

시물레이션과 수리모델을 이용한 생산-분배 계획

A HYBRID SIMULATION -ANALYTIC METHOD FOR PRODUCTION-DISTRIBUTION PLANNING

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ABSTRACT

Production-distribution planning is the most important part in supply chain management (SCM). To solve this planning problem, either analytic or simulation approach has been developed. However these two approaches have their own demerits in problem solving. In this paper, we propose a hybrid approach which is a specific problem solving procedure combining analytic and simulation method to solve production-distribution problems in supply chain. The machine capacity and distribution capacity constraints in the analytic model are considered as stochastic factors and adjusted by the proposed specific process according to the results from independently developed simulation model which includes general production-distribution characteristics.

1. INTRODUCTION

The production-distribution planning is one of the most important parts in supply chain management (SCM). To implement SCM in real logistic world, supply chains have been modeled in the analytic ways using deterministic or stochastic method. However, most of realistic problems are not simple to apply these analytic ways only. Simulation is preferred when an analytic solution can not give proper values for performance evaluations. Therefore, the hybrid procedure integrating analytic and simulation model for solving production-distribution planning problem is a useful idea.

The objective of this paper is to develop an integrated multi period, multi product, multi shop production and distribution model in supply chain to satisfy the retailer's demand while keeping inventories as low as possible. We formulated the problem as an analytic model which minimizes the sum of production cost, distribution cost, inventory holding and deficit costs, subject to

capacity and inventory balance constraints and propose a hybrid method combining mathematical programming and simulation model to solve this problem.

2. DESCRIPTION OF PRODUCTION-DISTRIBUTION PROBLEM IN SUPPLY CHAIN

The production-distribution in supply chain can take on many forms. In general, two distinctive models: production and distribution model, designed to be linked together are considered as a production-distribution model in supply chain. These model are operationally connected and closely related with each other.

The first shop of production model produces n different products that are used in the production of m different products at the second shop of production model. An example is a general manufacturing system where the first shop consists of fabricating machines machining a

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number of different types of parts and the second shop contains assembling machines that use the parts in producing several types of products.

The distribution model contains stack buffers where all products produced in production model are temporarily stored and intermediate warehouses storing all kinds of products and retailers that are the origin of demand. Products are transported in unit size from stack buffers

either to warehouses or to retailers directly or moved from warehouses to retailers to satisfy their demands. The problem is to meet the production and distribution requirements at minimum costs of production, distribution and inventory, subject to various resource constraints. The structure of a multi period, multi product, multi shop production and distribution system in supply chain environment is described in Fig. 1.

