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Estimation of Parameters in Fuzzy Time Series Model with Triangular Fuzzy Numbers

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Abstract

Using the fuzzified coefficients, ARMA processes can be extended to fuzzy time series model. In this paper, the estimation of parameters in the fuzzy time series model with asymmetric triangular fuzzy coefficients is studied. Nonlinear programming is applied to get solutions of parameters.

Keywords : Fuzzy number, Fuzzy time series, ARMA process

1. Introduction

In order to model some dynamic process and to apply the models for forecasting under the fuzzy environments in which historical data are of linguistic values like 'very hot', 'quite good' or 'very very cold'. In these cases, fuzzy time series model related to ARIMA process have been considered by many authors. Song, Chissom and Leland[2-5] provided a definition of fuzzy time series and proposed a model by means of fuzzy relational equations to deal with forecasting problems. The fuzzy regression model was suggested by Tanaka et al.[6-8]. Watada[9] also suggested a fuzzy times series using fuzzy regression modeling. Tzeng et al.[10] gave a method combining the advantages of the concepts of fuzzy regression model and SARIMA process.

2. Proposed Model

Consider the seasonal AR process SAR(p,P) by

$$W_t = \sum_{i=1}^{p} \phi_i W_{t-i} + \sum_{i=1}^{P} \Phi_i W_{t-is} - \sum_{i=1}^{p} \phi_i \Phi_1 W_{t-s-i} - \sum_{i=1}^{p} \phi_i \Phi_2 W_{t-2s-i} - \cdots - \sum_{i=1}^{p} \phi_i \Phi_P W_{t-Ps-i} + a_t$$

where W_t is the time series, ϕ, Φ's are coefficients and a_t is a white noise process with mean 0 and variance σ_a^2. This SAR process can be generalized to fuzzy SAR model with fuzzy numbers as coefficients. A triangular fuzzy number A is determined as $A = [\, u, m, v]$ with a membership function

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$$\mu_A(x) = \begin{cases} 1-(m-x)/u & , \ m-u < x \le m \\ 1-(x-m)/v & , \ m < x \le m+v \\ 0 & , \ otherwise \end{cases}$$

where $m(-\infty < m < \infty)$ is the center, and $u(v)$ is the left(right) width.

The fuzzy time series model with triangular fuzzy numbers can be expressed by

$$\widetilde{W}_t = \sum_{i=1}^{p} \widetilde{\beta}_i W_{t-i} + \sum_{i=1}^{P} \widetilde{\beta}_{p+i} W_{t-is} + \sum_{j=1}^{P} \sum_{i=1}^{p} \widetilde{\beta}_{jp+P+i} W_{t-js-i}$$

The fuzzy parameters are in the form of triangular fuzzy numbers: $\beta_i = (c_{i,L}, a_i, c_{i,R})$ with a membership function

$$\mu_{\widetilde{W}}(W_t) = \begin{cases} 1-\dfrac{W_t - \widetilde{W}_{t,C}}{C_{L,W_t}} & , \ \widetilde{W}_t - C_{L,W_t} \le W_t \le \widetilde{W}_t \\ 1+\dfrac{W_t - \widetilde{W}_{t,C}}{C_{R,W_t}} & , \ \widetilde{W}_t + C_{R,W_t} \ge W_t \ge \widetilde{W}_t \\ 0 & , \ otherwise \end{cases}$$

where

$$\widetilde{W}_{t,C} = \sum_{i=1}^{p} a_i W_{t-i} + \sum_{i=1}^{P} a_{p+i} W_{t-is} + \sum_{j=1}^{P} \sum_{i=1}^{p} a_{jp+P+i} W_{t-js-i}$$

$$C_{L,W_t} = \sum_{i=1}^{p} c_{i,L} \mid W_{t-i} \mid + \sum_{i=1}^{P} c_{p+i,L} \mid W_{t-is} \mid + \sum_{j=1}^{P} \sum_{i=1}^{p} c_{jp+p+i,L} \mid W_{t-js-i} \mid$$

$$C_{R,W_t} = \sum_{i=1}^{p} c_{i,R} \mid W_{t-i} \mid + \sum_{i=1}^{P} c_{p+i,R} \mid W_{t-is} \mid + \sum_{j=1}^{P} \sum_{i=1}^{p} c_{jp+p+i,R} \mid W_{t-js-i} \mid$$

$\mu_{\widetilde{W}}(W_t) \ge h$ for a chosen $h(0 \le h \le 1)$ and $t = 1, 2, \cdots, k$.

In order to estimate fuzzy parameters, i.e. the values which minimize the vagueness S of the model are to be obtained by following nonlinear programming problem :

Minimize S^n where $S = \sum_{t=1}^{k} (C_{L,Z_t} + C_{R,Z_t})$, $n = 1, 2, \cdots$

subject to $\widetilde{W}_{t,C} + (1-h)C_{L,Z_t} \ge W_t$,

$\widetilde{W}_{t,C} - (1-h)C_{R,Z_t} \le W_t$, $t = 1, 2, \cdots, k$

$c_i \ge 0$, $^{\forall} i = 1, 2, \cdots, (p+1)P + p$.

The nonlinear programming is used for the estimation of parameters and found that this is better than the linear programming for getting appropriate solutions.

3. Simulation Study

Simulation study is performed for the estimation of the proposed model. For example, the fuzzy AR(1) process, noted by FAR(1), is considered. As results, (1) the fuzzy time series are generated from the equation: $W_t = (200, 1700, 400) + (0.3, 0.7, 0.2)\, W_{t-1}$. By the fuzzy regression procedure, the estimated coefficients with the proposed model FAR(1) are

$$W_t = (231.43, 1157.69, 231.31) + (0.16, 079, 0.15)\, W_{t-1},$$

and that of SAR method is $W_t = 2890 + 0.982\, W_{t-1}$. (2) the sum of squared residuals are 8.56E+06 for the fuzzy time series model and 2.80E+08 for SAR model.

As a further research, the scheme of nonlinear programming problem, including the choice of the value of n, which does not have crystal clear outcome need to be studied.

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