Hypersonic flow calculations using AUSMPW+ and Shock-Aligned Grid Technique

AUSMPW+ 수치기법과 충격파 정렬 격자 기법을 이용한 극초음속 유동장 해석

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극초음속 유동장의 정확한 해석을 위해 AUSMPW+ 수치기법과 충격파 포착시 생기는 수치오차를 제거하기 위해 충격파 정렬 기법(Shock-Aligned Grid Technique)을 개발하였다. AUSMPW+ 수치기법은 자체 수치점성이 적은 수치기법으로 점성 경계층 계산시 정확한 계산결과를 보여주며 기존의 AUSM 계열이 가지는 문제점인 물성치의 진동 현상을 제거한 수치기법이다. 원통형과 무딘 물체 주위의 극초음속 유동장 해석을 통해 공력이 진동현상 없이 정확하게 계산됨을 확인하였다. 그리고 충격파 정렬 기법의 특성을 파악하기 위해 충격파 반사문제와 충격파-충격파 상호작용 문제를 해석하여 수치오차 없이 충격파를 포착할 수 있음을 보였다. 또한 화학적 평형 비평형 유동 영역까지 충격파 정렬 격자 기법을 확장하였다.

1. Introduction

There are commonly two problems in the computation of hypersonic flow problems. One is the inaccuracy of the physical modeling and the other is an error due to inaccurate numerical dissipation. Until now the distribution of species can not calculate accurately in the region of the strong interaction between vibration and chemical reaction. The computation in large expansion region such as nozzle is also inaccurate due to the inaccurate reaction rate coefficient. From the view point of numerical difficulty, a stiff pressure discontinuity may cause large numerical oscillations and degenerate robustness, convergence and accuracy since numerical errors propagate through the flow field. Numerical dissipation is especially sensitive in boundary layer since it can easily contaminate the physical viscosity, leading to the inaccurate resolution of skin friction or heat transfer coefficients. Especially, as in case of shock-wave/boundary-layer interaction problem where two difficulties interact each other, it is very difficult to calculate an accurate solution. In order to solve hypersonic problems accurately the scheme is required to be robust and accurate.

In this paper we focus on the issue of numerical discretization. A newly improved scheme of AUSMPW, so called AUSMPW+, and Shock-Aligned Grid Technique (SAGT) are proposed for an accurate computation of hypersonic flows. Compared to AUSMPW, AUSMPW+ scheme is more efficient to implement while it maintains the same level of robustness and accuracy in capturing the stiff pressure discontinuity or boundary layer. AUSMPW+ scheme has high resolution in calculating oblique shocks.

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Shock-aligned grid technique can exactly align several shocks with structured grid system. SAGT combined with AUSMPW+ can capture the oblique shock as well as the normal shock with little numerical dissipation

2. AUSMPW+ (AUSM by Pressure-based Weight functions) shceme

The purpose of AUSMPW+ is to reduce calculation time and to capture shocks with less numerical dissipation in improving the AUSMPW[1,2].

AUSMPW+ is summarized as follows;

$$\mathbf{F}_{\frac{1}{2}} = \overline{M}_L^+ c_{\frac{1}{2}} \Phi_L + \overline{M}_R^- c_{\frac{1}{2}} \Phi_R + \left(\mathrm{P}_L^+ \mathbf{P}_L + \mathrm{P}_R^- \mathbf{P}_R\right) \tag{1}$$

i) $0 \le m_{\frac{1}{2}} < 1$: $\overline{M}_L^+ = M_L^+ + M_R^- \times ((1-w)\cdot(1+f_R) - f_L)$, $\overline{M}_R^- = M_R^- \times w(1+f_R)$

ii) $-1 \le m_{\frac{1}{2}} < 0$: $\overline{M}_L^+ = M_L^+ \times w(1+f_L)$,

$\overline{M}_R^- = M_R^- + M_L^+ \times ((1-w)\cdot(1+f_L) - f_R)$

where

$$w(p_L, p_R) = 1 - \min\left(\frac{p_L}{p_R}, \frac{p_R}{p_L}\right)^3, \qquad f_{L,R} = \begin{cases} \dfrac{p_{L,R}}{p_s} - 1, & |M_{L,R}| < 1 \\ 0, & elsewhere \end{cases} \tag{2}$$

where $p_s = P_L^+ p_L + P_R^- p_R$

The split Mach number and the split pressure of AUSMPW+ at the cell-interface are written as follows;

$$M^\pm = \begin{cases} \pm\frac{1}{4}(M \pm 1)^2, & |M| \le 1 \\ \frac{1}{2}(M \pm |M|), & |M| > 1 \end{cases}, \qquad P^\pm = \begin{cases} \frac{1}{4}(M \pm 1)^2 (2 \mp M), & |M| \le 1 \\ \frac{1}{2}(1 \pm sign(M)), & |M| > 1 \end{cases} \tag{3}$$

Thus, the interpolation functions of AUSMPW+ are much simplified. The Mach number of each side is defined as follows;

$$M_{L,R} = \frac{U_{L,R}}{c_{\frac{1}{2}}}, \tag{4}$$

The definition of the speed of sound at a cell interface directly influences the resolution of physical discontinuities.