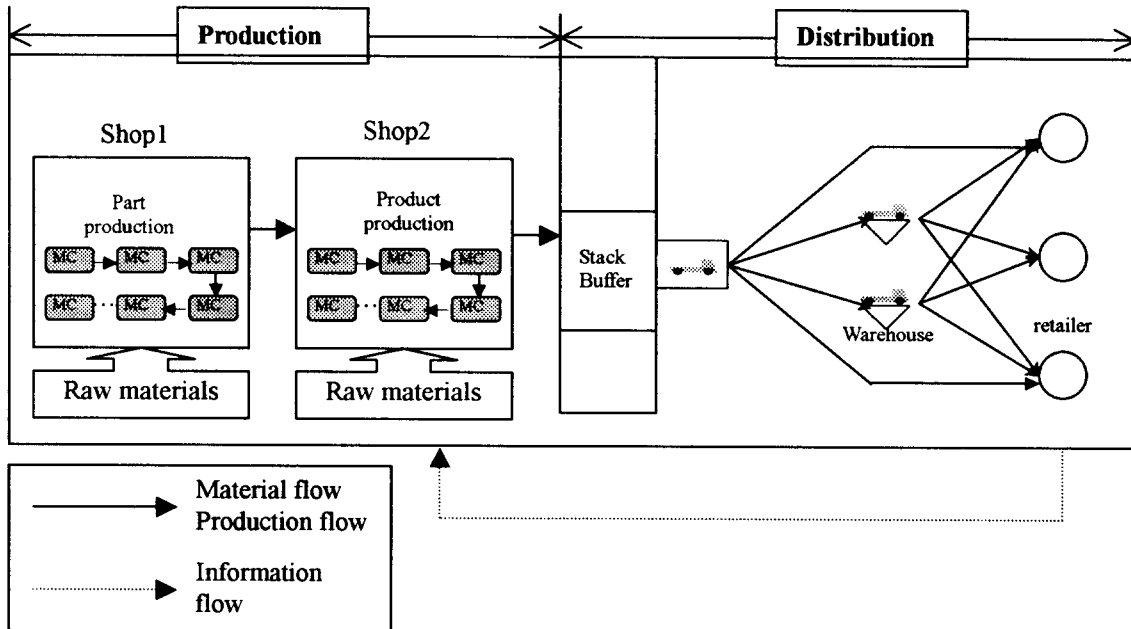


Figure. 1. A Proposed System in Supply Chain Environment

3. ANALYTIC MODEL DEVELOPMENT

Production and distribution model are connected and interrelated with each other. Therefore, these models should be modeled in an integrated way. We formulated this situation as a linear program explained in Appendix.

4. A HYBRID SIMULATION - ANALYTIC APPROACH

The hybrid simulation-analytic approach consists of building independent analytic and simulation model of the total system, developing their solution procedures, and using their solution procedures together for problem solving.

Generally, assigning the system capacity which is total operation time for production and distribution and which also represents machine center capacity and distribution center capacity in the analytic model is a difficult task. Therefore in

many researches, the capacity is known and fixed. But in the real systems significant difference exists between system capacity and the required time to achieve the production and distribution plan.

The procedure of the hybrid simulation-analytic approach is based on imposing adjusted capacities derived from the simulation model results. The procedure consists of the following steps:

- Step 1. Obtain production and distribution rates from the analytic model.
- Step 2. Input the production and distribution rates to the independently developed simulation model.
- Step 3. Simulate the system subject to realistic operational policies.
- Step 4. If the simulation model results show that production and distribution rates obtained from step 1 can be produced and distributed within the system capacity, then go to step 6. Otherwise go to step 5.

- Step 5. Adjust system capacity constraints for the analytic model based on the simulation results from step 3 and regenerate production and distribution rates and go to step 3.
- Step 6. Production and distribution rates given by the analytic model may be

- considered to be optimal solutions.
- Step 7. Stop.
The system capacity is the right hand side of constraint equation (7), (8), (19), (20) of analytic model.
This solution procedure is illustrated by a flow diagram in Fig. 2.

