극소불량 공정보증을 위한 모형연구
- Model for Process Quality Assurance When the Fraction Nonconforming is Very Small-

김 종 결
Jong-Gurl Kim

ABSTRACT

There are several models for process quality assurance by quality system (ISO 9000), process capability analysis, acceptance control chart and so on. When a high level process capability has been achieved, it takes a long time to monitor the process shift, so it is sometimes necessary to develop a quicker monitoring system. To achieve a quicker quality assurance model for high-reliability process, this paper presents a model for process quality assurance when the fraction nonconforming is very small. We design an acceptance control chart based on variable quality characteristic and time-censored accelerated testing. The distribution of the characteristics is assumed to be normal of lognormal with a location parameter of the distribution that is a linear function of a stress. The design parameters are sample size, control limits and sample proportions allocated to low stress. These paramaters are obtained under minimization of the relative variance of the MLE of location parameter subject to APL and RPL constraints.

1. INTRODUCTION

1.1 Control Charts for PPM

Modern production processes, particularly those occurring in the electronics industry, are of high quality when the fraction nonconforming production is in the parts per million(PPM) range. For control of such processes, the procedures

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성균관대학교 시스템경영공학부

generally assume that the production is 100% inspected[Quesenberry 1995[16]].

This control strategy may not be very effective or not useful when the 100% inspection is subject to test errors, destructive test or long-time to measure.

When the proportion of nonconforming(or nonconformity) product is extremely small, say, in the parts per million range, the p-chart(or c-chart) computed with managerable sample sizes will be of little use[as noted by Mongomery(1991)[10]].

For the process with small proportion of nonconforming, there are some approaches. Nelson(1994) proposed using 3σ control charts based on a power transformation of X(the number of items inspected until a nonconforming item is found) chosen so that Y is approximately normal[12]. Lucas(1985) proposed the counted data CUSUM to control chart for quality in the ppm range.[8]

McCool and Joyner-Motley(1998) proposed a chart based on $Z = \ln(X)$ comparable Nelson's Y chart[9]. Kittlitz, JR(1999) proposed other chart based on fourth root transformation $Y = X^{\frac{1}{4}}$.[7]

1.2 Lot Quality Assurance

Pesotchinsky(1987) suggested plans for very low fraction conforming, ie he devised a scheme that includes both the strategies of 100% inspection of the units and sampling inspection of lots formed together with certain criteria for switching between the strategies. Bevvington and Govindaraju(1998) provided corrected tables for the schemes considered by Pesotchinsky.

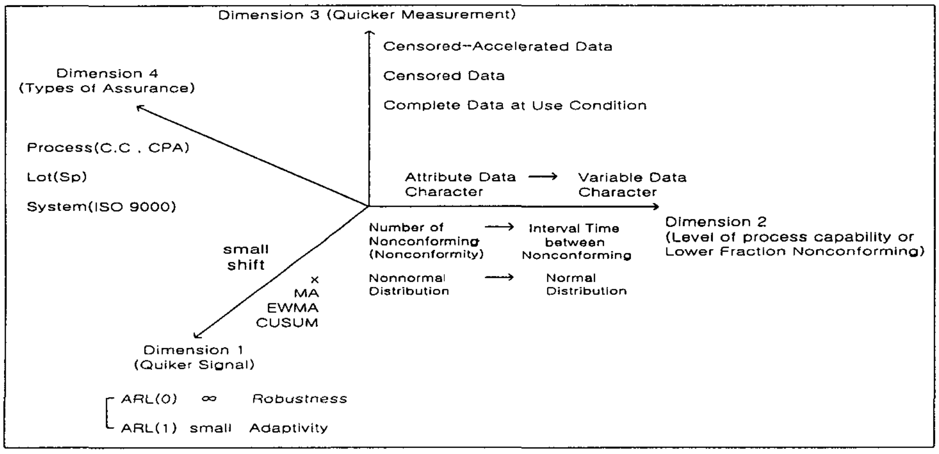

Figure 1. Development Dimensions for Quicker Process Quality Assurance

We should consider some dimensions for the development of quicker process quality assurance.; quicker measurement, quicker signal for small process shift, level of process quality and types of quality assurance(Figure 1).

As above, we have a brief review concerned on level of process quality and quicker signal. Next, we also adopt quicker measurement.

