한국해양공학회 2000년도 춘계학술대회 논문집 PP. 140~145

Numerical Simulation of Unsteady Inviscid Waves by Spectral Method

Jin Ho Lee*, Ho Hwan Chun**

ABSTRACT

The spectral method which is composed of an eigenfunction expansion of free modes in the wave number domain is used to produce two dimensional unsteady inviscid wave simulation such as progressive waves in a numerical pneumatic wave tank. A spatial and time dependent free surface elevation and the potential are calculated by integrating ODE derived from fully nonlinear kinematic and dynamic free surface boundary condition at each time step. The nonlinear characteristics in the waves by this method were notable as increasing wave steepness. This method is very useful and powerful in terms of saving computational time caused by rapid convergence exponentially with increasing number of nodes, even preserving accurate numerical results. Moreover, it will give us many possibilities to apply to naval and ocean engineering fields.

Key Words : Spectral Method, Free Motion, Numerical Pneumatic Wave Tank, Fully Nonlinear ODE

1. INTRODUNCTION

The investigation of unsteady wave characteristic as external forces to the marine vehicles such as ships or offshore structures is very important in the view point of the seakeeping or hydrodynamic. Especially, nonlinear characteristic in waves are focused to design the marine vehicles since these phenomena uniquely affect to the behavior of the ship or floating offshore structures. For examples, mean and slowly-varying wave loads (difference frequency loads) in the mooring system, added resistance of ships in waves and resonant oscillation of TLP's in heave, pitch and roll referred to as 'ringing' or 'springing' (sum frequency phenomena).

Experimental wave tank can be used to investigate these characteristics. However, it is more reasonable and useful to make accurate and cheap numerical wave tank in terms of saving the cost of construction, repair and labor expanded to the experimental tank.

Boundary Element Method (BEM) [5] [6] [7] or Finite Element Method (FEM) [8] [9] have been mainly used in a numerical wave tank. These methods are very precise to evaluate wave characteristic or wave-body interactions. However, the cost of computational time is expensive in the three dimensional numerical wave tank, especially, in the case of short waves adding to some complex numerical schemes such as high order boundary elements as the robust discretization for an body/free-surface interaction treatment in the BEM [2] .

In this paper, we introduce spectral method to cover the shortcomings of such kinds of tools, especially with respect to an expensive computational time in unsteady wave simulations while there exists the limitation to consider the complex geometry such as wave/body interactions (diffraction problems). However, the combination of robust numerical tool (BEM or FEM, etc) to this method may give solutions to extend to these of problems. We will leave this treatment as future works.

The potential in this method is consisted of an eigen function expansion of free modes in the wave number domain, which is satisfied the Laplace equation, each side wall and bottom boundaries condition except the Dirichlet free surface conditions. This method has rapid energy convergence exponentially with increasing the number of nodes so that computational time must be saved. The time derivative of mode coefficients is obtained by directly substituting an eigenfunction expansion to the free surface boundary condition so that a fixed number of free waves in the wave number domain will be transformed into that of in the physical domain.

The mode coefficients or the potential and free surface elevation at each time step will be evaluated from the governing equations in the prescribed potential and wave elevation.

Using this potential and perturbation scheme, two dimensional nonlinear gravity waves were successfully computed with increasing the order of the potential [1]. They used a stokes wave and the surface pressure as the initial conditions and their numerical nonlinear waves which gradually steepened were compared with those of using fully nonlinear MEL calculation. Recently, by including the wave maker potential to this free mode potential, two dimensional non-periodic water waves were demonstrated [3].