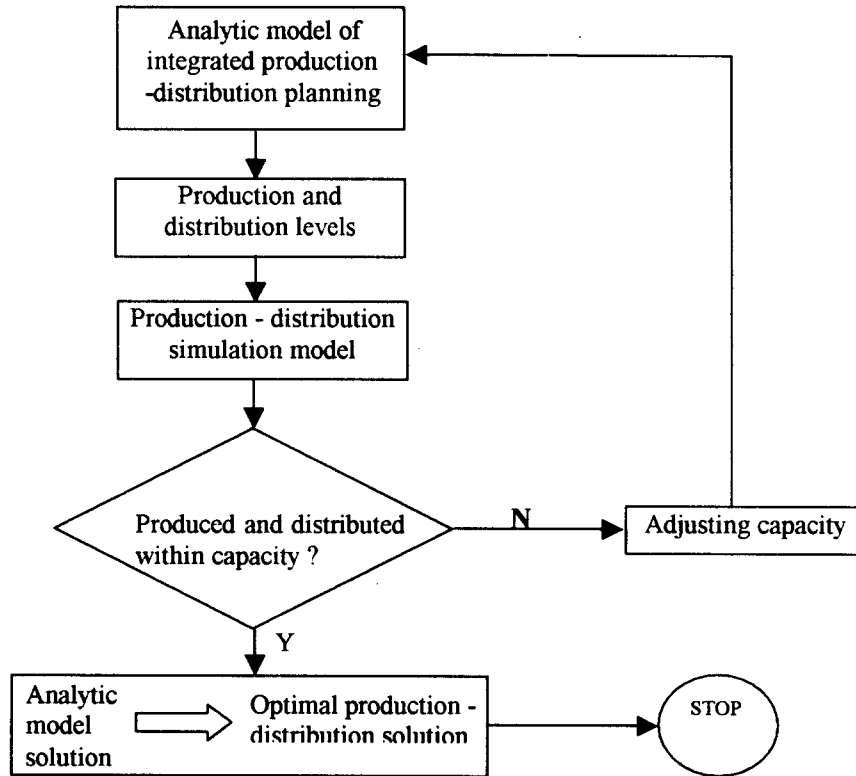


Figure. 2. Hybrid Solution Procedure

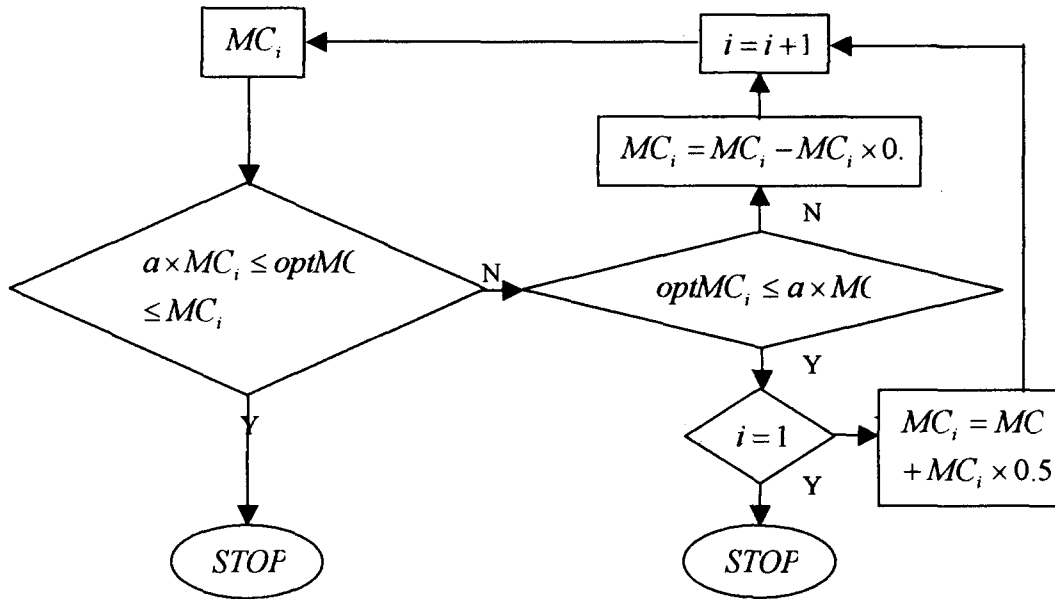
System capacity in the analytic model is considered as a stochastic factor and this factor is properly adjusted by a adjusting process to obtain the optimal system capacity in which production and distribution rates of analytic model can be produced and distributed through simulation.

Capacity adjustments are made through the following procedures.

- Step 1. System capacity is adjusted. Initial capacity of the system is given.
- Step 2. If the consumed simulation time to perform production and distribution rates of analytic model lies between the selected (criteria) capacity ($= 0.95 \times$ adjusted system capacity) and the

adjusted system capacity from step 1, then stop. Otherwise, go to step 3

- Step 3. If the consumed simulation time is less than or equal to the selected capacity then go to step 4. Otherwise go to step 5.
- Step 4. If the replication of simulation is the first one then stop. Otherwise go to step 6
- Step 5. New system capacity = system capacity - $0.5 \times$ system capacity. Update the replication number and go to step 1.
- Step 6. New system capacity = system capacity + $0.5 \times$ system capacity. Update the replication number and go to step 1.
- This adjusting procedure is illustrated in Fig. 3.



