Proceedings of the 14th
KACC, Octorber 1999

Robust Time-Optimal Control for Coarse/Fine Dual-Stage Systems

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Abstract

A robust and time optimal control strategy for dual-stage servo system is presented. The time optimal trajectory for a mass-damper system is determined and given as a reference input to the servo system. The feedback controller is constructed so that the fine stage tracks the coarse stage errors and robustly designed as the "perturbation compensated sliding mode control(PCSMC)" law, a combination of sliding mode controller(SMC) and perturbation observer(PO). In addition, a null motion controller which regulates the fine stage at its neutral position is designed based on the "dynamic consistency". So, the overall dual-stage servo system exhibits the robust and time-optimal performance. The inherent merit and performance of the dual-stage system will be shown.

1 Introduction

As the industrial need for high speed and high accuracy positioning devices increases, a new concept in servo system design appeared: the dual-stage servo system, which is defined as a combination of coarse/fine(or macro/micro) actuation stages for fast and precise positioning. In dual-stage systems, the coarse actuator is used for coarse and large range motions while the fine actuator for fine and small range motions.

The conventional actuator(e.g., electrical motor) has the following limitations: mechanical resonance at high frequencies, and bearing friction increase due to low speed in low frequency regions. These characteristics limit the system performance. The fine actuator(e.g., piezo-electric transducer(PZT)) is a solution to increase the servo bandwidth. It is limited in motion range and power but the high speed characteristic enables high frequency command following.

The representative examples which adopt the dual actuation concept are disk drives[1], macro/micro robot manipulators[2],[3], and X-Y linear positioning tables[4]. In this paper, we consider the robust control problem with time optimality for the dual-stage system composed of coarse/fine co-linear actuators.

2 Dual-Stage Model Description

As a dual-stage construction, we consider a system which is composed of two ball-screw driven linear motion stages in Fig. 1. We assume that the 2nd stage mounted on the 1st stage has the characteristics of fine actuators, *i.e.*, high resolution and high bandwidth.

The mathematical model of a ball-screw driven linear stage, which is in most current use in industry, is described as a mass-damper system:

$$J_e\dot{\omega}_m + B_e\omega_m = \tau_m - \tau_f \quad \text{and} \quad \dot{y} = p\omega_m \tag{1}$$

$$\rightarrow J_e\ddot{y} + B_e\dot{y} = p(\tau_m - \tau_f) \tag{2}$$

where $p = \frac{\ell}{2\pi}$: the angular to linear motion conversion factor, ℓ : lead of screw, J_e : effective inertia of linear stage

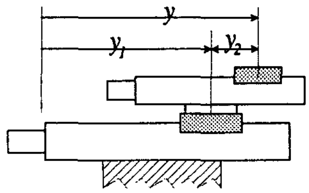


Fig. 1: Coordinate definitions of the dual-stage system.

w.r.t. rotary motor shaft B_e : effective viscous damping coefficient w.r.t. motor shaft τ_m : motor torque, τ_f : effective nonlinear friction torque w.r.t. motor shaft.

For a two stage ball-screw driven system, the decoupled model of the following is available.

$$\begin{bmatrix} J_{e_1} & 0 \\ 0 & J_{e_2} \end{bmatrix} \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} + \begin{bmatrix} B_{e_1} & 0 \\ 0 & B_{e_2} \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} p_1(\tau_1(t) - \tau_{f_1}) \\ p_2(\tau_2(t) - \tau_{f_2}) \end{bmatrix} \tag{3}$$

$$\rightarrow \boldsymbol{A}\ddot{\boldsymbol{q}} + \boldsymbol{B}\dot{\boldsymbol{q}} = \boldsymbol{\Gamma} \tag{4}$$

$$y = y_1(\text{coarse motion}) + y_2(\text{fine motion}) \tag{5}$$

Remark: Inertial property (reduced effective inertia). The inherent feature of the dual-stage system is that the effective inertia in any direction of motion is less than that of the fine actuator. Applying the theorem in [3] to the dual-stage model of Eq. (3)-(5), the effective inertia is readily derived as follows, which proves the theorem.

$$J_{eff} = \frac{J_{e1}J_{e2}}{J_{e1} + J_{e2}} \leq J_{e2} \tag{6}$$

3 Reference Trajectory Generation

The time optimal control is a critical issue in positioning systems since it is directly associated with productivity. A lot of applications using the dual-stage system require the *time optimal performance*, for example, in track seeking mode of disk drives, and also in chip mounting device.