Todays manufacturers are facing strong pressure to develop newer, higher technology products in record time, while improving productivity, product field reliability, and overall quality. This has motivated the development of methods like concurrent engineering and encouraged wider use of designed experiments for product and procss improvement efforts. The requirements for higher reliability have increased the need for more up-front testing of materials, components and systems. This is in line with the generally accepted modern quality philosophy for producing high reliability products: achieve high reliability by improving the design and manufacturing processes; more away from reliance of inspection to achieve high reliability.

1.3 QC based on ALT

Estimating the long-term performance of components of high reliability products is particularly difficult, because it takes too much time to test at use condition. In this point, censored accelerated life testing quickly provides useful data on the life of products[11,13]. Singly censored data arise when units are started together at a test condition and the data are analyzed before all units fail. Such data are singly time censored if the censoring time is fixed; then the number of failures in that fixed time is random. Data are singly failure censored if the test is stopped when a sepecifed number of failures occurs. The time to that fixed number of failures is random. Time censored acelerated life testing is designed to test a products in stress conditions - high temperature, high voltage, high pressures, etc. - and terminates the test in predetrmined time. We can save money and time by time censored accelerated life testing. For the design of ALT, see[14].

We can apply ALT to quality control. Kim[2] and Bai et al.[3,4] developed acceptance sampling plans based on ALT. In this paper we consider applying it to statistical process control. especially we design an acceptance control chart based on time-censored ALT.

1.4 Acceptance Control Chart

A typical control chart is a graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number of time. In most situations in which control charts are used, the focus is on statistical control of the process reduction of variability, and continuous process improvement. When $\overline{X}$-chart is used to control the fraction of conforming units produced by the process rather than to satisfy the traditional SPC objective of detecting assignable cause, the acceptance control chart can be employed. Freund[6] developed an acceptance control chart which was designed to judge the acceptance or nonacceptance of the process. Some authors[5,17,19] considered the extension and improvement of acceptance control charts. So long as both process dispersion and process mean was held in control, virtually all product would meet specifications. Generally, acceptance control charts are concerned with specification of the product and used to control products with high reliability.

The following notations will be used.

S_0, S_1, S_2	Stress levels (use, low, high)
ξ	Standardized stress level; $\xi=(s-s_0)/(s_1-s_0)$
ξ_0, ξ_1, ξ_2	Standardized use condition, low, high stresses
β_0, β_1	Parameters involved in the stress-life relationship model
n	Sample size
π	Sample proportion allocated to s_1 : $0<\pi<1$
η	Censoring time
X, Y	Lifetime and log lifetime of products : Y=log(X)
μ, σ	Location and scale parameters of distribution of Y
α, β	Producer's risk and consumer's risk : $0<\alpha, \beta<1$
k	Standardized limit constant
Asvar(·)	Asymptotic variance
$\Phi(\cdot), \phi(\cdot)$	Cdf and pdf of standard normal distribution
APL	Acceptable process level
RPL	Rejectable process level
LCL, UCL	Lower, upper control limits
LSL, USL	Lower, upper specification limits
Z_α, Z_β	α - and β - Percentiles of the standardized normal distribution

2. PROCESS QUALITY ASSURANCE MODEL

Assuming that the variance of the process is known, we determine the standardized limit constant k, the sample proportion π allocated to the low stress and the optimum sample size n.