MC_i : System capacity of i^{th} replication for the system adjusted by adjusting procedure

$optMC_i$: Consumed simulation time in i^{th} replication for the system

a : Rate for capacity for comparison decided by planner (0.95 in this paper)

Figure. 3. Capacity Adjustment Procedure

5. EXPERIMENTS

The hybrid simulation-analytic approach is applied to a cost minimization problem of the production-distribution system in supply chain. The production system consists of 2 shops, each shop having 3 machine centers, each machine center having 1 machine and 1 input buffer and 1 output buffer. Parts and products are moved between machine centers by non-accumulating belt conveyors. The distribution system comprises of 1 stack and 2 warehouses and 3 retailers. Products are transported along the routes by the unit vehicle which load one unit of product. Stack and each warehouse have a unit vehicle. Capacity of total system is initially 3000minutes per period. The analytic part of the hybrid procedure is modeled as a linear program (LP). GAMS (General Algebraic Modeling System), a LP solver is adopted to implement the formulation. The demand and various cost data, process routings and process times, transportation times according to routes are given.

The simulation model of the system was developed by using simulation tool, ARENA, a

C++ based, simulation tool. The simulation model accommodates the stochastic characteristics of production-distribution system in supply chain, such as machine and vehicle breakdowns, repair times, queuing and transportation delays, delivery priority, routings which are difficult to be included in the analytic model. Therefore, the iterative hybrid procedure provides more realistically feasible optimal solutions.

The resulting optimal production and distribution plans through the iterations for each period are given in Table 1. The results are the same after the fifth iteration, so we considered the solution at the fifth iteration is the optimal solution. This shows that the simulation analysis of the system has not validated the initial optimal results of the analytic model. We observe that demand fill rate in distribution is lower than demand fill rate in production. Eventually, comparing with analytic solutions, demand fill rate for retailers in production is decreased down to 45.2% and demand fill rate for retailers in distribution is dwindled to 27.4%. In order to satisfy retailer's demand fully, system capacity should be increased. We may increase the number

of machine or vehicle or total system capacity.

		Periods	Demands	Initial Analytic Model Solutions	Iterative Solutions for Production						
					1	2	3	4	5	6	
Production Levels	Part	1	1	645	540	540	336	302	245	245	
			2	645	600	613	336	302	245	245	
			3	330	480	467	336	302	245	245	
	2	1	1	968	810	810	503	453	367	367	
			2	968	900	919	503	453	367	367	
			3	494	720	701	503	453	367	367	
	Product	1	1	75	75	75	75	34	30	24	24
				2	80	80	80	14	5	0	0
				3	60	60	60	54	50	44	44
2		1	60	86	60	60	50	45	37	37	
			2	70	81	70	73	70	61	61	
			3	60	23	60	57	30	25	17	17
Adjusted system capacity (min)				3000	2143.5	1531.5	838.8	754.9	611.5	611.5	
Average demand fill rate for retailer (%)				100	100	100	62.2	55.6	45.2	45.2	

		Products	Periods	Distribution Routes	Initial Analytic Model Solutions	Iterative Solutions for Distribution						
						1	2	3	4	5	6	
Distribution Plans	1	1	L-P	70	56	10	0	0	0	0		
				L-Q	5	0	0	0	0	0	0	
					P-Q	70	56	10	0	0	0	0
			2			L-P	17	75	64	30	30	30
				L-Q			43	0	0	0	0	0
					P-Q		17	60	60	30	30	30
			3			L-P	67	65	46	20	20	20
				L-Q			13	0	0	0	0	0
					P-Q		67	80	50	20	20	20
	2	1	L-P			53	73	85	51	45	36	36
				L-Q		17	0	0	0	0	0	0
					P-Q	53	70	70	51	41	29	29
		2	L-P			60	57	25	21	15	7	7
				L-Q		0	0	0	0	0	0	0
					P-Q	60	60	40	20	20	15	15
		3	L-P			50	60	45	31	25	17	17
				L-Q		10	0	0	0	0	0	0
					P-Q	50	60	45	30	25	17	17
Adjusted system capacity (min)						3000	2143.5	1531.5	838.8	754.9	611.5	611.5
Average distribution rates for initial analytic solutions (%)				100		95.3	67.9	37.3	33.6	27.4	27.4	

