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The Exact Formulation of the Green Integral Equation Applied to the Radiation-Diffraction Problem for a Surface Ship Advancing in Waves

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파중 전진하는 선체에 의한 방사파-산란파 문제의 해법에 적용되는 Green 적분방정식의 정확한 도출

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KEY WORDS: Time-harmonic forward-speed Green function 시간영역 전진속도 Green함수, Brard number, Green integral equation Green 적분방정식, Water wave radiation -diffraction problem with non-zero Froude number 전진하며 동요하는 선체에 의한 방사파-산란파 문제.

ABSTRACT: The Green integral equation for the calculation of the forward-speed time-harmonic radiation -diffraction potentials IS derived. The forward-speed Green function presented by Brard is used and the correct free surface boundary condition for the Green function is imposed. The cause of the mistakes in the existing Green integral equation is also pointed out.

1. Introduction

Nowadays, the three-dimensional boundary element methods in the frequency domain are in common use to calculate the motion of freely floating zero-speed surface ships in waves and wave loads as far as boundary conditions are linearized. The added mass and wave damping coefficients in the equation of motion of the zero-speed surface ship are calculated directly using the time-harmonic potential on the hull. The unknown radiation and diffraction potentials can be obtained from the solution of the improved Green integral equation using Kelvin-type Green function for all frequencies (Hong 1987).

Unfortunately, the three-dimensional solution techniques for the ship motion problem with forward speed in the frequency domain, which is more important in the field of navel hydrodynamics, are not applicable to practical problems yet meanwhile strip methods are employed for linear seakeeping design(Gerritsma and Beukelman 1967, Ogilvie and Tuck 1969, Salvesen et al 1970, Newman and Sclavounos 1980). The time-harmonic forward-speed Kelvin-type three-dimensional Green function(denoted Brard's Green function hereafter) has already been presented(Brard 1948) and various three-dimensional numerical methods to solve a source integral equation using this Green function have been followed to calculate the motion of surface ship advancing in waves(Chang 1977, Bougis 1980, Inglice and Price 1981, Chan 1990). The recent drastic improvement of computer technology has removed the restriction on the memory space and computing time. But no serious comparative studies on their numerical results were carried out. It seems that all these three-dimensional methods in the frequency domain are not conclusive yet. Bougis presented also a Green integral equation where the unknown is the potential while the unknown in the source integral equation is the source density.

In this paper, the Green integral equation with Brard's Green function has been reviewed and revised through rational treatment of the free surface integral associated with the Brard number(Brard 1972).

2. Linearized Boundary-Value Problem in the Frequency Domain

A ship is moving with mean forward speed U in the free surface of deep water under gravity and in the presence of plane progressive sinusoidal incident wave of small amplitude a_0. Let $oxyz$ be a Cartesian co-ordinate system attached to the mean position of the ship, with z vertically upward , x in the direction of forward motion and o in the mean waterplane W_P. The ship performs simple harmonic oscillations of small amplitude about its mean position with circular frequency ω which is equal to the encounter frequency of incident wave. It is assumed that the disturbance of the free surface due to the forward motion is also small. It can be understood that the forward speed should be low for full-shaped ships and relatively high for fine-shaped ships.

With the usual assumptions of the incompressible, inviscid fluid and irrotational flow without capillarity, the fluid velocity can be given by the gradient of a velocity potential Φ which satisfies the Laplace equation,

$$\nabla^2\Phi = 0 \tag{1}$$

in the fluid region.

Under the assumptions given above, Ψ at P in the fluid region can be decomposed as follows:

$$\Phi(P,\ t) = \Phi_S(P) + Re\{\Psi(P)\, e^{-i\omega t}\} \tag{2}$$

where Φ_S denotes a steady potential known as the Neumann-Kelvin potential, Ψ a complexed-valued unsteady potential and ω the encounter frequency of the incident wave. The velocity potential of incident wave is as follows:

$$\Phi_0 = Re\{\Psi_0\, e^{-i\omega t}\} \tag{3}$$

where

$$\Psi_0 = -\frac{a_0 g}{\omega_0}\, e^{k_o[z + i(x\cos\beta + y\sin\beta)]} \tag{4}$$

for

$$\omega = (\omega_o - Uk_o\cos\beta) > 0 \tag{4a}$$

and

$$\Psi_0 = -\frac{a_0 g}{\omega_0}\, e^{k_o[z - i(x\cos\beta + y\sin\beta)]} \tag{5}$$

for

$$\omega = (Uk_o\cos\beta - \omega_o) > 0 \tag{5a}$$

where g is the gravitational acceleration, β the angle between the phase velocity of the incident wave and the forward velocity of the ship, ω_0 is the circular frequency of incident wave and $k_0 = \frac{\omega_0^2}{g}$ the wavenumber expressed in a space-fixed co-ordinate system $\overline{o\,x\,y\,z}$ given as follows:

$$\bar{x} = x + Ut, \quad \bar{y} = y, \quad \bar{z} = z \tag{6}$$

The equation of the mean free surface is

$$z = 0 \tag{7}$$

and the linearized free surface boundary condition for Φ on $z = 0$ is as follows:

$$[(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x})^2 + g\frac{\partial}{\partial z}]\Phi = 0 \quad on \quad z = 0 \tag{8}$$

