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Magnetic Dipole Model in an Eddy Current Flow Detection for a Nondestructive Evaluation

S. G. Han, J. Y. Kim, and J. H. Kang
Department of physics, University of Inchon, Inchon 402-743, Rep. of Korea

Abstract

A SQUID magnetometer or a SQUID gradiometer can be used to measure the field or gradient distribution, respectively. We describe the magnetic dipole model of the eddy currents in a nondestructive evaluation. Such a theoretical calculation of the magnetic dipole fields produced by a deep flaw in metallic materials can be used in aerospace and transportation industries.

I. Introduction

The detection and characterization of subsurface anomalies in conducting structures are an important problem for both geophysics and nondestructive evaluations (NDE). The works in NDE are mostly covered by the classical methods, such as ultrasonics, electromagnetics and visual inspections. Historically, progresses in NDE have been closely related to the development of the science in that all the physical phenomena are the potential candidates for nondestructive measurements of the material properties or the detection of flaws. In the case of electromagnetic NDE methods, the use of eddy current phenomena for metals classification is of the same age as Maxwell's treatise [1].

However, a lot of the related literatures were not written until 1970s. Since then widespread, sustained effort has gone into the development of and the modeling of NDE techniques [2]. This reappearance of interest has been brought about by the use of metal structures in the most demanding environments and the accompanying expenses, both economic and social, of large failures, particularly in the aerospace, transportation and power industries.

Traditional electromagnetic NDE methods cover a wide range of frequency from DC to microwave. The common industrial techniques are limited to active DC, and eddy current forms of excitation ; all low frequency phenomena for which displacement current effects can be neglected.

In this paper, a magnetic dipole model has been used to apply to the electronic NDE methods. The use of such a dipole moment model could gain the insights of material flaw characterization or the detection and the test conditions of the eddy current methods. This is shown by; 1) the development of the relations between the material flaw dimensions and the most adequate frequency, 2) the development of the relationships between the frequency and the sample material. This is largely accomplished by the determination of the effect of the flaws and sample conductor slab thickness. Such methods are useful in many industrial applications. These applications include integrity testing and performance or quality assurance for aircraft and nuclear reactor components, as well as critically important highly stressed components of machinery of all sorts (aircraft engines, turbine blades, etc).

II. magnetic dipole model

Maxwell's equations form the basis of the formulation of this analysis. Namely, the formulation in the presence of a nonconducting flaw is given by

$$\nabla \times E = i\omega\mu H \qquad (1)$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}. \qquad (2)$$

In equation (2), J is the conduction current and $\frac{\partial D}{\partial t}$ is the displacement current which can be ignored yielding

$$\nabla \times H = J, \qquad (3)$$

where $J = \sigma E$. It is important to realize that the $\frac{\partial D}{\partial t}$ term represents a redistribution of electric charges. The current involved in this redistribution is small and may be neglected in low frequency applications. However the electric field due to the charge involved in the displacement current significantly affects the electric field and therefore the eddy currents.

One calculates the magnetic field intensity H from equations (1) and (3) at the measurement point when no conductors are present.

$$H = H_0 \exp\left(-\frac{E}{\delta} - j\frac{z}{\delta}\right) \qquad (4)$$

where δ is the skin depth and z is the thickness of the conductor that shields. The exponential decay, $\exp\left(-\frac{z}{\delta}\right)$ is appropriate for an incident plane wave, where the incident field is uniform over the entire surface. The magnetic dipole field produced by flaws, however, cannot be utilized as a plane wave since the detector is placed in its near zone region.

The skin depth is given by as the following:

$$\delta = \sqrt{\frac{2}{\omega \mu_0 \mu_r \sigma}}. \qquad (5)$$

In equation (5), $\omega = 2\pi f$ is the angular frequency, σ is the conductivity, μ_r is the relative permeability of the conducting material and μ_0 is the free-space permeability.