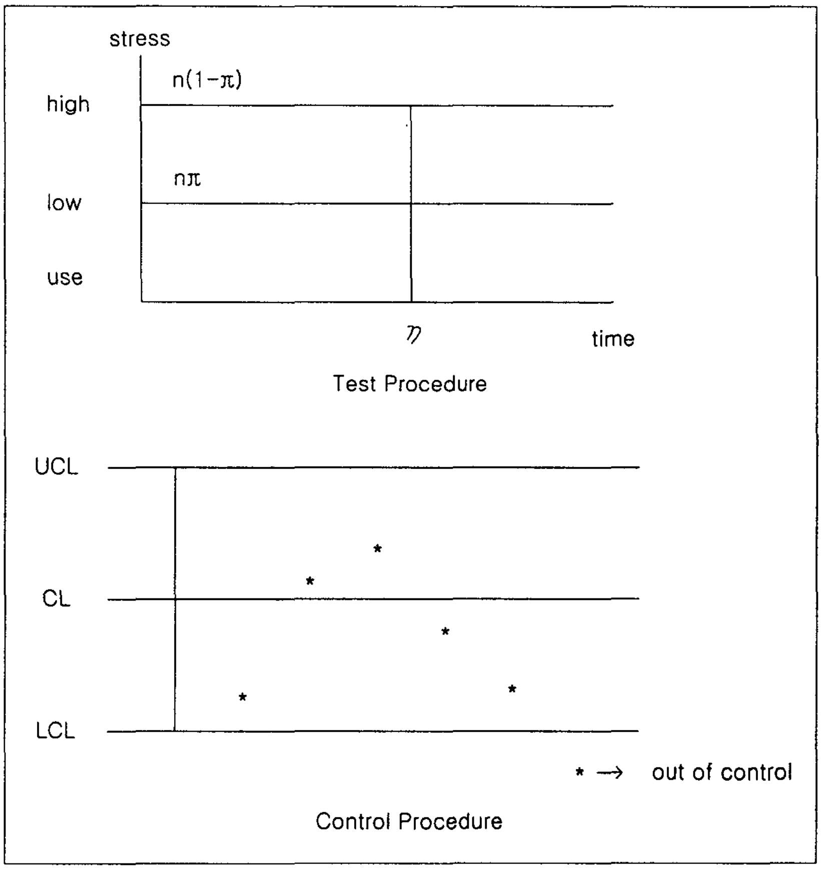

Figure 2. Process Quality Assurance Model

The distribution of the characteristics is assumed to be normal or lognormal with a location parameter(μ) of the (log) distribution that is a linear function of a stress(S)

$$\mu = r_0 + r_1 \cdot S \tag{1}$$

Where $r_{0,} r_1$ are unknown constant. In two stress levels($S_{1,} S_2$), the number of $n\pi$ items are tested in low stress level(S_1) and the rest $n(1-\pi)$ of the items are tested in tested in high stress level(S_2) simultaneously until censoring time η (Figure 2).

We find the plot statistic $\hat{\mu}$(MLE of the location parameter μ at use condition) from censored accelerated test. Them we decide that the process is not acceptable if the accelerated mean is fall ourside the control limits, like above Figure 2. The process quality is assured by determining. k, π and n so that

$\Pr[LCL < \hat{\mu} < UCL | \mu = APL] = 1 - \alpha$ and $\Pr[LCL < \hat{\mu} < UCL | \mu = RPL] = \beta$.

3. ASYMPTOTIC DISTRIBUTION OF PLOT STATISTIC

We can standardize the stress by following transformation

$$\xi = \frac{(S - S_0)}{(S_2 - S_0)} \tag{2}$$

For the use condition stress $S = S_0$, $\xi = \xi_0 = 0$, for the low stress level $S = S_1$, $\xi = \xi_1$, $(0 < \xi_1 < 1)$, and for the high stress level $S = S_2$, $\xi = \xi_2 = 1$.

Then allocation parameter(μ) will be the following equation

$$\mu(\xi) = \beta_0 + \beta_1 \cdot \xi\ . \tag{3}$$

In use condition, there is no stress, i.e. $\xi = 0$, then the location parameter is $\mu_0 = \beta_0$. Thus the plot statistic is

$$\widehat{\mu(\xi_0)} = \hat{\mu} = \hat{\beta}_0 \tag{4}$$

3.1 Fisher Information Matrix

We have to find the variance of $\hat{\mu}$ from data by using Fisher information matrix. The lifetime X of a test unit at stress level ξ is assumed to have a lognormal distribution, the p.d.f of which is given by

$$f(x) = \frac{1}{(2\pi)^{1/2} \cdot \sigma x} \cdot \exp[-\frac{1}{2}(\frac{\log(x) - \mu}{\sigma})],\ x > 0. \tag{5}$$

Let Y(=log X) be an observation of an item tested at stress ξ_i. If we regard η as Type I censoring time, the elements of the Fisher Information matrix for an observation are the expectations[14].