L: Stack, P: Warehouse, Q: Retailer
Optimal system capacity: 611.5 minutes

Table. 1. A Hybrid Simulation-Analytic Results

6. CONCLUSIONS

In this paper, we present a hybrid method combining the analytic and simulation model for an integrated production-distribution system in supply chain environment. The hybrid method uses advantages of both modeling methods while avoiding demerits of both methods. The solution procedure of independently developed analytic and simulation models are used together to solve the problem. We also present an efficient algorithm that can find more realistic and optimal

system capacity for production and distribution. Through experiments of the hybrid method, we verified that the initial analytic solutions can not be accepted in the real world system having stochastic characteristics which are not included in analytic model. Hybrid method provides more realistic optimal solutions for the integrated production-distribution planning in supply chain that are quite different from the initial analytic solutions. Finding the optimal capacities through hybrid method satisfying fluctuated customer demands in supply chain during certain time

period under real world situation is a further
APPENDIX

The mathematical formulation

Index

t : period index ($t = 1, 2, 3, \dots, T$)
 i : part index in shop1 of production model ($i = 1, 2, 3, \dots, N$)
 j : product index in shop2 of production model ($j = 1, 2, 3, \dots, M$)
 v : machine index for shop1 ($v = 1, 2, 3, \dots, V$)
 u : machine index for shop2 ($u = 1, 2, 3, \dots, U$)
 k : raw material index for shop1 ($k = 1, 2, 3, \dots, K$)
 r : raw material index for shop2 ($r = 1, 2, 3, \dots, R$)
 l : stack point index
 p : warehouse index ($1, 2, 3, \dots, P$)
 q : retailer index ($1, 2, 3, \dots, Q$)
 D_{jt} : demand for product j in period t
 a_{ij} : number of units of part i used to make one unit of product j
 d_{ki} : number of units of raw material k used to make one unit of part i
 g_{rj} : number of units of raw material r used to make one unit of product j
 b_{kt} : available amount of raw material k in period t
 b_{rt} : available amount of raw material r in period t
 C_{it} : cost to produce a unit of part i in period t
 C_{jt} : cost to produce a unit of product j in period t
 C_{kt} : cost to buy a unit of raw material k in period t
 C_{rt} : cost to buy a unit of raw material r in period t
 H_{it} : cost to hold a unit of part i in period t
 H_{jt} : cost to hold a unit of product j in period t
 H_{kt} : cost to hold a unit of raw material k in period t
 H_{rt} : cost to hold a unit of raw material r in period t
 π_{it} : unit cost of sales lost for part i in period t
 π_{jt} : unit cost of sales lost for product j in period t

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π_{kt} : unit cost of deficit for raw material k in period t
 π_{rt} : unit cost of deficit for raw material r in period t
 α_{iu} : processing time to produce a unit of part i on machine center u
 α_{jv} : processing time to produce a unit of product j on machine center v
 MC_{ut} : capacity of machine center u in period t
 MC_{vt} : capacity of machine center v in period t
 DEM_{qjt} : demand for product j from retailers q in period t
 SL_{jt} : storage cost of product j at stack point in period t
 SP_{pjt} : storage cost of product j at warehouse p in period t
 SQ_{qjt} : storage cost of product j at retailer q in period t
 SLL_{jt} : shortage cost of product j at stack point in period t
 SPP_{pjt} : shortage cost of product j at warehouse p in period t
 SQQ_{qjt} : shortage cost of product j at retailer q in period t
 LPC_p : the cost of transporting any product from stack point to warehouse p
 LQC_q : the cost of transporting any product from stack point to retailer q
 PQC_{pq} : the cost of transporting any product from warehouse p to retailer q
 TQ_{qt} : product holding capacity at retailer q in period t
 TP_{pt} : product holding capacity at warehouse p in period t
 SB_t : product holding capacity at stack buffer in period t
 TC_t : the distribution capacity at stack point in period t
 TC_{pt} : the distribution capacity at warehouse p in period t

a_p : distribution time to transport any product from stack point to warehouse p

b_q : distribution time to transport any product from stack point to retailer q

c_{pq} : distribution time to transport any product from warehouse p to retailer q