Substitution of (2) to (8) yields the following free surface boundary conditions for Φ_S and Ψ respectively:

$$[U^2\frac{\partial^2}{\partial x^2} + g\frac{\partial}{\partial z}]\Phi_S = 0 \quad \text{on} \quad z = 0 \tag{9}$$

$$[(-i\omega - U\frac{\partial}{\partial x})^2 + g\frac{\partial}{\partial z}]\Psi = 0 \quad on \quad z = 0 \tag{10}$$

The forward speed U is of $O(1)$. Under the assumption of small amplitude oscillation, the displacement vector $\vec{A}(M)$ of a point M on the wetted surface S of the ship in its mean position is of $O(\varepsilon)$ where ε, being as small as the wave slope. is the measure of smallness in the present study. The expression of $\vec{A}(M)$ is as follows:

$$\vec{A}(M) = Re\{\vec{a}(M)\, e^{-i\omega t}\}, \qquad M \in S \tag{11}$$

$$\vec{a}(M) = \sum_{k=1}^{3} a_k\, \vec{e}_k + \vec{\theta} \times \overrightarrow{OM}, \qquad M \in S \tag{11a}$$

$$\vec{\theta} = \sum_{k=4}^{6} a_k\, \vec{e}_{k-3} \tag{11b}$$

where $a_k (k = 1,2,,,6)$ denotes complex valued amplitude of surge, sway, heave, roll, pitch, yaw respectively and O the center of rotation of the ship.

It should be noted that the time-harmonic quantities correspond to the real part of terms involving $e^{-i\omega t}$ and it will not be shown unless its presence is necessary.

Applying impermeability condition on S, the following body boundary condition can be found:

$$(\vec{n} + \vec{\theta} \times \vec{n}) \cdot \nabla(\Phi_S + \Psi) = (\vec{n} + \vec{\theta} \times \vec{n}) \cdot (U\vec{e_1} - i\omega\vec{\alpha}) \quad (12)$$

where $\vec{n}$ denotes a unit normal to S directed into the fluid region, at its mean position and $(\vec{n} + \vec{\theta} \times \vec{n})$ the Taylor expansion of the normal at its instantaneous position. The above condition can also be found from its alternative expression given by Timman and Newman (Timman and Newman 1962).

Assuming Φ_S is of $O(\varepsilon)$ and neglecting second-order quantities, the following linearized body boundary condition for Φ_S and Ψ can be found respectively:

$$\frac{\partial \Phi_S}{\partial n} = U n_1 \quad on \quad S \quad (13)$$

$$\frac{\partial \Psi}{\partial n} = -i\omega\vec{\alpha} \cdot \vec{n} + U(a_5 n_3 - a_6 n_2) \quad on \quad S \quad (14)$$

With these linearized boundary conditions on S and on $z = 0$, the unsteady potential problem and Neumann-Kelvin problem can be solved independently and the latter will be dropped from the present study.

The unsteady potential Ψ can further be decomposed as follows:

$$\Psi = \Psi_0 + \Psi_7 + \Psi_R \quad (15)$$

where the sum of Ψ_0 and Ψ_7 is known as the diffraction potential and Ψ_R the radiation potential which can be decomposed as follows:

$$\Psi_R = -i\omega \sum_{k=1}^{6} a_k \Psi_k - U(a_6 \Psi_2 - a_5 \Psi_3) \quad (16)$$

Then the body boundary conditions for $\Psi_k (k = 1,2,,,7)$ are

$$\frac{\partial \Psi_k}{\partial n} = n_k \quad on \quad S, \quad k = 1,2,3 \quad (17a)$$

$$\frac{\partial \Psi_j}{\partial n_0} = (\vec{e_{j-3}} \times \vec{OM}) \cdot \vec{n} \quad on \quad S, \quad \text{for } j = 4,5,6 \quad (17b)$$

$$\frac{\partial \Psi_7}{\partial n} = -\frac{\partial \Psi_0}{\partial n} \quad on \quad S \quad (18)$$

The potentials $\Psi_k (k = 1,2,,,7)$ also satisfy the free surface boundary condition given by the equation (10):

$$[(-i\omega - U\frac{\partial}{\partial x})^2 + g\frac{\partial}{\partial z}]\Psi_k = 0 \quad on \quad F, \quad \text{for } k = 1,2,,,7 \quad (19)$$

It is also assumed that they vanish at infinity as $\frac{1}{r^{\infty}}$ where r^{∞} denotes the distance from the ship. They must also satisfy the radiation condition presented by Brard(Brard 1948).

