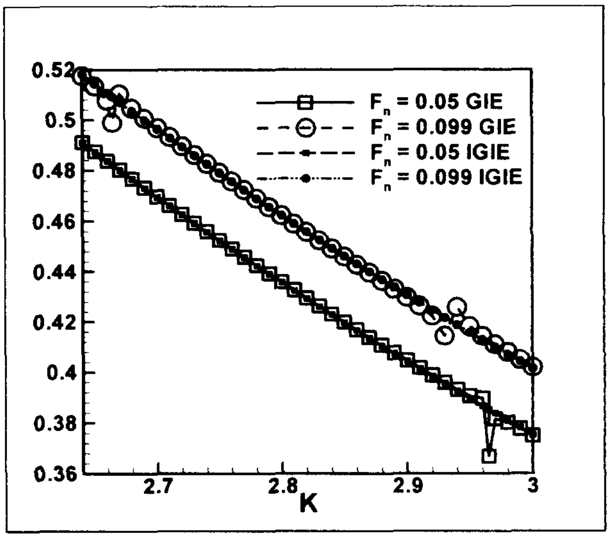

Fig. 1 Irregular frequencies in the sway wave-damping coefficients

The hydrodynamic coefficients computed by making use of the improved Green integral equation has been presented in Figures (2) and (3).

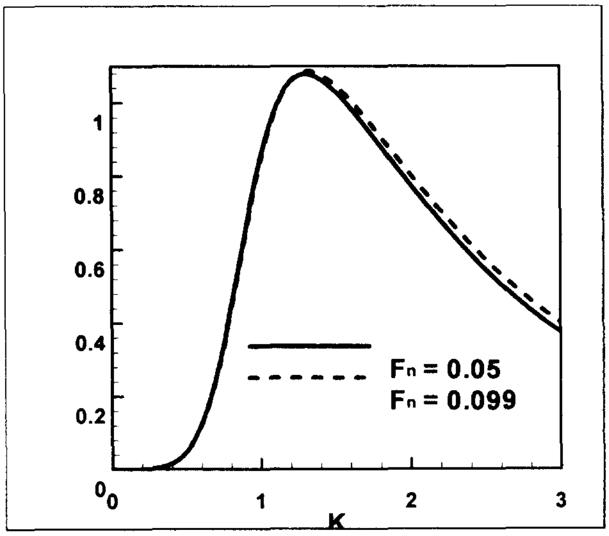

Fig. 1 Sway wave-damping coefficients

where $E_1(\zeta)$ is the complex exponential integral.
It should be noted that the present simplification is valid for $\nu \le 1/2$.
The above Green function $G(P,M)$ satisfies the following equations:

$$\nabla_P^2 G(P,M) = 0 \quad \text{for} \quad P \neq M \tag{19}$$

$$\left(-\omega^2 + 2i\omega U\frac{\partial}{\partial x_P} + g\frac{\partial}{\partial y_P}\right) G(P,M) = 0 \quad \text{for} \quad y_M < 0 \quad \text{and} \quad y_P \le 0 \tag{20}$$

The adjoint free surface condition for $G(P,M)$ is as follows :

$$\left(-\omega^2 - 2i\omega U\frac{\partial}{\partial x_P} + g\frac{\partial}{\partial y_M}\right) G(P,M) = 0 \quad \text{for} \quad y_P < 0 \quad \text{and} \quad y_M \le 0 \tag{21}$$

Applying Green's theorem to the potential Ψ and the Green function G over the fluid region D_e, the following Green integral equation for weak current can be obtained :

$$\frac{\Psi}{2} + \int_S \Psi \frac{\partial G}{\partial n} dl - 2i\nu [(\Psi G)_C - (\Psi G)_D] = \int_S \frac{\partial \Psi}{\partial n} G dl \quad on \quad S \tag{22}$$