Since the time optimal *feedback law* has no margin on the plant model and control input, a reasonable "sub-optimal" strategy is desirable for performance robustness. For this purpose, we are to determine the time optimal solution for mass-damper systems in open loop sense and give it as a reference trajectory for the dual-stage system to robustly follow rather than directly apply the time optimal feedback law.

For the nominal model of Eq.(2), $\ddot{y} + a\dot{y} = bu(t)$, based on the Pontrygin's minimum principle [8], the time optimal feedback law for the target condition, $\{y, \dot{y}\} = \{x_1, x_2\}(t_f) = \{0, 0\}$ is readily determined as $u(t) = -\text{SGN}(s(\boldsymbol{x}))$ where the switching function is

$$s(\boldsymbol{x}) = x_1(t) - \frac{x_2}{|x_2|}\frac{b}{a^2}\ln\left\{1 + \frac{a}{b}|x_2|\right\} + \frac{1}{a}x_2(t). \tag{7}$$

Using the switching function, the open loop reference trajectory for the initial condition $\{x_1, x_2\}(t(0)) = \{0, 0\}$ and the target condition $\{x_1, x_2\}(t_f) = \{x_{1f}, 0\}$ is derived as:

1) when $t \leq t_s$ (acceleration interval, $u = +1$)

$$t_s = \ln\left\{1 - \sqrt{1 - \exp\{-(a^2/b)x_{1f}\}}\right\} \quad (8)$$

$$x_1(t) = (b/a)t + (b/a^2)\left(e^{-at} - 1\right) \quad (9)$$

$$x_2(t) = (b/a)\left(1 - e^{-at}\right) \quad (10)$$

$$y(x_2) = -\frac{b}{a^2}\ln\{1 - \frac{a}{b}|x_2|\} - \frac{1}{a}x_2(t) \quad (11)$$

2) when $t_s < t \leq t_f$ (deceleration interval, $u = -1$)

$$t_f = t_s + (1/a)\ln\{1 + (a/b)x_2(t_s)\} \quad (12)$$

$$x_1(t) = x_1(t_s) - (b/a)(t - t_s) \quad (13)$$

$$+ (1/a)\left[x_2(t_s) + (b/a)\right]\left[1 - e^{-a(t-t_s)}\right]$$

$$x_2(t) = x_2(t_s)e^{-a(t-t_s)} - \frac{b}{a}\left[1 - e^{-a(t-t_s)}\right] \quad (14)$$

$$y(x_2) = x_{1f} + \frac{b}{a^2}\ln\{1 + \frac{a}{b}|x_2|\} - \frac{1}{a}x_2(t) \quad (15)$$

4 Control strategy for dual-stage systems

Basics of dual-stage servo design

Since the dual-stage system has a redundancy in actuation(see Eq.(5)), each actuator has infinite number of control solutions for a specified end position. Therefore, to achieve the purpose of dual-stage system, *i.e.*, fast and fine positioning with large range of motion, the servo controller should be carefully designed so that the characteristics of coarse and fine actuators are fully utilized. The basics of dual-stage servo design may be remarked as follows.

1) The nominal trajectory should be tracked by the coarse actuator while, due to the limited motion range, the fine actuator compensates perturbation including friction effects using its high resolution and high speed capacity.

2) From the viewpoint of frequency domain, at low frequencies where large motion is dominant, tracking should be performed primary by coarse actuator. On the contrary, the main operation of the fine actuator should be at high frequencies where perturbations are fast but small in magnitude and the resonance of coarse actuator limits the performance.

Dynamically consistent null motion control

To satisfy the above strategy, the dual-stage controlle should be structured so that the coarse stage error be th *reference* for the fine stage to follow and the fine stage com pensates the coarse stage tracking errors. Since the mo tion range of fine actuator is very small(e.g., usually unde $100\mu m$ in PZT), the fine stage motion would be easily sat urated if not properly controlled. While the fine stage i saturated, it loses its ability to compensate the high fre quency perturbations. So, it is very important to assig the controller the property for the fine stage to restore t its neutral position as quickly as possible while the end po sition not perturbed due to the *null motion*. So far, littl attention has been given to the point in dual-stage systems In robotics field, the null motion control and the dynami decoupling between the task space motion and null spac motion are general topics and well established. For re dundant manipulators, general form of the relationship be tween operational force($\boldsymbol{F}$) and joint torque($\boldsymbol{\Gamma}$) is that[3]

$$\boldsymbol{\Gamma} = \boldsymbol{J}^T(q)\boldsymbol{F} + \underbrace{[\,\boldsymbol{I} - \boldsymbol{J}^T(q)\boldsymbol{J}^{T^+}(q)\,]\boldsymbol{\Gamma}_o}_{\text{null motion control vector}} \quad (16)$$

where $\boldsymbol{J}$ is the Jacobian between joint velocity and end effector velocity. and $\boldsymbol{\Gamma}_o$ is an arbitrary generalized joint torque vector.