$$E\left\{-\frac{\partial L}{\partial\beta_j \partial\beta_k}\right\} = (\xi_j \xi_k/\sigma^2)\left\{(\psi(\zeta_i) - \phi(\zeta_i))[\zeta_i - \frac{\phi(\zeta_i)}{1-\psi(\zeta_i)}\right\},\quad \text{j}, k = 0,\ 1$$

$$E\left\{-\frac{\partial L}{\partial\beta_j \partial\sigma}\right\} = (\xi_j/\sigma^2)\left\{-\phi(\zeta_i)[1 + \zeta_i(\zeta_i - \frac{\phi(\zeta_i)}{1-\psi(\zeta_i)})]\right\},\quad \text{j}=0,\ 1 \tag{6}$$

$$E\left\{-\frac{\partial L}{\partial \sigma^2}\right\}=(1/\sigma^2)\left\{2\psi(\zeta_i)-\zeta_i\phi(\zeta_i))[1-\zeta_i^2-\frac{\zeta_i\phi(\zeta_i)}{1-\psi(\zeta_i)}]\right\}$$

where $\zeta=(\eta-\beta_0-\beta_1\xi_1)/\sigma$, $\xi_1=\xi_{i.}$ Then the Fisher information matrix for $n\pi_1$ items at low stress and $n\pi_1$ items at low stress and $n\pi_h$ at high stress is

$$F(\widehat{\beta_o},\widehat{\beta_1},\hat{\sigma})=n\pi_1\cdot F_{\xi_i=\xi_1}(\widehat{\beta_0},\widehat{\beta_1},\hat{\sigma})+n\pi_2\cdot F_{\xi_i=\xi_2}(\widehat{\beta_0},\widehat{\beta_1},\hat{\sigma})$$

$$=\sum_{1=1}^{h}\frac{n}{\sigma^2}\cdot\pi_i\begin{bmatrix} A_i & A_i\xi_i & B_i \\ A_i\xi_i & A_i\xi_i^2 & B_i\xi_i \\ B_i & B_i\xi_i & C_i \end{bmatrix}=\frac{n}{\sigma^2}\cdot(F_{jk}),\quad \text{j}, k\text{=1,2,3} \tag{7}$$

$$=\frac{n}{\sigma^2}\cdot F_1$$

where A_i , B_i and C_i are the factors in the braces of the right hand side of formulae (6) respectively, which depend only on ALT parameters. It is a function of n, π_1 and ξ_1.

3.2 Asymptotic Variance of Plot Statistic $\widehat{\beta_0}$

Thus variance-covariance matrix for the MLEs of the β_0, β_1 and σ can be obtained by inverting the Fisher information matrix

$$\begin{aligned} Var(\widehat{\beta_0},\widehat{\beta_1},\hat{\sigma}) &= F^{-1}(\widehat{\beta_0},\widehat{\beta_1},\hat{\sigma}) \\ &=(C_{jk})/Det[F(\widehat{\beta_0},\widehat{\beta_1},\hat{\sigma})] \qquad (8) \\ &=(\frac{\sigma^2}{n})^3(C_{jk})/Det[F_1],\quad \text{j}, k = 1,\ 2,\ 3 \end{aligned}$$

where C_{jk} and Det[·] indicate the cofactors and the determinant of a matrix respectively. From above, the plot statistic $\widehat{\beta_0}$ has asymptotic normal distribution with the expectation

$$E(\widehat{\beta_0})\doteq\beta_0=\mu_0 \tag{9}$$

and variance

$$Var(\widehat{\beta_0})\doteq\frac{\sigma^2}{n}(Q_{11})/Det[F_1]$$

hence $$\widehat{\beta}_0 \sim N(\beta_0, \frac{\sigma^2}{n} \cdot Q_{11}/Det[F_1]) \tag{10}$$

where,

$$Q_{11} = B_2^2 - A_2C_2 - (C_1A_2 + 2B_2^2 - 2A_2C_2 - 2B_2B_1\xi_1 - A_2C_2\xi_1^2)\pi_1 \\ + C_1A_2 + B_2^2 - A_2C_2 - 2B_1B_2\xi_1 + (B_1^2 - A_1C_1 + A_1C_2)\xi_1^2, \tag{11}$$

$$Det[F_1] = -\pi_1(\pi_1 - 1)[-A_1B_2^2 + A_1A_2C_2 + (A_1C_1A_2 - B_1^2A_2 \\ + A_1B_2^2 - A_1A_2C_2)\pi_1 \cdot (\xi_1 - 1)^2 \tag{12}$$

4. OPTIMAL DESIGN

4.1 APL, RPL Requirements

Let β_0 follow above distribution, we will accept the process when

$$\frac{\widehat{\beta}_0 - LSL}{\sigma} \geq k \quad \text{i.e.} \quad \widehat{\beta}_0 > \widehat{\beta}_L = LCL = LSL + k \cdot \sigma \tag{13}$$