Here, the suffixes C and D denote the two intersecting points of S and $y=0$ where two-dimensional line integrals are to be calculated(Hong 2000).
According to the theory of integral equation, an integral equation must contain all the boundary conditions of the boundary value problem in question. But, it seems that some boundary conditions are missing in equation (22). Let the surface in contact with the fluid be the positive side of the boundary surface and the other side of the same surface outside D_e the negative side of the surface. The wetted surface will be denoted by S^+ hereafter. According to the potential theory, the potential jump across S which has been incorporated in (22) implies that the condition $\Psi=0$ is imposed on S^-, the negative side of S. In fact, it was necessary to impose $\Psi(P)=0$ when P lies on the negative side of the free surface F_e as it is done in the Green integral equation with the Rankine-type Green Function. Thus, in order to ensure the uniqueness of the solution, it is necessary to impose the following condition:

$$\Psi = 0 \quad on \quad F_e^- \tag{23}$$

The condition in the infinity can be omitted since Ψ vanishes there. But, since the integral over the boundary surface F_e was already replaced by line integrals, it is not desirable to reintroduce F_e into the present Green integral equation. Instead, let us impose the following supplementary condition for Ψ which can compensate for the condition (23):

$$\Psi(P) = 0 \quad \text{for} \quad P \in W \tag{24}$$

This condition is equivalent to

$$\int_S \Psi \frac{\partial G}{\partial n} dl - 2i\nu [(\Psi G)_C - (\Psi G)_D] = \int_S \frac{\partial \Psi}{\partial n} G dl \quad on \quad W \tag{25}$$

Combining the equation (25) with (22), we have the following integral equation, say, the improved Green integral equation of overdetermined type:

$$\alpha(P)\Psi(P) + \int_S \Psi(M) \frac{\partial G(P,M)}{\partial n_M} dl_M - 2i\nu [(\Psi(M)G(P,M))_C - (\Psi(M)G(P,M))_D] = \int_S \frac{\Psi(M)}{\partial n_M} G(P,M) dl_M \tag{26}$$

where

$$\alpha(P) = \begin{cases} \frac{1}{2} & \text{for} \quad P \in S \\ 0 & \text{for} \quad P \in W \end{cases} \tag{26a}$$

4. B-Spline Higher Order Panel Method

We will represent the potential as a weighted sum of B-spline basis functions as follows;

$$\Psi = \sum_{j=0}^{N^v-1} \Psi_j^v N_j(u) \quad on \quad S \cup W \tag{27}$$

where $N_j(u)$ are the p-th degree B-spline basis functions, Ψ_j^v the potential control vertices.

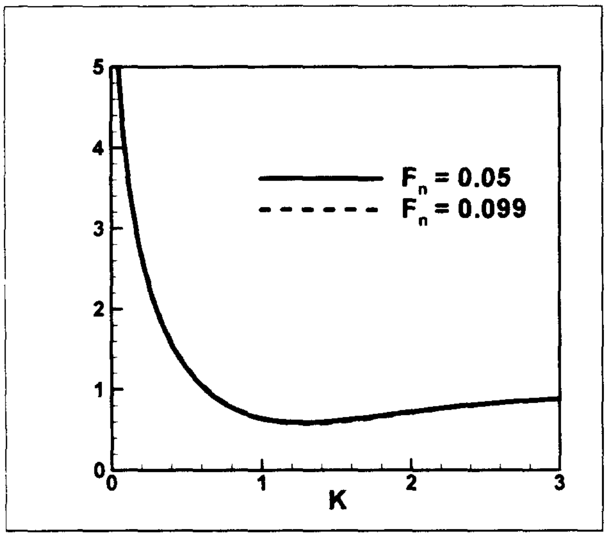

Fig.3. Heave added-mass coefficients

It should be noted that the improved Green integral equation which was presented by Hong(2000) without numerical validation is different from the improved Green integral equation of overdetermined type presented by (26). The supplementary boundary condition for Ψ on W presented by Hong(2000) was not complete.

6. Conclusions

1. An improved Green integral equation of overdetermined type as well as a Kelvin-type Green function for the radiation potential due to an oscillating surface-piercing cylinder in the presence of weak current, have been presented.

2. It has been shown that there exist irregular frequencies in the solution of the two-dimensional forward-speed Green integral equation for a surface-piercing cylinder.

3. The solution of the improved Green integral equation is shown to be free of irregular frequencies.

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