A No-Charge Integral Equation Formulation for Three-Dimensional Electromagnetic Modeling

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1. Introduction

Volume integral equation (IE) approaches represent a three-dimensional (3-D) inhomogeneity in a layered earth host with an equivalent scattering current, and the secondary, or scattered, field is specified as a volume integral over the scattering, or source current. If the scattering current can be approximated with pulse basis functions, which are the most usual and simple, the numerical solution will fail as the host conductivity becomes small or the operating frequency goes down. There are two main numerical difficulties.

In the IE approach, the inhomogeneity is discretized into a finite number of volume cells with discretized electric fields defined inside the cells. This implies, among other things, that the boundary condition of continuity of the tangential components of the electric field at conductivity discontinuities is not fulfilled; spurious charges are created even in a homogeneous region due to artificial discontinuities made by the basis functions. It is easily seen that relaxing this condition will diminish the ability to produce induction currents in a conducting body in a resistive host. By continuity, the strong electric field in the resistive host forces strong currents to flow on the surface of the body.

The other numerical difficulty associated with solving scattering currents is due to a disparity in sizes of the current and charge operators. Lajoie and West (1976) avoided the disparate-operator problem by solving for curl-free and divergence-free scattering currents on a thin 3-D plate in a conductive half-space. Weidelt (1981) further modified their approach to greatly improve the stability of modeling process. He derived that the contribution of the divergence-free potential to the charge term of Green's tensors disappeared. Recently, Song et al. (1998) modeled high-frequency responses of thin sheets using the Weidelt's approach.

In this paper, we discuss the spurious charge and disparate-operator problems associated with IE modelings using the simple pulse basis functions. In the IE formulation divergence of electric fields is forced to be free in the homogeneous 3-D body. This avoids the creation of spurious charges within the body, and is effective for solving the disparate-operator problem. Finally, we derive a no-charge IE formulation of the thin sheet approximation for EM scattering problems.

2. IE formulation

We denote the distribution of electrical conductivity with the function $\sigma(\mathbf{r})$. Neglecting displacement currents and choosing a time dependence of $e^{+i\omega t}$, an integral equation for electric fields $\mathbf{E}(\mathbf{r})$ can be represented by (Hohmann, 1988)

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_p(\mathbf{r}) + \mathbf{E}_s(\mathbf{r}) = \mathbf{E}_p(\mathbf{r}) - \hat{z}\int_{r'} \underline{\underline{\mathbf{G}}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_s(\mathbf{r}')d\mathbf{r}', \qquad (1)$$

where $\mathbf{E}_p(\mathbf{r})$ is the primary electric field one would obtain for the background media of σ_b, $\mathbf{E}_s(\mathbf{r})$ the secondary, or scattered, field calculated via scattering currents $\mathbf{J}_s(\mathbf{r})$, $\hat{z} = i\omega\mu$ the impedivity, μ the magnetic permeability which is assumed to be equal to that of free space, and $\mathbf{G}(\mathbf{r},\mathbf{r}')$ the Green's tensor relating the electric field at $\mathbf{r}$ to a current element at $\mathbf{r}'$.

The scattering current is defined by

$$\mathbf{J}_s(\mathbf{r}) = \Delta\sigma(\mathbf{r})\mathbf{E}(\mathbf{r}), \qquad (2)$$

with the conductivity anomaly $\Delta\sigma(\mathbf{r}) = \sigma(\mathbf{r}) - \sigma_b$. The Green's tensor can be decomposed into current and charge accumulation terms (Song et al., 1998):

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$$\underline{\underline{\mathbf{G}}}(\mathbf{r},\mathbf{r}') = \underline{\underline{\mathbf{S}}}(\mathbf{r},\mathbf{r}') + \frac{1}{k_h^2}\nabla\mathbf{\Phi}(\mathbf{r},\mathbf{r}'), \quad (3)$$

where $k_h^2 = -\hat{z}\sigma_h$ denotes the wave propagation constant. Substituting equation (3) into equation (1) yields

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_p(\mathbf{r}) - \hat{z}\int_V \underline{\underline{\mathbf{S}}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_s(\mathbf{r}')d\mathbf{r}' + \frac{1}{\sigma_h}\int_V \nabla\mathbf{\Phi}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_s(\mathbf{r}')d\mathbf{r}'. \quad (4)$$

This equation clearly shows the disparate-operator problem. Also, if equation (4) were discretized using the pulse basis functions, we would not be free from the spurious charge effect.

3. Thin sheet approximation

Consider a thin conductive plate, which is described by the unit vector $\hat{\mathbf{a}}$ in strike direction and $\hat{\mathbf{b}}$ in dip direction. For this thin sheet in a layered half-space, the Green's tensor and scattering current are represented by (Weidelt, 1981; Song et al., 1998)

$$\underline{\underline{\mathbf{G}}}_S = \underline{\underline{\mathbf{S}}} + \frac{1}{k_h^2}\nabla_S\mathbf{\Phi}, \qquad \text{and} \qquad \mathbf{J}_S = \nabla_S\times(\hat{\mathbf{c}}\psi) + k_h^2\nabla_S\varphi, \quad (5)$$

where $\hat{\mathbf{c}} = \hat{\mathbf{a}}\times\hat{\mathbf{b}}$ and $\nabla_S = \partial\hat{\mathbf{a}}/\partial a + \partial\hat{\mathbf{b}}/\partial b$. Note that the vortex current $\nabla_S\times(\hat{\mathbf{c}}\psi)$ is confined in the sheet so that the potential ψ is constant along the edge of the surface S; the vortex current component of the secondary field vanishes for the thin sheet model. Hence, substituting equation (5) into equation (4) yields

$$\mathbf{E}_S(\mathbf{r}) = \mathbf{E}_S^p(\mathbf{r}) - \hat{z}\int_S\left\{\underline{\underline{\mathbf{S}}}\cdot\left[\nabla_S\times(\hat{\mathbf{c}}\psi) + k_h^2\nabla_S\varphi\right] + \nabla_S\mathbf{\Phi}\cdot\nabla'_S\varphi\right\}dS. \quad (6)$$

This is the same IE derived by Weidelt (1981), and has no disparity problem in sizes of the current and charge operators.

A more advanced IE for the thin sheet model can be obtained from enforcing the divergence of scattering currents to be free as

$$\mathbf{E}_S(\mathbf{r}) = \mathbf{E}_S^p(\mathbf{r}) - \hat{z}\int_S\underline{\underline{\mathbf{S}}}\cdot\left[\nabla_S\times(\hat{\mathbf{c}}\psi) + k_h^2\nabla_S\varphi\right]dS - \hat{z}\oint_{\partial S}\hat{\mathbf{n}}'_l\cdot(\nabla'_S\varphi\mathbf{\Phi})dl, \quad (7)$$

where $\hat{\mathbf{n}}_l$ indicates the unit vector normal to the edge of the sheet. This equation has no spurious charge effect through introducing the line integral along the edge of the sheet, which is expected to enhance the accuracy of the solution.

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