We can design an acceptance control chart by solving these equations if APL, RPL requirements are defined by (APL, $1-\alpha$) and (RPL, β). (Figure 3)

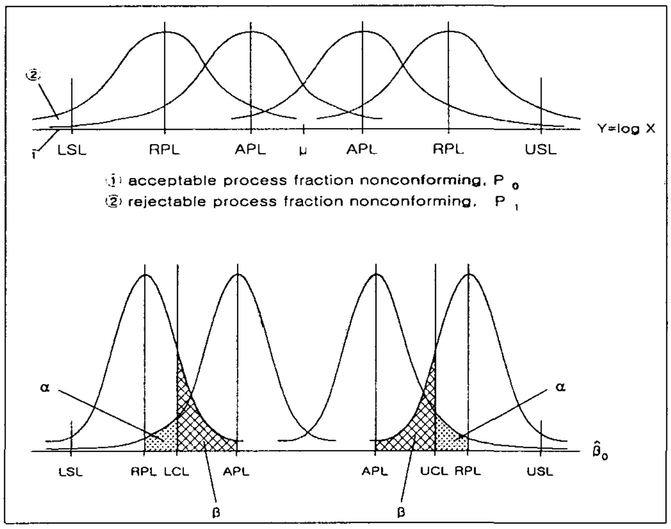

Figure 3 Requirements on APL and RPL

$$P\{\widehat{\beta}_0 \leq LCL | \mu = APL\} = \alpha$$

$$\frac{LCL-APL}{\frac{\sigma}{\sqrt{n}}\cdot\sqrt{Q_{11}/Det[F_1]}}=-Z_\alpha \tag{14}$$

$$P\{\widehat{\beta}_0\ge LCL|\mu=RPL\}=\beta$$

$$\frac{LCL-RPL}{\frac{\sigma}{\sqrt{n}}\cdot\sqrt{Q_{11}/Det[F_1]}}=Z_\beta \tag{15}$$

4.2 Determination of k^*, π^*, n^*

We can determine the standardized limit constant k and sample size n by solving(14) and (15).

$$\begin{aligned}k&=\left(\frac{Z_{p_0}\cdot Z_\beta+Z_{p_1}\cdot Z_\alpha}{Z_\alpha+Z_\beta}\right)\\&=\left(\frac{APL\cdot Z_\beta+RPL\cdot Z_\alpha}{Z_\alpha+Z_\beta}-LSL\right)/\sigma\end{aligned} \tag{16}$$

$$\begin{aligned}n&=\left(\frac{Z_\alpha+Z_\beta}{APL-RPL}\right)^2\cdot\sigma^2\cdot Q_{11}/Det[F_1]\\&=\left(\frac{Z_\alpha+Z_\beta}{Z_{p_0}-Z_{p_1}}\right)^2\cdot Q_{11}/Det[F_1]\end{aligned} \tag{17}$$

Note that k does not depend on ALT parameters and k is direct to the optimal value k^*. Next we determine the sample proportion π allocated to low stress. We find π, in the point of minimizing the sample size n. in formulae (17) Z_α, Z_β, APL and RPL are predetermined values, but the variance of plot statistic is a function of π from (11) and (12). By minimizing asymptotic variance of plot statistic, we determine the optimal sample allocation proportions. Let π^* be the optimum value which minimize $[Q_{11}/Det[F_1]]$. We compute the optimal sample size n^* by evaluating (17) at $\pi=\pi^*$.

A procedure for designing an acceptance control chart is described as steps.

Step 1. Choose APL, RPL requirements (APL, $1-\alpha$) and (RPL, β)

Step 2. Choose the stress level (S_1, S_2).

Step 3. Compute the asuymptotic mean and variance of plot statistic from (9) and (10).

Step 4. Compute standardized limit constant k from (16).

Step 5. Compute the sample proporation π^* allocated to low stress level by minimizing $[Q_{11}/Det[F_1]]$.

Step 6. Compute the optimal sample size n^* from (17) and π^*.

5. CONCLUDING REMARKS

After designing an acceptance control chart, it is desirable to design an R-chart and go with acceptance control chart, because we must always pay attention to the stability of variance of a process.

In further study, it is desirable to study ALT acceptance control chart for other distributions and it is necessary to provide the implementation model for practical use.

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