Solution to Lamb's Problem by Finite Difference Midpoint-Averaging Scheme

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1. Introduction

The numerical realization of Lamb's problem is a difficult problem in elastic wave equation modeling by ordinary FD scheme. Inaccurate incorporation of the free-surface boundary conditions results in decaying of the amplitude of Rayleigh wave. Therefore, We propose here the finite-difference midpoint-averaging scheme and present the solution to 2-D Lamb's problem to verify the developed scheme.

2. Numerical Experiment

To derive the finite-difference approximations, we begin our treatment with the integration method approach that is known as effective one in treating Neumann-type boundary conditions. Then, the free-surface condition can be easily incorporated by treating them as interface conditions. Furthermore, we also incorporate the weighted-averaging sense into these approximation procedures to maximize the efficiency of computational grids without further increasing the numerical bandwidth. As a result, we have a finite-difference scheme that uses not only the collocation point but also nearby midpoints. The finite-difference approximation of spatial derivatives at a collocation point are given by the averages of the approximation at the collocation point and/or those at nearby midpoints as shown in figure 1. For the consistency of the problem, we also approximate the mass acceleration at the collocation point with the weighted-sum of all masses at nine grid points within the computational grid.

Figure 2 shows the horizontal and vertical displacements at 500m apart from the source for a halfspace model of which the velocity of P- and S-wave are given by 2000m/s and 1000m/s, respectively. To test the accuracy of the solutions, we compared them with analytical solutions and other numerical solutions by 25-point weighted-averaging scheme(Min, 1998) and by FEM. As observed in figure, our solutions show good agreement with analytical solution with better degree of accuracy than others. Especially, the agreement in Rayleigh wave arrivals is remarkable when considering that ordinary finite-difference schemes fail to match it.

3. Conclusion

We have presented the solution to 2-D Lamb's problem and have compared the results with analytical solutions and other numerical solutions. Our solutions have shown good agreement with analytical solution. Thus, we can conclude that (1) the midpoint scheme can be an alternative in incorporating the free-surface boundary conditions into elastic modeling by FDM, and that (2) it can be a useful tool in earthquake engineering and shallow seismic application where the accurate implementation of stress-free conditions and the accurate simulation of Rayleigh wave are of great important issues.

References

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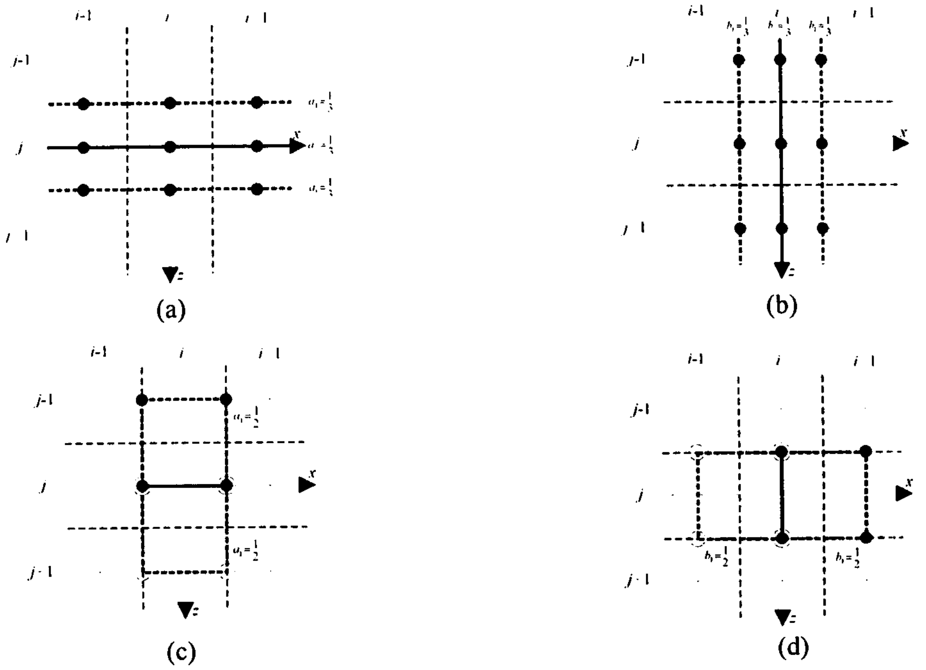