Here, the potential boundary-value problems for $\Psi_k (k = 1,2,,,7)$ will be solved by making use of the Green integral equation.

3. Brard's Green Function

The Green function derived by Brard(Brard 1948) characterizes the potential induced at P where $z_P \leq 0$, by a pulsating source of unit strength at M where $z_M < 0$, advancing under the free surface with uniform velocity $U\vec{e_1}$. The point M is the so-called source point and P the field point. It has been obtained as follows:

$$G(P,M,t) = Re\{G(P,M)e^{-i\omega t}\} \quad (20)$$

where

$$G(P,M) = G_0(P,M) - G_i(P,M) + G_f(P,M) \quad (21)$$

where

$$G_0(P,M) = -\frac{1}{4\pi}\frac{1}{r} \quad (22)$$

$$G_i(P,M) = -\frac{1}{4\pi}\frac{1}{r_1} \quad (23)$$

$$G_f(P,M) = \frac{1}{4\pi^2}(H_1 + H_2) \quad (24)$$

$$H_1 = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\theta \int_0^{\infty} \frac{1}{D_1} e^{\bar{\zeta}} gk \, dk \quad (25)$$

$$H_2 = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\theta \int_0^{\infty} \frac{1}{D_2} e^{\zeta} gk \, dk \quad (26)$$

$$r = \{(x_P - x_M)^2 + (y_P - y_M)^2 + (z_P - z_M)^2\}^{\frac{1}{2}} \quad (27)$$

$$r_1 = \{(x_P - x_M)^2 + (y_P - y_M)^2 + (z_P + z_M)^2\}^{\frac{1}{2}} \quad (28)$$

$$D_1 = (\omega - Uk\cos\theta)^2 - gk + i\nu(\omega - Uk\cos\theta) \quad (29)$$

$$D_2 = (\omega + Uk\cos\theta)^2 - gk + i\nu(\omega + Uk\cos\theta) \quad (30)$$

$$\zeta = k\{z_P + z_M + i[(x_P - x_M)\cos\theta + (y_P - y_M)\sin\theta]\} \quad (31)$$

where ν is an artificial damping parameter infinitely small, positive, which will determine the path of integration in the complex plane K associated with the variable k shown in the expressions of H_1 and H_2.

The function G_0 is the Rankine-type Green function which is singular when $P=M$ and regular otherwise. The function G_i is regular for $z_P \leq 0$. The function G_f is regular for $z_P \leq 0$ and $z_M < 0$ or $z_P < 0$ and $z_M \leq 0$ and is not defined when $z_P = z_M = 0$.

Brard's Green function satisfies the following equations:

$$\nabla_P^2 G(P,M) = 0 \quad \text{for} \quad z_P \leq 0 \quad \text{and} \quad z_M < 0 \quad \text{and} \quad P \neq M \tag{32}$$

$$\nabla_P^2 G(P,M) = 0 \quad \text{for} \quad z_P < 0 \quad \text{and} \quad z_M \leq 0 \quad \text{and} \quad P \neq M \tag{33}$$

$$\left[\left(-i\omega - U\frac{\partial}{\partial x_P}\right)^2 + g\frac{\partial}{\partial z_P}\right] G(P,M) = 0 \quad \text{for} \quad z_M < 0 \quad \text{and} \quad z_P \leq 0 \tag{34}$$

$$\left[\left(-i\omega + U\frac{\partial}{\partial x_M}\right)^2 + g\frac{\partial}{\partial z_M}\right] G(P,M) = 0 \quad \text{for} \quad z_P < 0 \quad \text{and} \quad z_M \leq 0 \tag{35}$$

It has been shown by Brard that the radiation condition for $G(P,M)$ is satisfied when the artificial damping parameter is present in the denominators D_1 and D_2. In fact, the so-called damped free surface condition given below was used to construct Brard's Green function $G(P,M)$:

$$\left[\left(-i\omega - U\frac{\partial}{\partial x_P}\right)^2 + \nu\left(-i\omega - U\frac{\partial}{\partial x_P}\right) + g\frac{\partial}{\partial z_P}\right] G(P,M) = 0 \quad \text{for} \quad z_M < 0 \quad \text{and} \quad z_P \leq 0 \tag{36}$$

Since the Green function $G(P,M)$ is of $O(\frac{1}{r})$, it tends to zero as $r \rightarrow \infty$.