We devised two dimensional numerical wave tank by this method. Three dimensional simulation is in progress and must be easily extended from

* Hydrodynamic Team, Samsung Heavy Ind.
** Dept. of Naval Architecture & Ocean Engineering, Pusan National Univ.

two dimensional simulation. It is not so easy in this method to generate progressive waves by a piston or flap type wave maker since Neuman condition of the moving boundary condition at the side wall as the wave maker will not be satisfied any more. To overcome this disadvantage, the waves can be generated by the pressure patch forced at the end of side wall in the wave basin as a pneumatic wave maker including a useful wave absorbing zone at the opposite side of pressure distributions. The maximum pressure have to be chosen carefully to obtain a target wave amplitude correspond to the length and shape of patch. We found that this pressure was affected by the wave period so that the transfer function between a maximum pressure and wave amplitude measured inside effective zone of the tank were considered. The spatial and time dependent progressive waves in linearized condition were compared with those of in fully nonlinear condition. We found that the nonlinear characteristics in the waves were notable in the higher wave steepness.

Each section in this paper was composed as like this. In section 2, the mathematical formulation was represented. The numerical scheme was shown in section 3. Numerical results of the simulation by this method were shown in section 4. Conclusion and future works in section 5.

2. MATHEMATICAL FORMULATION

We consider the irrotational motion of a homogeneous, incompressible and inviscid fluid in the domain Ω under a free surface in finite depth h. Surface tension is not considered. The origin is located at the mean water level and the vertical axis z is positive upward as shown in figure 1. For simplicity, the gravity acceleration, g and water depth, h have chosen so that wave elevation, $\eta(\vec{x},t)$ and time, t are unity. The flow can be described by a velocity potential, $\phi(\vec{x},z,t)$ which satisfies the following fully nonlinear-initial-boundary-value problem.

$$\nabla^2\phi + \phi_{zz} = 0 \qquad \text{in } \Omega \qquad (1)$$

$$\frac{d\eta}{dt} = -\nabla\phi\cdot\nabla\eta + \phi_z - \upsilon(x)\eta \quad \text{at } z=\eta$$

$$\phi_t = -\frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}\phi_z^{\ 2} - \eta - \upsilon(x)\phi - P$$

$$\frac{d\phi}{dt} = \phi_t + \eta_t\phi_z \qquad \text{at } z=\eta \qquad (2)$$

$$\phi_z = 0 \qquad \text{at } z=-h \qquad (3)$$

$$P(i,j,t) = \frac{A_o}{H(\omega)}\sin(j\Delta y\,k\sin\theta - \omega t)\cos^{\beta}\frac{\pi(i-1)}{2(I-1)}$$

(i=1,2,----,I),(j=1,2,---,n2) (4)

I : Maximum number to decide a length of pressure patch in the x direction

β : Shape coefficient of pressure patch (1,2,---)

$$\phi(\vec{x},0,0) = 0$$
$$\eta(\vec{x},0) = 0 \qquad (5)$$
$$P(x,y,0) = 0$$

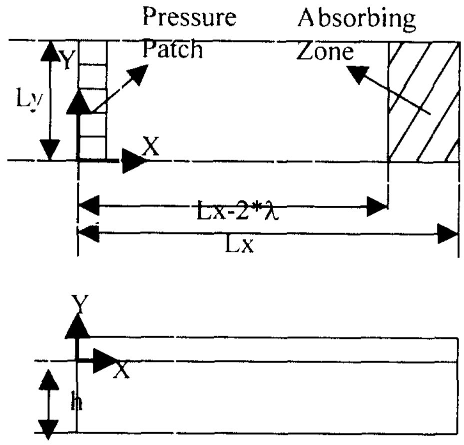

Figure 1. Co-ordinate system and sketch of a numerical wave tank

Here $\nabla = (\frac{\partial}{\partial x},\frac{\partial}{\partial y})$, $\vec{x} = (x,y)$ is a vector in the horizontal plane, P(x,y,t) represents time dependent pressure patch distributions at the end of side walls to generate progressive. We consider a proper pressure patch by a trial and error in order to make stable progressive waves. The length and shape of pressure patch affect to evaluation accurate numerical progressive waves depending on the target wave period. A_o is the target wave amplitude, $H(\omega)$ is transfer function between pressure distribution and measured wave amplitude in the wave basin θ is the propagating wave direction, ω is an angular frequency of pressure forced periodically. For decreasing wave reflections at the far from wave generation region, $\upsilon(x)$, damping coefficient exists in the region of opposite side wall of pressure distributions as a damping layer. We consider a simple shape function as shown in equation (6) to minimize reflection of propagating waves.