Detailed modelling of the field inside the metal is especially valuable where the skin depth or the electromagnetic penetration depth is larger than, or comparable with, the flaw depth.

Application of a magnetometer or a gradiometer in NDE relies on the fact that structural defects will distort current flow in the testing materials, This will alter the field and gradient distribution. This principle can be illustrated by using a current applied to a rectangular metal slab. In a long metal slab, such a current can be assumed to be evenly distributed across the uniform slab. When a flaw is present in the metal, it will distort the current distribution at its site as shown in Fig. 1(a). This distortion can be viewed as a superposition of three parts as in Fig. 1(b) : an undisturbed straight current flow, a clockwise current flow loop and offset by a distance characteristic of

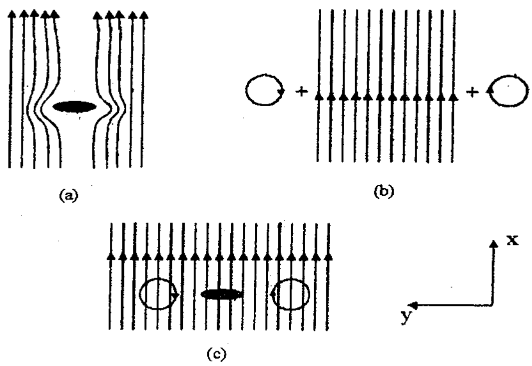

Fig. 1. Diagram of a structural defect viewed as a superposition of three parts.

the flaw dimensions, a counterclockwise current flow loop. The two loops generate two anti-parallel magnetic dipole moment as in Fig. 1(c). Thus we can use a SQUID magnetometer or a SQUID gradiometer to measure the strength or gradient of such fields, respectively.

As a simple illustration of the effects of coil on the eddy current field, we now consider the case of an infinitesimal current loop or equivalently, an oscillating magnetic dipole, above a solid aluminum plate of finite thickness. As shown in Fig 2, the magnetic field generated by a current flowing a coil of diameter 2a can be described as the field generated by a current flow around the flaw. A metal plate of finite thickness is assumed between the sensor (SQUID) and the coil (flaw). Considering the boundary conditions for the inserted metal plate, we need more specific formulae. Differential

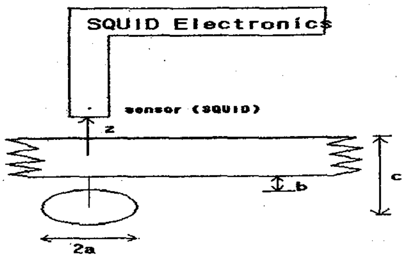

Fig. 2. A plate with finite thickness inserted between a SQUID sensor and a circular coil of radius a.

equations for the vector potential can be derived from Maxwell's equations, with the assumption of cylindrical symmetry. To obtain the solution, a sinusoidal driving current and a linear, isotropic, and homogeneous medium are assumed. We could divide the problem into the three regions.

The differential equation in air (region 1; between the sensor and the plate, region 3; between the plate and the circular coil) is

$$\frac{\partial^2 A}{\partial r^2}+\left(\frac{1}{r}\right)\frac{\partial A}{\partial r}+\frac{\partial^2 A}{\partial z^2}-\frac{A}{r^2}=0. \qquad (6)$$

The differential equation in a conductor (region 2; inside the plate) is

$$\frac{\partial^2 A}{\partial r^2}+\left(\frac{1}{r}\right)\frac{\partial A}{\partial r}+\frac{\partial^2 A}{\partial z^2}-\frac{A}{r^2}-j\omega\mu\sigma A=0. \qquad (7)$$

