이산시간 TS 퍼지 시스템의 추종 제어기 설계
Output Tracking Controller Design of Discrete-Time TS Fuzzy Systems

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Abstract

In this paper, an output tracking control technique of discrete-time Takagi-Sugeno (TS) fuzzy systems is developed. The TS fuzzy system is represented as an uncertain multiple linear system. The tracking problem of TS fuzzy system is converted into the stabilization problem of a uncertain multiple linear system. A sufficient condition for asymptotic tracking is obtained in terms of linear matrix inequalities (LMI). A design example is illustrated to show the effectiveness of the proposed method.

1 Introduction

Many frameworks in real world have hard nonlinearity and uncertainty, so a lot of control techniques have been developed and the fuzzy control is one of the major nonlinear control theories. However, the main drawback of fuzzy control is that it is difficult to analyze the stability of a fuzzy system.

The Takagi-Sugeno (TS) fuzzy model is widely used, since it is possible to apply the systematic linear control theory to design a controller. Tanaka proposed some sufficient condition for the stability of TS fuzzy model [4]. However, the common positive definite matrix that guarantees their stability condition of the controlled system is difficult and time-consuming to be found. On the other hand Cao et. al. developed the switching type controller design technique by applying the multiple linear system theory [3]. This approach is more suitable since there is no need to find common positive definite matrix.

For a few years, the stabilization problem of TS fuzzy system is extensively studied [4–6]. In real industrial process, such as robot manipulators, the tracking control is also a challenging problem. The contribution of this paper is that some new systematic design technique is developed for output tracking control of the discrete-time TS fuzzy system. In this paper, We first develop the uncertain multiple linear system which represents the discrete-time TS fuzzy system. The sufficient condition of output tracking with guaranteed-cost is derived and formulated in linear matrix inequalities (LMI) framework. The advantage of the studied results in this paper are verified from the computer simulation of the truck trailer system.

2 Preliminaries

Consider a discrete-time uncertain nonlinear system of the form:

$$x(t+1) = f(x(t)) + g(x(t))u(t)\,, \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $f(x(t))$ and $g(x(t))$ are nonlinear vector functions. The nonlinear system (1) can be modeled as the following TS fuzzy system:

Plant Rule i

If $x_1(t)$ is Γ_1^i and $\cdots$ and $x_n(t)$ is Γ_n^i

THEN $x(t+1) = A_i x(t) + B_i u(t)$ (2)

where Γ_j^i $(j = 1, \ldots, n,\quad i = 1, \ldots, q)$ is the fuzzy set, Rule i denotes the ith fuzzy inference rule. The defuzzified output of this TS fuzzy system (2) is represented as follows:

$$x(t+1) = \sum_{i=1}^{q} \mu_i(x(t))(A_i x(t) + B_i u(t))\,. \tag{3}$$

where

$$\omega_i(x(t)) = \prod_{j=1}^{n} \Gamma_j^i(x_j(t))\,, \qquad \mu_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{q} \omega_i(x(t))}\,.$$

in which $\Gamma_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in Γ_j^i. In ith subspace divided by the fuzzy membership functions, the TS fuzzy system has much highly nonlinear interaction among the fuzzy rules, which complicates the analysis and control of the TS fuzzy system [3]. In order to get rid of these theoretical difficulties, we represent the uncertain TS fuzzy system as an uncertain multiple linear system with the following subspace [1].

$$\Theta_i = \{x(t)|\mu_i(x(t)) \geq \mu_j(x(t)), \quad j = 1,2,\ldots,q, \quad i \neq j\} \qquad i = 1,2,\ldots,r. \quad (4)$$

The characteristic function of Θ_i is defined by

$$\eta_i(x(t)) = \begin{cases} 1, & x(t) \in \Theta_i \\ 0, & x(t) \notin \Theta_i \end{cases}, \qquad \sum_{i=1}^{r} \eta_i = 1. \quad (5)$$

Then, on every subspace the fuzzy system (2) can be represented with an uncertain multiple linear system as follows:

$$x(t+1) = \sum_{i=1}^{r} \eta_i(x(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i))u(t). \quad (6)$$

where

$$\Delta A_i = \sum_{j=1, j\neq i}^{r} \mu_j(x(t))\Delta A_{ij}, \quad \Delta B_i = \sum_{j=1, j\neq i}^{r} \mu_j(x(t))\Delta B_{ij},$$
$$\Delta A_{ij} = A_j - A_i, \quad \Delta B_{ij} = B_j - B_i, \quad i = 1,2,\ldots,q.$$

3 Problem Statement

This section deals with the output tracking controller design problem for the discrete-time TS fuzzy system. The state-space representation of the fuzzy system can be described as follows:

$$x(t+1) = \sum_{i=1}^{q} \mu_i(x(t))(A_i x(t) + B_i u(t)), \quad (7)$$

This TS fuzzy system can be represented by the uncertain multiple linear system of the form:

$$x(t+1) = \sum_{i=1}^{r} \eta_i(x(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)) \quad (8)$$