$$\upsilon(x) = \alpha(x - x_o)^2 \qquad x \ge x_o$$
$$\upsilon(x) = 0 \qquad x \langle x_o \qquad (6)$$
$$\alpha = 0.5 \qquad x_o = L_x - 2\lambda$$

λ is wave length

3. NUMERICAL SCHEME

We represent the potential ϕ, as an eigenfunction (ψ_{mn}) expansion of free modes which satisfy at each boundary conditions all but the Dirichlet free surface conditions (2).

$$\phi = \sum_{m=0}^{M} \sum_{n=1}^{N} a_{mn} \psi_{mn} \qquad (7)$$

a_{mn} are mode coefficients, Eigenfunction (ψ_{mn}) can be represented as below equation for the finite depth h.

$$\psi_{mn} = \frac{\cosh(k_{mn}(z+h))}{\cosh(k_{mn}z)} \cos(\vec{k}_{mn} \cdot \vec{x}) \qquad (8)$$

Here, $\vec{k}_{mn} = (\frac{m\pi}{L_x}, \frac{n\pi}{L_y})$ is a wave number vector and k_{mn} is an amplitude of wave number, L_x, L_y is the length and width of the basin. We can easily evaluate the vertical or horizontal plane derivative of the potential (ϕ_x, ϕ_y, ϕ_z) as like this.

$$\phi_x = \sum_{m=0}^{M} \sum_{n=1}^{N} -\left|\vec{k}_{mn}\right|_x a_{mn} \frac{\cosh(k_{mn}(z+h))}{\cosh(k_{mn}z)} \sin(\vec{k}_{mn} \cdot \vec{x}) \qquad (9)$$

$$\phi_y = \sum_{m=0}^{M} \sum_{n=1}^{N} -\left|\vec{k}_{mn}\right|_y a_{mn} \frac{\cosh(k_{mn}(z+h))}{\cosh(k_{mn}z)} \sin(\vec{k}_{mn} \cdot \vec{x}) \qquad (10)$$

$$\phi_z = \sum_{m=0}^{M} \sum_{n=1}^{N} k_{mn} a_{mn} \frac{\sinh(k_{mn}(z+h))}{\cosh(k_{mn}z)} \cos(\vec{k}_{mn} \cdot \vec{x}) \qquad (11)$$

at $z = \eta$

The procedure to evaluate the spatial and time dependent free surface elevation is mainly divided two parts. First, mode coefficients in the potential must be evaluated. To do this, the $m \times n$ time derivative of mode coefficients, $\partial a_{mn} / \partial t$ in wave number domain were evaluated by transformed them to those of in the physical domain consisted of $m \times n$ linear algebraic equation in the equation (12).

$$\sum_{m=0}^{M} \sum_{n=1}^{N} \frac{\partial a_{mn}(t)}{\partial t} \frac{\cosh(k_{mn}(z+h))}{\cosh(k_{mn}z)} \cos(\vec{k}_{mn} \cdot \vec{x}) = \phi_t + \eta_t \phi_z$$

at $z = \eta$ (12)

The computational time must be saved in the linearized condition since the kernel, $\cos(\vec{k}_{mn} \cdot \vec{x})$ is identical and time invariant at each time. However, much more computational time must be taken with moving free surface in the kernel at each time step in the fully nonlinear condition.