We now write the expressions for the vector potential in each region[3]:

$$A^{1\theta}=\mu_0\int_0^\infty\left[\frac{I}{2}e^{-\frac{kz}{a}}J_i(k)+L(k)e^{\frac{kz}{a}}\right]J_1\left(\frac{kr}{a}\right)dk, \qquad (8)$$

$$A^{2\theta}=\mu_0\mu_r\int_0^\infty\left[M(k)e^{-\frac{qz}{a}}+N(k)e^{\frac{qz}{a}}\right]J_1\left(\frac{kr}{a}\right)dk, \qquad (9)$$

and

$$A^{3\theta}=\mu_0\int_0^\infty P(k)e^{-\frac{kz}{a}}J_1\left(\frac{kr}{a}\right)dk, \qquad (10)$$

where J_1 is the Bessel function of the first kind, a is the assumed radius of the current loop (flaw) that causes the magnetic dipole field, and $L(k)$, $M(k)$, $N(k)$, $P(k)$ are constants to be determined by the boundary conditions.

We define q as

$$q=\sqrt{k^2+jp^2}, \qquad (11)$$

where $p=\sqrt{\mu_0\mu_1\omega\sigma a^2}$ or $p=\sqrt{\frac{2a^2}{\delta^2}}$.

Using the boundary conditions in each region gives

$$A^{1\theta}=\frac{\mu_0 I}{2}\int_0^\infty e^{\frac{-kb}{a}}J_1(k)J_1\left(\frac{kr}{a}\right)\left[e^{\frac{-K(|z|-b)}{a}}+\frac{k\mu_r-\sqrt{k^2+jp^2}}{k\mu_r+\sqrt{k^2+jp^2}}e^{\frac{K(z-b)}{a}}\right]dt, \qquad (12)$$

$$A^{2\theta} = \mu_0 \mu_r I \int_0^\infty e^{\frac{-kb}{a}} J_1(k) J_1\left(\frac{kr}{a}\right) \frac{(k\mu_r+q)\, e^{\frac{q(z-b)}{a}} - (k\mu_r-q)\, e^{\frac{-q(d-z)}{a}}}{(k\mu_r+q)^2 e^{\frac{qd}{a}} - (k\mu_r-q)^2 e^{\frac{-qd}{a}}} kdt, \quad (13)$$

and the vector potential at both surfaces of the inserted slab yields.

$$A^{3\theta} = 2\mu_0 \mu_r I \int_0^\infty \frac{qk J_1(k)\, e^{\frac{k(d-z)}{a}} J_1\left(\frac{kr}{a}\right)}{(k\mu_r+q)^2 e^{\frac{qd}{a}} - (k\mu_r-q)^2 e^{\frac{-qd}{a}}} dk. \quad (14)$$

where $d = c - b$.

The Z- component of the magnetic field can be written as

$$H^{1z} = \mu_0 I \int_0^\infty e^{\frac{-kb}{a}} J_1(k) J_1\left(\frac{kr}{a}\right) \left[e^{\frac{-K(|z|-b)}{a}} + \frac{k\mu_r - \sqrt{k^2+jp^2}}{k\mu_r + \sqrt{k^2+jp^2}}\, e^{\frac{K(z-b)}{a}} \right] \frac{b}{a} dk, \quad (15)$$

$$H^{2z} = 2\mu_r I \int_0^\infty e^{\frac{-kb}{a}} J_1(k) J_0\left(\frac{kr}{a}\right) \frac{(k\mu_r+q)\, e^{\frac{q(d-z)}{a}} - (k\mu_r-q)\, e^{\frac{-q(d-z)}{a}}}{(k\mu_r+q)^2 e^{\frac{qd}{a}} - (k\mu_r-q)^2 e^{\frac{-qd}{a}}} \frac{k^2}{a} dk, \quad (16)$$

$$H^{3z} = 2\mu_r I \int_0^\infty \frac{qk J_1(k)\, e^{\frac{k(d-z)}{a}} J_0\left(\frac{kr}{a}\right)}{(k\mu_r+q)^2 e^{\frac{qd}{a}} - (k\mu_r-q)^2 e^{\frac{-qd}{a}}} \frac{k}{a} dk. \quad (17)$$

