Optimization of Thinned Antenna Arrays using a Least Square Method

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Abstract

This paper concerns a least square approach to optimizing a thinned antenna array with respect to antenna spacing to improve the sidelobe performance. A least square method based on a modified version of the modified perturbation method is proposed to efficiently synthesize an optimum pattern in a thinned array. It is demonstrated that the array performance improves with the proposed method, compared with the conventional method.

1. Introduction

The thinned antenna array is an efficient array system which prevents the degradation of array performance due to mutual coupling effects and also reduces the array cost by employing less number of antenna elements compared to a half-wavelength spaced filled array. The thinned array has been widely investigated in such areas as radar[1], astronomy[2] and satellite communication[3]. If the number of elements in a filled array is reduced, the sidelobe performance is degraded due to the less of degrees of freedom to control the beam pattern. The problem in the thinned array is how to synthesize an optimum pattern with reduced number of elements which satisfies given design specifications while the performance is comparable to that of the filled array.

In this paper, it is concerned that the thinned array is designed such that the sidelobe level is equalized in a Dolph-Chebyshev sense to counteract the interferences uniformly distributed over the array visual range. A certain set of element spacings is found by a least square approach with an iterative perturbation of element spacings with uniform array weights.

2. Perturbation Method [4]

The array factor of a symmetric thinned array of $2N$ elements is given by

$$H(\omega) = 2\sum_{i=1}^{N} a_i \cos(\omega d_i) \qquad (1)$$

where the weights a_i are assumed to be uniform, $\omega = \pi \sin\theta$, θ is the angle from the array normal, d_i is the element spacing from the array center and i is element index.

The perturbation method[4] equalizes the sidelobes iteratively by perturbing the element spacings using an arbitrary initial spacing. Suppose that L sidelobes are located at ω_i , $1 \leq i \leq L$. Since the derivative of $H(\omega)$ with respect to each sidelobe w_i will be zero, we have the following equations.

$$H(\omega_j) = \frac{1}{N}\sum_{i=1}^{N}\cos(\omega_j d_i) \tag{2}$$

$$\sum_{i=1}^{N} d_i \sin(\omega_j d_i) = 0, \quad 1 \leq j \leq L \tag{3}$$

In (2), the array weights are normalized such that the maximum gain is one at the array normal. The element spacings d_i are perturbed iteratively such that N sidelobes get closer to a specified threshold level. Assuming a small perturbation, the nonlinear equations in (2) and (3) with respect to d_i may be linearized via a Tayler series expansion. If the kth spacing is perturbed by Δd_i^k, the $(k+1)$th spacing is given by

$$d_i^{k+1} = d_i^k + \Delta d_i^k \tag{4}$$

Accordingly, the $(k+1)$th sidelobe location and level is given by

$$\omega_j^{k+1} = \omega_j^k + \Delta\omega_j^k \tag{5}$$

$$H^{k+1} = H^k + \Delta H^k \tag{6}$$

respectively, where H denotes $H(\omega_j)$.

If (4),(5), and (6) are substituted into (2) and (3) and simplified by a first-order approximation, we have the following equations as

$$-2f\omega_j^k \sum_{i=1}^{N} a_i \Delta d_i^k \sin(f d_i^k \omega_j^k) = \Delta H_j^k, \tag{7}$$

$$f\Delta\omega_j^k \sum_{i=1}^{N} a_i d_i^{k2} \cos(f d_i^k \omega_j^k) + f\omega_j^k \sum_{i=1}^{N} a_i d_i^k \Delta d_i^k \cos(f d_i^k \omega_j^k) + \sum_{i=1}^{N} a_i \Delta d_i^k \sin(f d_i^k \omega_j^k) = 0 \tag{8}$$

Assuming that ΔH_j^k is a small fraction of the difference of the actual sidelobe and a specified threhold level, the element spacings are perturbed iteratively until all the sidelobes are equalized. It is to be noted that to ensure a unique solution of Δd_i and $\Delta\omega_j$ at each iteration, the number sidelobes to be controlled should be equal to the number of element spacings N.

In the perturbation method, the initial element spacings should be half-wavelength to have the same number of sidelobes as the number of element spacings, which is not the case in the thinned array. Also, if the number of sidelobes in the initial pattern is more than the number of element spacings, some of the sidelobes can not be controlled which results in poor sidelobes performance. Thus, the perturbation method is not suitable for the design of the thinned array.

3. Least Square Approach

In the modified perturbation method[5], the sidelobe location at each iteration is found numerically instead of calculating it algebraically using (8). Thus, the same number of highest sidelobes as the number of spacings can be located at each iteration so that the sidelobes are controlled properly to achieve a uniform sidelobe level.