Second, free surface elevation at each time step is evaluated by integrating kinematic and dynamic free surface boundary condition in equation (2). We used runge-kutta 4^{th} order scheme with initial value of the potential, free surface elevation and even pressure distribution in the case of progressive waves. The calculation will be done in sequence so that the potential and free surface elevation in the physics domain can be evaluated at the advanced time step.

Spatial derivative of free surface elevation, η_x, η_y was evaluated for fully nonlinear condition based on the second order central difference approximation in equation [4].

4. NUMERICAL RESULTS

Progressive waves generated by pressure patch forced at the end of wall were produced in a numerical wave tank, $(L_x/h, L_y/h, h/h) = (6,2,1)$. The length and shape of pressure patch or a mesh size ($\Delta x' = \Delta x/h$) must be chosen to make accurate numerical results.

We investigated characteristics of progressive waves with respect to $\Delta x'=0.1$ and 0.15 in the condition of an appropriate length and shape of patch. In addition to this, the wave steepness ($\varepsilon=0.05$ and 0.1) was changed in the given wave period, $T'=T\sqrt{g/h}=2.818$ to see the nonlinearity in waves. An absorbing zone corresponding to equation (6) was created in order to minimize the reflection of propagating waves. The effectiveness of this absorbing zone was shown in figure 2 (a-d).

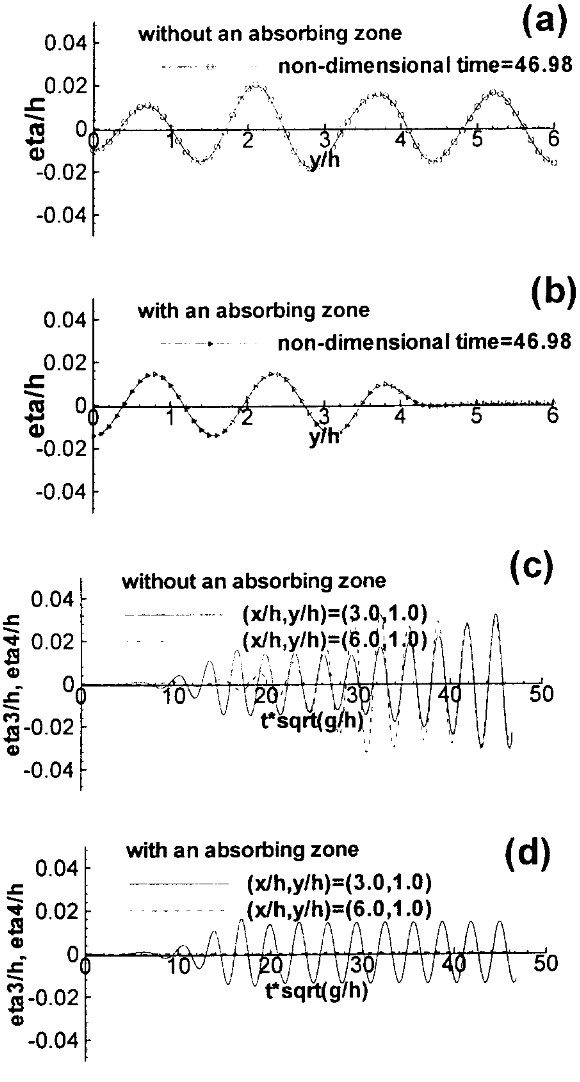

Figure 2. Effectiveness of a damping layer (Lx/h,Ly/h,h/h)=(6,2,1), Δx/h=0.1, Po=0.01, β=4, $\Delta t\sqrt{g/h}=0.1566$, Length of patch / h=0.6

4.1 Numerical results depending on a mesh size

The time dependent free surface elevation at four different points (x/h=0.0, 1.8, 3.0 and 6.0) in figure 3 and the spatial free surface elevation at a given non-dimensional time $t'=t\sqrt{g/h}$ in figure 4 were shown with two different $\Delta x'=0.1$ and 0.15 and same length of pressure patch (the length / h = 0.6). We found that the spatial free surface in $\Delta x'=0.15$ could not realize a precise sinusoidal wave profile comparing with that of in $\Delta x'=0.1$. However, the time series of free surface wave in both of them was shown accurate results at the points x/h=0.0 and x/h=1.8 except some deviation at the point, x/h=3.0 resulted from the influence of wave reflection.