Fig. 1 Grid points used to approximate non-mixed and mixed spatial derivatives: (a) for $\frac{\partial}{\partial x}\left(\alpha\frac{\partial u}{\partial x}\right)$, (b) for $\frac{\partial}{\partial z}\left(\beta\frac{\partial u}{\partial z}\right)$, (c) for $\frac{\partial}{\partial x}\left(\alpha'\frac{\partial w}{\partial z}\right)$ and (d) for $\frac{\partial}{\partial z}\left(\beta'\frac{\partial w}{\partial x}\right)$.

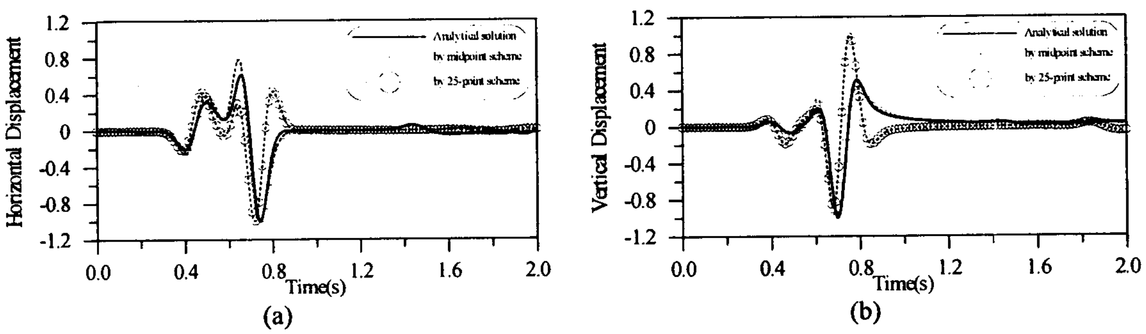

Fig. 2 The comparison of solutions to 2-D Lamb's problem at the location apart from the source: (a) for horizontal and (b) for vertical displacement. The number of grid points used to obtain the numerical solutions are 10 for midpoint scheme and 3.3 for 25-points scheme, respectively.

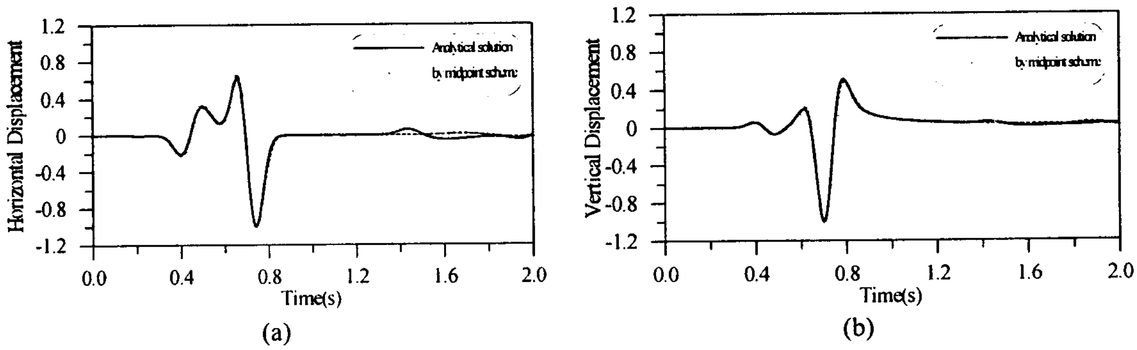

Fig. 3 The comparison of solutions to 2-D Lamb's problem at the location 500m apart from the source: (a) for horizontal and (b) for vertical displacement. Numerical solutions are obtained by using 15 grid points per minimum wavelength.