III. Discussion

As shown in Fig. 3, the calculated attenuation of plane wave is much larger than the attenuation of a magnetic dipole field. In other words, the attenuation of a magnetic dipole field inside a conductor is significantly smaller than the calculated value of the incident plane wave equation given by eq. (4). The reason of the smaller attenuation is attributed to the fact that the attenuation of the field is produced by the shielding effect of the induced current which is normally in opposite phase to the source current. The small circular shielding current induced by a magnetic dipole at the low frequency has a different phase shift. Thus when the dipole moment is along the z-axis, the orthogonal component of the shielding current provides no significant shielding effect. Therefore attenuation is smaller than that calculated for a incident plane wave.

When a current flow circumvents a flaw, it causes a pair of positive and negative dipole fields similar to a quadrapole field. Because of the positive and negative counterbalance, the total

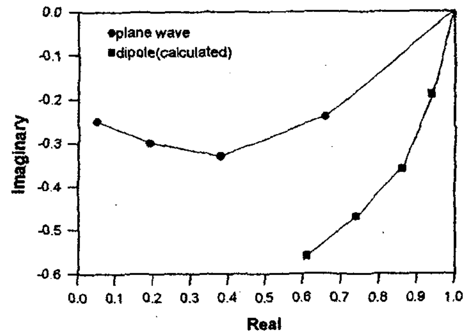

Fig. 3. Magnetic dipole field and plane wave attenuation curves. The SQUID sensor was assumed 3/2" above the defective aluminum slab. The current was set 500 mA/cm^2 at 10 Hz. Both curves have five data points. These points represent thicknesses increased of aluminum slab at 0, 1/4", 1/2", 3/4" and 1", respectively.

magnetic field H_z along the z orientation of the symmetric coordinate will be very small. Therefore, the signal measured by the SQUID gradiometer will be mostly the magnetic field gradient $\frac{\nabla H_z}{\nabla x}$ which is estimated to be twice the strength of a single dipole.

An example is shown in Fig 4, where we show the magnetic dipole field as a function of the depth below the surface for lift-off distances corresponding to coil radii (flaw radii). From the figure it is immediately obvious that the than fall off of magnetic dipole field is much larger than for the smaller coil (smaller flaw size), where dipole field associated with larger coils fall off more slowly with depth than those produced by smaller coils in this calculation. We might also note that different results are given for magnetic $\exp(-\frac{|z|}{\delta})$, appropriate for an incident plane the incident field is spread over a larger area.

As shown in the figure is the exponential decay, wave, where the incident field is uniform

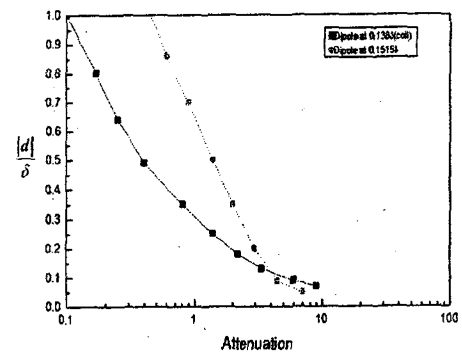

Fig. 4. Attenuation of the current density as a function of subsurface depth for dipole field. (d is the distance between the conductor and circular current source.)

over the entire surface. We can see that the magnetic field attenuation through an aluminum alloy slab at various frequencies, as compared eqs. (15-17) with eq. (4). Therefore, it is expected that the devices based on the measurement of the magnetic dipole field in the near zone would be more effective for the detection of subsurface flaws.

IV. Conclusion

The main results of the work reported here is contained in the magnetic dipole field induced in a conducting half-space by a current source below the conductor. Since a structual defect distorts the current flow in the testing materials, the calculation of the magnetic dipole field due to a current source can be used for a flaw detection in the eddy current nondestructive evaluation.

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