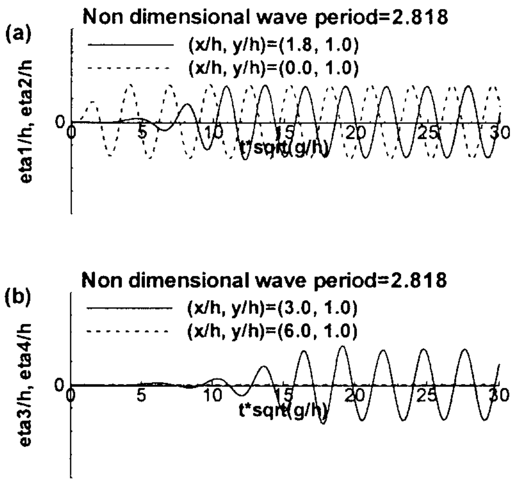

Figure 3 Time dependent wave profile at four points

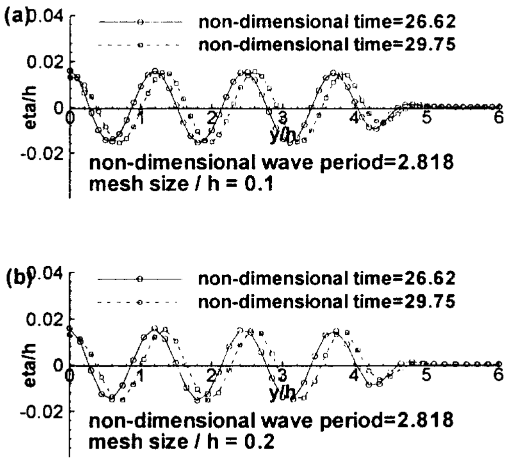

Figure 4 Special dependent wave profile

4.2 Numerical results depending on a boundary condition

The nonlinearity in propagating waves was investigated as changing wave steepness. We chose two different wave steepness, 0.064 and 0.096. The nonlinearity was strongly shown in the case of $\varepsilon = 0.096$ in figure 5. We found that phenomenon in the wave profile in fully nonlinear condition figured crest with sharp and trough with flat as increasing a wave steepness compared with that of in linearized condition.

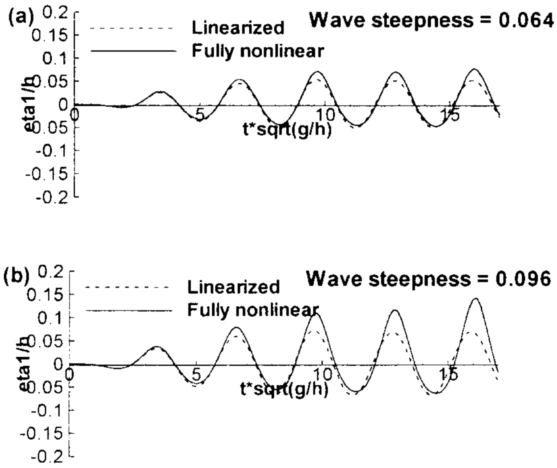

Figure 5 Time dependent wave profile changing with wave steepness

5. CONCLUSION AND FUTURE WORK

Using the spectral method, spatial and time dependent progressive waves in a numerical tank generated by pressure patch forced were simulated. The reasonable and efficient numerical results must be obtained by this method in the condition of an appropriate sampling time, mesh size or length and shape of pressure patch. Computational time was not so expensive compared with that of by other numerical tools. In the numerical wave tank, we devised pressure patch representing a pneumatic wave maker to generate waves since the limitation of this method to introduce a piston or flap type wave maker. The length and shape of pressure patch have to be carefully chosen to produce accurate wave simulation. However, we could not make the reliable energy convergence in the case of fully nonlinear condition because of mistreating the vertical derivative of potential in kinetic energy. It will be corrected well sooner or later. Moreover, the nonlinearity in propagating waves was corrected as adding to transformation term in the dynamic free surface condtion.