The condition (35) is the so-called adjoint free surface condition for $G(P,M)$ and can be derived from the free surface condition (34) according to the reasoning given by Brard(Brard 1972).

The integrations with respect to k in H_1 and H_2 can be done analytically by making use of the complex exponential integral $E_1(\zeta)$ as shown by Hong(Hong 1978) in his report on the radiation problem of a cylinder advancing under the free surface. This method of integration was generalized by Guevel(Guevel et al. 1979) and was applied to the three-dimensional radiation-diffraction problem with forward speed by Bougis(Bougis 1980).

4. Green Integral Equation

When a ship is present in the free surface, the fluid region D^e is bounded by the mean wetted surface of the ship S, the outer free surface $F^e = F - W_P$ where W_P is the waterplane at its mean position and some arbitrary surface S^∞ at infinity. Let C and C^∞ denote the closed intersection contours of F with S and S^∞ respectively. Applying Green's theorem to the potential Ψ and the Green function G over the fluid region D^e, the following integral identities can be obtained:

$$\Psi(P) = -\int\int_{S \cup F^e \cup S^\infty} \left[\Psi(M)\frac{\partial G(P,M)}{\partial n_M} - \frac{\partial \Psi(M)}{\partial n_M} G(P,M)\right] ds, \quad \text{for} \quad z_P < 0 \tag{37}$$

where $\vec{n}$ denotes a unit normal to the boundary surface directed into the fluid region D^e.

Since Ψ and G tend to zero as $\frac{1}{r^\infty}$, the integral over S^∞ vanishes in the limit and we have

$$\Psi(P) = -\int\int_S \left[\Psi(M)\frac{\partial G(P,M)}{\partial n_M} - \frac{\partial \Psi(M)}{\partial n_M} G(P,M)\right] ds - I_F^e \quad \text{for} \quad z_P < 0 \tag{38}$$

where

$$I_F^e = \int\int_{F^e} \left[\Psi(M)\frac{\partial G(P,M)}{\partial n_M} - \frac{\partial \Psi(M)}{\partial n_M} G(P,M)\right] ds, \quad z_P < 0 \tag{39}$$

Substitution of the free surface condition (19) and the adjoint free surface condition (35) into the normal derivatives of Ψ and G in (39) respectively, yields

$$I_F^e = I_\gamma + I_{Fn} \tag{40}$$

where

$$\begin{aligned} I_\gamma &= -2i\gamma \int\int_{F^e} \left[\Psi(M)\frac{\partial G(P,M)}{\partial x_M} + \frac{\partial \Psi(M)}{\partial x_M} G(P,M)\right] ds \\ &= -2i\gamma \int\int_{F^e} \frac{\partial}{\partial x_M}\left[\Psi(M) G(P,M)\right] ds, \quad \text{for} \quad z_P < 0 \end{aligned} \tag{41}$$

and

$$I_{Fn} = \frac{U^2}{g} \int\int_{F'} \frac{\partial}{\partial x_M} [\Psi(M) \frac{\partial G(P,M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M} G(P,M)] ds \quad \text{for} \quad z_P < 0 \tag{42}$$

The ν in (41) is a non-dimensional parameter known as the Brard number.

$$\nu = \frac{U\omega}{g} \tag{43}$$

Application of Stokes's theorem to (41) yields

$$I_\nu = -2i\nu \int_{C^\infty} \Psi(M) G(P,M) dy_M + 2i\nu \int_C \Psi(M) G(P,M) dy_M, \quad z_P < 0 \tag{44}$$

where the positive directions around both C and C^∞ are defined counterclockwise when one would see them from above the free surface.