Remark 1 The uncertain matrices $\Delta A_i, \Delta B_i$ represent the highly complex and nonlinear interaction among uncertain subsystems of TS fuzzy system by the fuzzy inference rules. These uncertainties are norm-bounded and can be decomposed of the form:

$$\begin{bmatrix}\Delta A_i & \Delta B_i\end{bmatrix} = D_i F_i(t) \begin{bmatrix}E_{1i} & E_{2i}\end{bmatrix},$$

where D_i, E_{1i}, and E_{2i} are known real constant matrices of appropriate dimensions, and $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F_i(t)^T F_i(t) \leq I$, in which I is the identity matrix of appropriate dimension.

The objective in this section is to design a state feedback controller of the following form:

$$u(t) = \sum_{i=1}^{r} \eta_i(x(t)) K_i x(t) \quad (9)$$

4 Problem Statement

This Section formulates the output tracking problem of the discrete-time TS fuzzy system. The reference signal to be tracked is the output $r(t)$ generated by the exogenous system

$$w(t+1) = Fw(t) \quad (10)$$
$$r(t) = Gw(t) \quad (11)$$

where $w(t) \in R^k$ is the state vector of exogenous system and $r(t) \in R^l$ is the output vector to be tracked by the output of the TS fuzzy system (2). It is further required that the state of this exogenous system should be uniformly bounded.

Problem 1 The objective in this paper is to design a TS fuzzy-model-based controller which stabilize the plant (2) and track the reference signal vector $r(t)$ such that the tracking error

$$e(t) := y(t) - r(t) = Cx(t) - Gw(t) \quad (12)$$

asymptotically to be zero.

In order to construct the error system, a new state vector is defined as

$$z(t) := x(t) - T_i w(t), \quad \text{for all } x(t) \in \Theta_i, \quad i = 1,2,\ldots,q. \quad (13)$$

where T_i is a solution to the following matrix equations:

$$\begin{bmatrix} A_i & B_i \\ C & 0 \end{bmatrix} \begin{bmatrix} T_i \\ L_i \end{bmatrix} = \begin{bmatrix} T_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix} \quad (14)$$
$$i = 1,2,\ldots,q$$

Assumption 1 For the exogenous signal model The matrix equations (14) are assumed to be solvable. To solve (14), following assumption must be satisfied [7].

$$\text{rank} \begin{bmatrix} A_i & B_i \\ C & 0 \end{bmatrix} = n + l$$

This assumption is satisfied if each nominal subsystem in the multiple linear system (6) is controllable and the number of outputs is less than or equal to the number of inputs, i.e., $m \geq l$.

Assuming that T_i and L_i have been found to satisfy (14), consider the tracking control law of the following form:

$$u(t) = \sum_{i=1}^{q} \eta_i(x(t))(L_i w(t) + v(t)) \quad (15)$$

where $v(t)$ remains to be defined. Using this control law, and the matrix equations (14), the newly defined state vector $z(t)$ in (13) satisfies

$$z(t+1) = \sum_{i=1}^{q} \eta_i(x(t))((A_i + \Delta A_i)z(t) + (B_i + \Delta B_i))v(t) \quad (16)$$
$$e(t) = Cz(t) \quad (17)$$

If the newly constructed system (16) is globally asymptotically stable, then the tracking error $e(t)$ converge to zero. Therefore Problem 1 is equivalent to the following problem statement.

Problem 2 The objective in this paper is to design a TS fuzzy-model-based state feedback controller $v(t)$ which asymptotically stabilize the dynamic system (16).

The main result on the output tracking control of the discrete-time TS fuzzy system is summarized in the following theorem.

Theorem 1 *If there exist symmetric positive definite matrices, P_i, a symmetric positive definite matrix, Q, R and matrices, K_i such that the following LMIs are satisfied, then the TS fuzzy system (7) is asymptotically stabilizable via TS fuzzy-model-based controller (9) with guaranteed-cost.*

$$\begin{bmatrix} -W_i & * & * & * & * & * \\ A_iW_i + B_iM_i & -W_i & * & * & * & * \\ E_{1i}W_i + E_{2i}M_j & 0 & -\epsilon_i I & * & * & * \\ 0 & D_i^T & 0 & -\epsilon_i^{-1} I & * & * \\ W_i & 0 & 0 & 0 & -Q^{-1} & * \\ M_i & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0, \quad (18)$$

$$i = 1, 2, \ldots, r.$$

*where $W_i = P_i^{-1}$, $M_i = K_i P_i^{-1}$, and * denotes the transposed elements in the symmetric positions.*

proof: The proof is omitted due to lack of space.