As future works, we can also devise and compare wave simulation by this method with that of by other numerical tool such as numerical generation of transient waves [9] or long crested irregular waves in two dimensional region. Variable wave generation will be possible by extending this method to 3D unsteady wave simulation such as oblique waves [10], freak waves, short crested irregular waves [11]. We can enjoy more the merit of this method such as cheap computational time as extending to 3D simulation. Additional to this, wave diffraction problem induced by wave/body interaction will be treated by combining other robust numerical tool.

ACHNOWLEDEMENT

This research subject was supported financially by C.I.E.S (Foundation in France) for 6 months (15^{th} / Sep. / 1999 – 14^{th} / Mar. / 2000).

The author would like to acknowledge Dr. Pierre Ferrant for his cordial advice to perform this subject in Ecole Centrale de Nantes (Department of Naval Hydrodynamic) as a postdoctoral position.

REFERENCE

[1] Dommermuth, Douglas G. and Yue, Dick K.P., "A High-Order Spectral Method for the Study of Nonlinear Gravity Waves", J. Fluid Mech. (1987). Vol. 184, pp. 267-288

[2] Tsai, Wu-Ting and Yue, Dick K.P., "Computational of Nonlinear Free-Surface Flows", Annu. Rev. Fluid Mech. (1996). 28, pp. 249-278

[3] Agnon, Yehuda and Bingham, Harry B., "A Non-Periodic Spectral Method with Application to Nonlinear Water Waves", Eur. J. Mech. B/Fluids, 18,n°3, 1999, pp. 527-534

[4] Buchmann, Bjarne, "A 2-D Numerical Wave Flume Based on a Third Order Boundary Element Model", M.Sc Thesis at Technical University of Demark, Institute of Hydrodynamics and Hydraulic Engineering, 1995

[5] Ferrant, Pierre, "Simulation of Strongly Nonlinear Wave Generation and Wave-Body Interaction using a 3D MEL Model", Proc. 21th ONR Symposium, 1996, Trondheim, Norway

[6] Ferrant ,Pierre and Guillerm, Pierre-Emmanuel "Interaction of Second Order Wave Packets With a Veritical Cylinder", Proceedings of the eigth International Offshore and Polar

Engineering Conference, Montreal, Canada,1998, PP.

[7] Sung, Hong G. and Choi, Hang S., "A Numerical Approach to Three-Dimensional Diffraction Problem in Non-Linear Waves", Proceedings of the Ninth International Offshore and Polar Engineering Conference, Brest, France,1999, PP. 297-303

[8] Arai, M., Paul, U.K. and Inoue, Y., "Wave Generation in 3D Numerical Wave Tanks", Journal of the Society of Naval Architects of Japan, Vol. 174, pp.253-259.

[9] Clauss, Gunther F. and Steinhagen, Ulrich, "Numerical Simulation of Nonlinear Transient Waves and its Validation by Laboratory Date", Proceedings of the Ninth International Offshore and Polar Engineering Conference, Brest, France,1999, PP. 368-375

[10] Williams, A.N, Crull, W.W., "Simulation of directional waves in a numerical basin by a desingularized integral equation approach", Ocean Engineering 27 (2000) PP. 603-624

[11] Kim, M.H., Park, J.C., Hong, S.Y and Tavassoli, A., "Fully Nonlinear Multi-Directional Waves by a 3D Viscous Numerical Wave Tank", Proceedings of the Ninth International Offshore and Polar Engineering Conference, Brest, France,1999, PP. 368-375