Decision Variables

X_{it} : number of units of part i at shop1 in period t

Y_{jt} : number of units of product j at shop2 in period t

I_{it}^+ : amount of end of period inventory of part i in period t

I_{it}^- : amount of end of period deficit of part i in period t

I_{jt}^+ : amount of end of period inventory of product j in period t

I_{jt}^- : amount of end of period deficit of product j in period t

E_{kt} : number of units of raw material k in period t

F_{rt} : number of units of raw material r in period t

I_{kt}^+ : amount of end of period inventory of raw material k in period t

I_{kt}^- : amount of end of period deficit of raw material k in period t

I_{rt}^+ : amount of end of period inventory of raw material r in period t

I_{rt}^- : amount of end of period deficit of raw material r in period t

LP_{pjt} : amount of product j transported from stack point to warehouse p in period t

LQ_{ajt} : amount of product j transported from stack point to retailer q in period t

PQ_{pajt} : amount of product j transported from warehouse p to retailer q in period t

L_{jt} : amount of product j stored at stack point in period t

P_{pjt} : amount of product j stored at warehouse p in period t

Q_{ajt} : amount of product j stored at retailer q in period t

L_{jt}^+ : amount of end of period inventory of product j at stack point in period t

L_{jt}^- : amount of end of period deficit of product j at stack point in period t

Q_{ajt}^+ : amount of end of period inventory of product j at retailer q in period t

Q_{ajt}^- : amount of end of period deficit of product j at retailer q in period t

P_{pjt}^+ : amount of end of period inventory of product j at warehouse p in period t

P_{pjt}^- : amount of end of period deficit of product j at warehouse p in period t

Objective Function

$$\text{Min } Z = \sum_{t=1}^T \left\{ \begin{aligned} & \sum_{i=1}^N C_{it} X_{it} + H_{it} I_{it}^+ + \pi_{it} I_{it}^- + \sum_{j=1}^M C_{jt} Y_{jt} + H_{jt} I_{jt}^+ + \pi_{jt} I_{jt}^- \\ & \sum_{k=1}^K C_{kt} E_{kt} + H_{kt} I_{kt}^+ + \pi_{kt} I_{kt}^- + \sum_{r=1}^R C_{rt} F_{rt} + H_{rt} I_{rt}^+ + \pi_{rt} I_{rt}^- \\ & \sum_j \sum_t SL_{jt} L_{jt}^+ + SLL_{jt} L_{jt}^- + \sum_p \sum_j \sum_t SP_{pjt} P_{pjt}^+ + SPP_{pjt} P_{pjt}^- \\ & + \sum_q \sum_j \sum_t SQ_{qjt} Q_{qjt}^+ + SQQ_{qjt} Q_{qjt}^- + \sum_p \sum_j \sum_t LPC_p LP_{pjt} \\ & + \sum_q \sum_j \sum_t LQC_q LQ_{qjt} + \sum_p \sum_q \sum_j \sum_t PQC_{pq} PQ_{pqjt} \end{aligned} \right\}, \forall i, j, p, q, t$$

Subject to:

$$I_{jt} = I_{jt-1} + Y_{jt} - D_{jt}, \forall j, t \quad (1)$$

$$I_{it} = I_{it-1} + X_{it} - \sum_{j=1}^M a_{ij} Y_{jt}, \forall i, t \quad (2)$$

$$I_{rt} = I_{rt-1} + F_{rt} - \sum_{j=1}^M g_{rj} Y_{jt}, \forall r, t \quad (3)$$

$$I_{kt} = I_{kt-1} + E_{kt} - \sum_{i=1}^N d_{ki} X_{it}, \forall k, t \quad (4)$$