If the number of sidelobes that are involved in the perturbation process is more than the number of unknown spacings, (7) will become an overdetermined system of linear equations whose optimum solution can be obtained by a least square method. If we formulate (7) in matrix from as

$$A\Delta D = \Delta H \tag{9}$$

where the jth row and ith column component of A is

$$[A_{ji}] = -\frac{1}{N}\omega_j^k \sin(\omega_j^k d_i^k) \quad 1 \le j \le L,\ 1 \le i \le N,\ L > N \tag{10}$$

$$\Delta D = [\Delta d_1^k \ \ \Delta d_2^k \ \ \cdots \ \ \Delta d_N^k]^T \tag{11}$$

$$\Delta E = [\Delta H_1^k \ \ \Delta H_2^k \ \ \cdots \ \ \Delta H_N^k]^T \tag{12}$$

Then the least square solution is expressed as

$$\Delta D = (A^T A)^{-1} A^T \Delta H \tag{13}$$

Using (13), we can find Δd_i , $1 \le i \le N$ at each iteration.

A thinned array of 31 elements is formed in the array length of a 101-element filled array in which the number of unknown spacings is 14. The modified perturbation method (14 sidelobes are involved) and the least square method(16 sidelobes are involved) are simulated and the resulting patterns are shown in Figs. 1 and 2. It is observed that the least square method yields a better sidelobe performance than the modified perturbation method. It is shown that the sidelobe performance is degraded for some cases of more sidelobes than 16.

4. Conclusion

A least square approach was proposed for synthesis of an optimum beam pattern with uniform sidelobes in a thinned antenna array. It was shown that the proposed method performs better than the conventional one.

References

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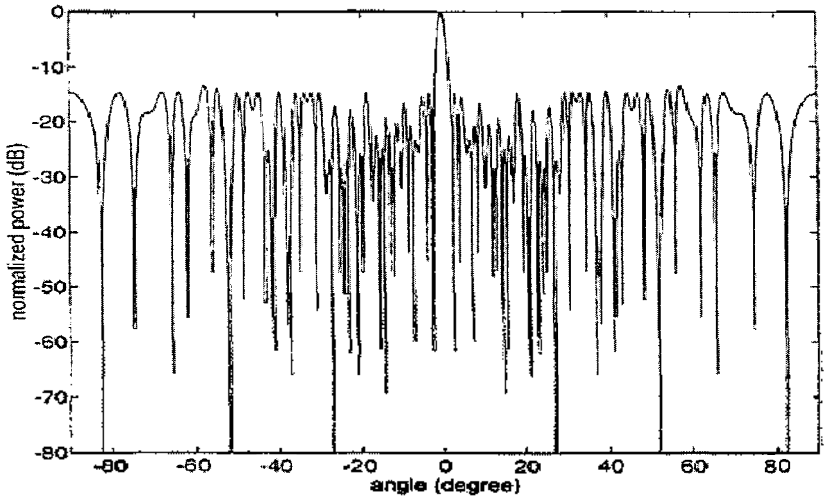

Fig 1. Beam pattern of a 31-element thinned array by modified perturbation method.

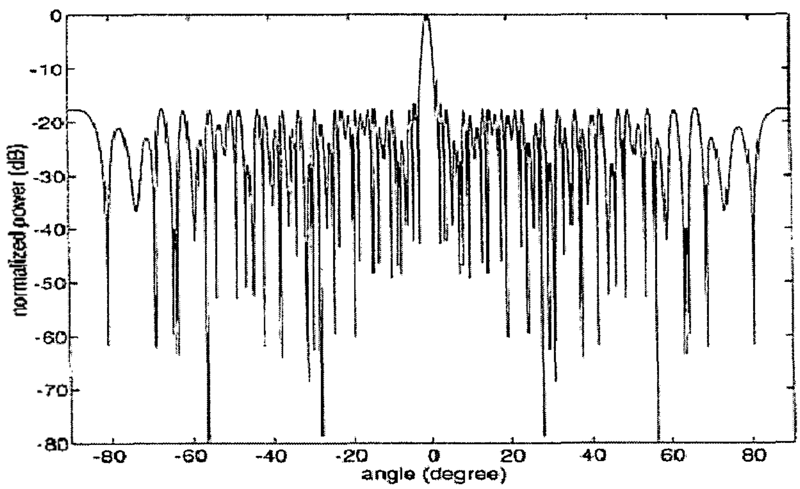

Fig 2. Beam pattern ofan 31-element symmetric thinned array by Least square approach with 16 sidelobes.