5 An Example

We now apply the above design technique to the control of a computer simulated truck trailer. The control objective of this simulation is to design a controller such that the truck trailer follows backward the given reference trajectory. We use the following truck trailer model formulated in [4]:

$$\begin{aligned} x_1(t+1) &= (1 - v \cdot t/L)x_1(t) + v \cdot t/l \cdot u(t) \\ x_2(t+1) &= x_2(t) + v \cdot t/L \cdot x_1(t) \\ x_3(t+1) &= v \cdot t \cdot \sin(x_2(t) + v \cdot t \cdot x_1(t)/2L) + x_3(t) \end{aligned}$$

The following fuzzy model is used to design a fuzzy controller:

$$\begin{aligned} R^1 &: \text{If } x_2(t) + v \cdot t/L \cdot x_1(t) \text{ is about } 0 \\ &\quad \text{Then}, x(t+1) = A_1 x(t) + B_1 u(t) \\ R^2 &: \text{If } x_2(t) + v \cdot t/L \cdot x_1(t) \text{ is about } \pi \\ &\quad \text{Then}, x(t+1) = A_2 x(t) + B_2 u(t) \end{aligned}$$

where

$$A_1 = \begin{bmatrix} 1 - v \cdot t/L & 0 & 0 \\ v \cdot t/L & 0 & 0 \\ (v \cdot t)^2/2L & v \cdot t & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1 - v \cdot t/L & 0 & 0 \\ v \cdot t/L & 0 & 0 \\ g \cdot (v \cdot t)^2/2L & g \cdot v \cdot t & 1 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} v \cdot t/l \\ 0 \\ 0 \end{bmatrix}$$

and $l = 0.2, L = 0.32, v = -0.1, t = 0.5$, and $g = 20^{-1}/\pi$. The membership functions are

$$\mu_1(x(t)) = \frac{\sin(x_2(t) + v \cdot t/2L \cdot x_1(t)) - g \cdot x_2(t) + v \cdot t/2L \cdot x_1(t)}{(x_2(t) + v \cdot t/2L \cdot x_1(t))(1-g)}$$

$$\mu_2(x(t)) = 1 - \mu_1(x(t))$$

The reference signal model is defined as follows:

$$w(t+1) = Fw(t), \quad r(t) = Gw(t),$$

where

$$F = [1], \quad G = [1].$$

and $w(0) = 1$.

Solving (14), we get

$$T_1 = T_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$L_1 = 4.5475\text{e} - 013, \quad L_2 = -1.4552\text{e} - 011.$$

Based on Theorem 1, we obtain

$$P_1 = 1.0\text{e} - 5 \begin{bmatrix} 0.1057 & -0.1072 & 0.0675 \\ -0.1072 & 0.2796 & -0.1718 \\ 0.0675 & -0.1718 & 0.3562 \end{bmatrix},$$

$$P_2 = 1.0\text{e} - 4 \begin{bmatrix} 0.0010 & -0.0008 & 0.0009 \\ -0.0008 & 0.0021 & -0.0021 \\ 0.0009 & -0.0021 & 0.1889 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 2.2806 & -0.4016 & -0.0002 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 2.3171 & -0.4090 & 0.0002 \end{bmatrix},$$

$$Q = 1.0\text{e} - 7 \begin{bmatrix} 0.7497 & -0.1473 & 0.0710 \\ -0.1473 & 0.7043 & -0.2311 \\ 0.0710 & -0.2311 & 0.7208 \end{bmatrix},$$

$$R = 4.4272\text{e} - 8$$

The initial value is set to $x(0)^T = \begin{bmatrix} 0.5236 & -0.1745 & 0.6981 \end{bmatrix}^T$ Figure 1 shows the computer simulation results. From Fig. 1, the backward movement control based on the proposed technique in this paper is excellent.

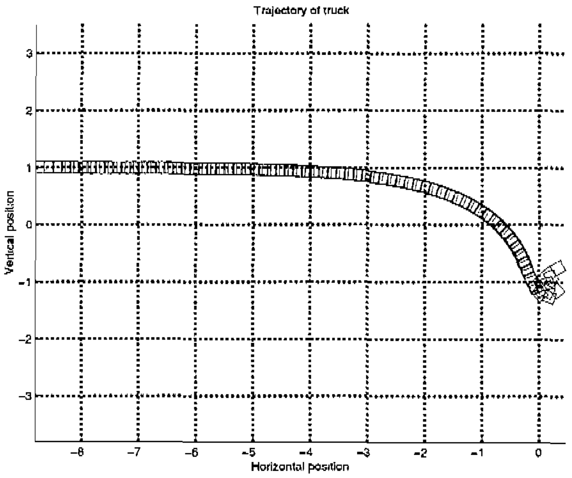

Figure 1: The controlled system response.

6 Conclusion

In this paper, the output tracking controller design technique for discrete-time TS fuzzy system is presented. The stabilization problem of uncertain TS fuzzy system was converted into the stabilization problem of the uncertain multiple linear system. The sufficient condition was formulated in LMI framework. The simulation example ensured us the feasibility of the developed design technique.

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