$$\sum_{j=1}^M g_{rj} Y_{jt} \leq b_r, \forall r, t \quad (5)$$

$$\sum_{i=1}^N d_{ki} X_{it} \leq b_k, \forall k, t \quad (6)$$

$$\sum_{j=1}^M a_{jv} Y_{jt} \leq MC_v, \forall v, t \quad (7)$$

$$\sum_{i=1}^N a_{iu} X_{it} \leq MC_u, \forall u, t \quad (8)$$

$$I_{jt} = I_{jt}^+ - I_{jt}^-, \forall j, t \quad (9)$$

$$I_{it} = I_{it}^+ - I_{it}^-, \forall i, t \quad (10)$$

$$I_{rt} = I_{rt}^+ - I_{rt}^-, \forall r, t \quad (11)$$

$$I_{kt} = I_{kt}^+ - I_{kt}^-, \forall k, t \quad (12)$$

$$\sum_p LP_{pjt} + \sum_q LQ_{qjt} - L_{jt-1} \leq Y_{jt} + L_{jt}, \forall j, t \quad (13)$$

$$LQ_{qjt} + \sum_p PQ_{pqjt} - Q_{qjt-1} \geq DEM_{qjt} + Q_{qjt}, \forall q, j, t \quad (14)$$

$$P_{pjt-1} + LP_{pjt} = \sum_q^Q PQ_{pqjt} + P_{pjt}, \forall p, j, t \quad (15)$$

$$\sum_{j=1}^M Y_{jt} \leq SB_t, \forall t \quad (16)$$

$$\sum_{j=1}^M LQ_{qjt} + \sum_{p=1}^P \sum_{j=1}^M PQ_{pqjt} \leq TQ_{qt}, \forall q, t \quad (17)$$

$$\sum_{j=1}^M LP_{pjt} \leq TP_{pt}, \forall p, t \quad (18)$$

$$\sum_p^P \sum_j^M a_p \times LP_{pjt} + \sum_q^Q \sum_j^M b_q \times LQ_{qjt} \leq TC_t, \forall t \quad (19)$$

$$\sum_q^Q \sum_j^M c_{pq} \times PQ_{pqjt} \leq TC_{pt}, \forall p, t \quad (20)$$

$$L_{jt} = L_{jt}^+ - L_{jt}^-, \forall j, t \quad (21)$$

$$Q_{qjt} = Q_{qjt}^+ - Q_{qjt}^-, \forall q, j, t \quad (22)$$

$$P_{pjt} = P_{pjt}^+ - P_{pjt}^-, \forall p, j, t \quad (23)$$

$$I_{jt}^+, I_{jt}^-, Y_{jt} \geq 0, \forall j, t, I_{it}^+, I_{it}^-, X_{it} \geq 0, \forall i, t, I_{rt}^+, I_{rt}^-, F_{rt} \geq 0, \forall r, t, I_{kt}^+, I_{kt}^-, E_{kt} \geq 0, \forall k, t \quad (24)$$

$$L_{jt}^+, L_{jt}^-, P_{jt}^+, P_{jt}^-, Q_{jt}^+, Q_{jt}^- \geq 0, \forall j, t, LP_{pjt}, LQ_{qjt}, PQ_{pqjt} \geq 0, \forall p, q, j, t \quad (25)$$

The objective of the model is to minimize the overall cost: production cost, transportation cost, inventory cost, shortage cost. Constraint equation (1), (2) are the balance equations for the product and part inventory. Constraint equation (3), (4) are the balance equations for the inventory of raw materials for products and the inventory of raw materials for parts. Constraint equation (5), (6) are the resource availability constraints for product and parts. Constraint equation (7), (8) are machine center capacity constraints. Constraint equation (9), (10) are the net inventory equations for products and parts. Constraint equation (11), (12) are the net inventory equations for raw

materials for products and parts. Constraint equation (13) is the availability constraints for the stack. Constraint equation (14) is the demand constraint for retailers. Constraint equation (15) is the balance equation for the warehouse. Constraint equation (16), (17), (18) are the product holding capacity for stack buffer and warehouses and retailers. Constraint equation (19), (20) are the distribution capacity constraints for the stack and warehouses. Constraint equation (21), (22), (23) are the net inventory equations for the stack and warehouses and retailers. Constraint equations (24), (25) enforce the non-negativity restriction on the decision variables.

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