① $0.5*(U_L + U_R) > 0$ $\qquad c_{1/2} = c^{*2} / \max(U_L, c^*)$

② $0.5*(U_L+U_R)<0 \qquad c_{1/2}=c^{*2}/\max(U_R,c^*)$

where,

$$c^*=\sqrt{2(\gamma-1)/(\gamma+1)H_{normal}}, \qquad H_{normal}=0.5\times(H_{total,1}-0.5\times V_{t,1}^2+H_{total,2}-0.5\times V_{t,2}^2)$$

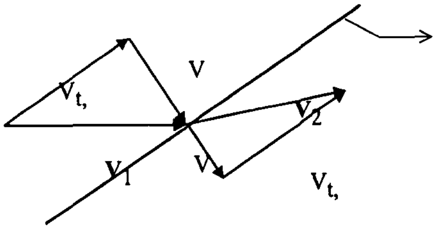

Fig. 1 Schematic of the oblique shock

The speed of sound ($=c^*$) satisfies Prandtl relation.

$$M_1^*\times M_2^*=1 \quad : \quad \text{Prandtl relation,} \qquad \frac{V_{n,1}}{c^*}\times\frac{V_{n,2}}{c^*}=1 \tag{5}$$

3. Characteristics of AUSMPW+ scheme

AUSMPW+ as well as AUSM+ is such an accurate scheme that it can capture the high resolution shock and calculate the accurate aerodynamic coefficients in the boundary-layer. AUSMPW+ consists of original AUSM plus additional terms.

Flux in cell-surface is written as follows;

$$\boldsymbol{F}_{1/2}=0.5(u_L\boldsymbol{\Phi}_L+u_R\boldsymbol{\Phi}_R)+\boldsymbol{D} \tag{6}$$

When the oblique shock is exactly on the line of grid,

$\mathbf{D}|_1=-0.5(\rho_R U_R-\rho_L U_L)=0$

$\mathbf{D}|_2=(\rho_R U_R U_R-\rho_L U_L U_L+p_R-p_L)\cdot\sin\theta+(\rho_R V_R U_R-\rho_L V_L U_L)\cdot\cos\theta=0$

$\mathbf{D}|_3=(\rho_R U_R U_R-\rho_L U_L U_L+p_R-p_L)\cdot(-\cos\theta)+(\rho_R V_R U_R-\rho_L V_L U_L)\cdot\sin\theta=0$

$\mathbf{D}|_4=-0.5\rho_L U_L(H_R-H_L)=0$

All dissipation terms of AUSMPW+ become zero and this means that AUSMPW+ can capture the shock without numerical diffusion in case of a stationary oblique shock problem.

Another advantage of AUSMPW+ scheme is the reduction of oscillations. The defect of AUSM+ is the numerical overshoots behind shocks because AUSM+ considers only the one-side cell properties.

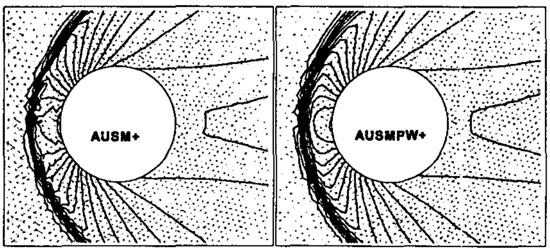

Fig. 2 Reduction of oscillations behind shocks

Thus AUSM+ conveys excessive or insufficient properties compared with AUSMPW+ and as a results the phenomena of oscillations occur. By introducing the function pw, AUSMPW+ considers the both side properties adequately and removes these oscillations.

4. Shock-Aligned Grid Technique (SAGT)

In calculating hypersonic flow problems there are pressure oscillations or numerical errors due to a strong shock. In modern upwind scheme these oscillations mostly result from grid system which is generated without considering shock position. In order to remove these oscillations two alternatives can be considered. First, an adequate numerical dissipation term is to be added to scheme. AUSMPW+ has additional numerical dissipation compared with AUSM+ in existence of the pressure oscillations.Second, the fundamental reason of oscillations, that is, the spatial discretization error is to be removed. Through a shock-aligned grid, shock wave can be captured without oscillation.

In structured grid, shock is commonly captured across grids with different index numbers as shown in Fig.1-(d). In this case, large numerical errors are induced and contaminate the results. Shock-Aligned Grid Technique(SAGT) is designed to overcome this problem.