The line integral of the product Ψ and G along C^∞ vanishes in the limit since both Ψ and G tend to zero as $\frac{1}{r^\infty}$ and we have

$$I_\nu = 2i\nu \int_C \Psi(M) G(P,M) dy_M, \quad z_P < 0 \tag{45}$$

Similarly, application of Stokes's theorem to (42) yields

$$I_{Fn} = -\frac{U^2}{g} \int_C [\Psi(M) \frac{\partial G(P,M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M} G(P,M)] dy_M, \quad z_P < 0 \tag{46}$$

Substitution of (45) and (46) into (40) yields

$$I_F^e = 2i\nu \int_C \Psi(M) G(P,M) dy_M - \frac{U^2}{g} \int_C [\Psi(M) \frac{\partial G(P,M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M} G(P,M)] dy_M \quad \text{for} \quad z_P < 0 \tag{47}$$

Substituting the final expression of I_F^e into the integral relation (38) and taking account of the potential jump across S, we have the following integral relations:

$$c\Psi(P) = -\int\int_S [\Psi(M) \frac{\partial G(P,M)}{\partial n_M} - \frac{\partial \Psi(M)}{\partial n_M} G(P,M)] ds - 2i\nu \int_C \Psi(M) G(P,M) dy_M + \frac{U^2}{g} \int_C [\Psi(M) \frac{\partial G(P,M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M} G(P,M)] dy_M \tag{48}$$

where the value of c is 1, $\frac{1}{2}$ and 0 according as the field point P is inside D^e, on S and outside D^e.

Knowing the body boundary condition on S, the following Green integral equation for Ψ can be found when P is on S:

$$\frac{1}{2}\Psi(P) + \int\int_S \Psi(M) \frac{\partial G(P,M)}{\partial n_M} ds + 2i\nu \int_C \Psi(M) G(P,M) dy_M - \frac{U^2}{g} \int_C [\Psi(M) \frac{\partial G(P,M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M} G(P,M)] dy_M = \int\int_S \frac{\partial \Psi(M)}{\partial n_M} G(P,M) ds \quad \text{for} \quad P \in S \tag{49}$$

The derivative of Ψ with respect to x_M can be decomposed as follows:

$$\frac{\partial \Psi(M)}{\partial x_M} = \vec{e_1} \cdot [\frac{\partial \Psi(M)}{\partial n_M} \vec{n_M} + \frac{\partial \Psi(M)}{\partial l_M} \vec{l_M} + \frac{\partial \Psi(M)}{\partial \tau_M} \vec{\tau_M}], \quad M \in S \tag{50}$$

where $\vec{l}$ is a unit vector tangent to C whose direction along which one, traveling in D^e, would proceed in keeping W_p to his left, is defined positive and τ a unit vector tangent to S forming a right-hand vector triad $\vec{\tau} = \vec{l} \times \vec{n}$.

Substitution of (50) into (49) yields

$$\frac{1}{2}\Psi(P) + \int\int_S \Psi(M) \frac{\partial G(P,M)}{\partial n_M} ds + 2i\nu \int_C \Psi(M) G(P,M) dy_M - \frac{U^2}{g} \int_C \{\Psi(M) \frac{\partial G(P,M)}{\partial x_M} - \vec{e_1} \cdot [\frac{\partial \Psi(M)}{\partial l_M} \vec{l_M} + \frac{\partial \Psi(M)}{\partial \tau_M} \vec{\tau_M}] G(P,M)]\} dy_M = \int\int_S \frac{\partial \Psi(M)}{\partial n_M} G(P,M) ds - \frac{U^2}{g} \int_C \frac{\partial \Psi(M)}{\partial n_M} G(P,M) \vec{e_1} \cdot \vec{n_M} dy_M \quad \text{for} \quad P \in S \tag{51}$$

Bougis presented a Green integral equation for Ψ (Bougis 1980). But, in the Green integral equation by Bougis, the sign before the line integral associated with the Brard number is minus since he used minus sign before the term involving U in the expression of the adjoint free surface condition (35). In fact, he used the following condition:

$$[(-i\omega - U\frac{\partial}{\partial x_M})^2 + g\frac{\partial}{\partial z_M}]G(P,M) = 0 \qquad (52)$$

for $z_P < 0$ and $z_M \le 0$

It is evident that Brard's Green function does not satisfy the condition (52) which is different from the adjoint free surface condition defined by (35).

Besides, he had to assume that

$$\int\int_{F'} \frac{\partial \Psi(M)}{\partial x_M} G(P,M)ds = 0 \quad \text{for} \quad z_P < 0 \qquad (53)$$

in order to obtain the line integral associated with the Brard number. There is no reason that the equation (53) holds. Thus his Green integral equation is wrong. More recently, the same mistake has been made by Hong in his study on the two-dimensional radiation problem of a cylinder advancing on the free surface(Hong 1995).

The equation (51) is the exact Green integral equation which contains the correct boundary conditions.

5. Conclusion

The Green Integral equation for the forward-speed radiation-diffraction problem of a surface ship has been derived with correct boundary conditions on the free surface. The cause of mistakes in the existing Green integral equation is also pointed out.

Numerical validation of the present Green integral equation will be followed soon.

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