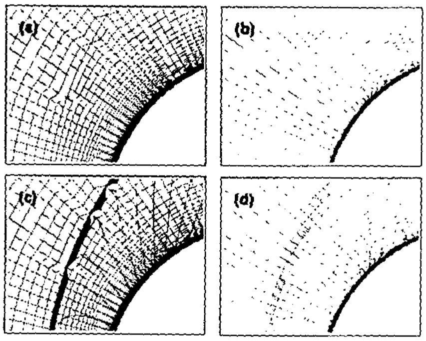

Fig. 3 Comparison shock-aligned grid with non-shock-aligned grid

The process of the shock-aligned grid technique is as follows;

1. Find index numbers of the cell interface at which numerical shock exist.
2. Join two grid points at which indexes of shock location are changed.(Fig. 3-(a))
3. Adjust the grid points to align the shock.
4. Converge the solution again using the reconstruction grid

5. Results

5.1 Reflection of shock and shock-shock interactions problems

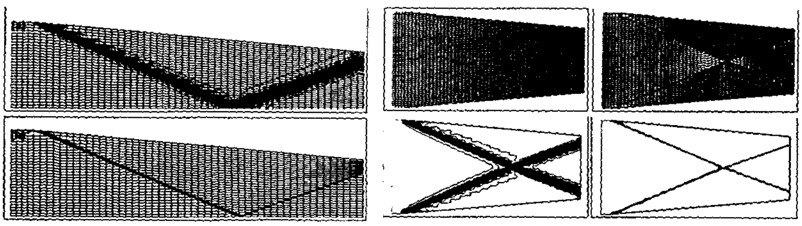

Fig. 4 Comparisons of pressure contours of the shock-aligned grid with the original grid

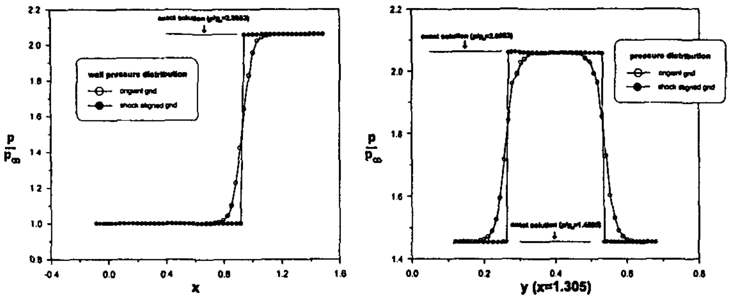

Fig. 5 pressure distributions of shock reflection and shock-shock interactions problems

The free stream Mach number is 3.0 and wedge angle is 5°. Figure 4 shows wall pressure distribution. In original grid the oblique shock is captured through 8 cell-interfaces, however, in the grid which the shock-aligned grid technique is applied, this shock is captured through only one cell-interface as shown in Fig.5.

5.2 *Nonequilibrium* flow around a cone

Free stream conditions are as follows;

$M_\infty = 15$, $T_\infty = 233.75K$, $\mathrm{Re} = 2.0\times10^5$, $p_\infty = 663.4\ N/m^2$, $T_w = 1168.7K$.

Figure 6 shows the comparison of standing shock distances of frozen, equilibrium, nonequilibrium gas in the same free steam conditions.

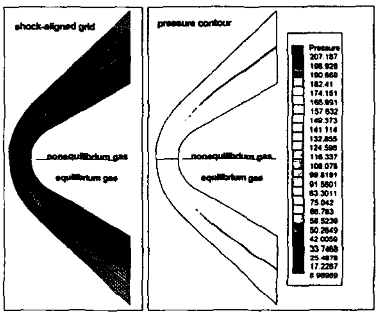

Fig. 5 Comparison of equilibrium and nonequilibrium results

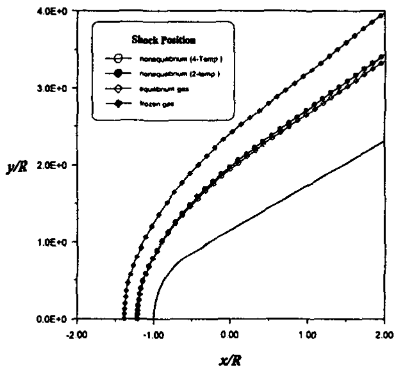

Fig. 6 Standing shock distances

6. Conclusions

In this paper, we propose a new scheme called AUSMPW+ improved from the previous AUSMPW scheme, and a Shock-Aligned Grid Technique.

Although AUSM+ is robust and accurate, it has overshoots or oscillations behind shocks and numerical oscillations near a wall. AUSMPW+ is designed to improve these shortcomings. It uses pressure-based weight functions to consider the properties of both cells of a cell interface. Exploiting these functions properly, AUSMPW+ successfully eliminates oscillations and overshoots, reduces the grid dependency, and improves the convergence. AUSMPW+ also shows no carbuncle phenomena, high resolution at discontinuities, and accuracy in boundary layer. Combining AUSMPW+ and SAGT, a stationary oblique shock can be captured without numerical dissipation. By removing the error induced by a non-aligned grid system, more accurate results can be obtained.

7. References

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