Theorem: dynamic consistency[3]: a generalized inverse which satisfies the dynamically consistent condition(*i.e.*, the joint torques not producing the end effector's acceleration) is unique and is given by

$$\boldsymbol{J}^{\#}(q) = \boldsymbol{A}^{-1}(q)\boldsymbol{J}^T(q)\boldsymbol{\Lambda}_o(q) \quad (17)$$

where $\boldsymbol{A}$ is the manipulator inertia matrix and $\boldsymbol{\Lambda}_o(q) = \left(\boldsymbol{J}\boldsymbol{A}^{-1}\boldsymbol{J}^T\right)^{-1}$. Then, the null motion control input which do not affect the end point motion is determined by

$$\boldsymbol{\Gamma}_n = [\,\boldsymbol{I} - \boldsymbol{J}^T(q)\boldsymbol{J}^{T\#}(q)\,]\boldsymbol{\Gamma}_o \quad (18)$$

For two degrees of freedom dual-stage system of Eq. (3)-(5), $\boldsymbol{J} = [1\ 1]$, and the null motion control input satisfying the dynamic consistency is readily derived as follows.

$$\boldsymbol{\Gamma}_n = \begin{Bmatrix} u_{n1} \\ u_{n2} \end{Bmatrix} = \begin{bmatrix} -\frac{1}{4}\left(\frac{J_{e1}+J_{e2}}{J_{e2}}\right) \\ 1 - \frac{1}{4}\left(\frac{J_{e1}+J_{e2}}{J_{e2}}\right) \end{bmatrix} \boldsymbol{\Gamma}_o \quad (19)$$

where the generalized torque for fine stage regulation can be constructed as a simple PD rule, $\boldsymbol{\Gamma}_o = k_p(y_{2r} - y_2)$ - $k_d\dot{y}_2$. This control action would prevent the range saturation of fine stage and so, enhance the overall performance robustness.

5 A Robust Feedback Controller Design

In general, a robust controller is designed with respect to the upper limit of model uncertainties. However, too excessive assumption on the limit will result in a conservative design with fixed high control gains and, as a result, it may be troublesome to apply to real plant. So, a smart approach to reduce the robust controller gains is necessary.

Following the philosophy of reducing the robust control gains, we propose the *Perturbation Compensated Sliding Mode Control with No Variable Structure*(PCSMC) as a robust control approach for dual-stage systems, This is a mixed approach of sliding mode control(SMC) and perturbation observer(PO). We can find similar works in [6],[9], where variable structure type SMC's were adopted for robust stability.

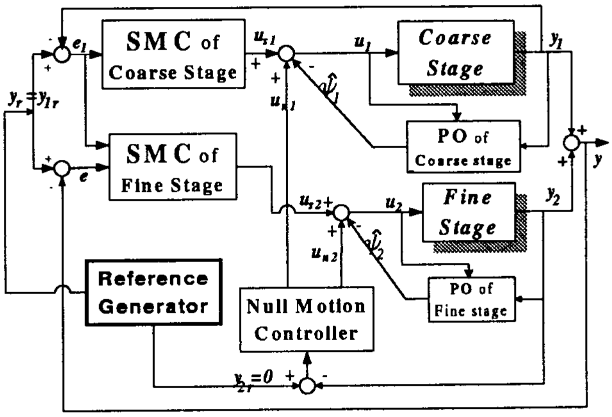


Fig. 2: The structure of PCSMC with null motion controller.

The overall controller structure for the dual-stage system is shown in Fig. 2 where the reference generator produces the time-optimal trajectory of Eq.'s (8)-(15) for coarse stage model.

Considering the coordinate relationships, $y_r = y_{1r}$, $y = y_1 + y_2$, $e = y_r - y$, $e_1 = y_{1r} - y_1$, and as a result, $e = e_1 - y_2$.

So, the direct feedback of end position error to the fine stage controller is the same as the situation that the fine stage controller accepts the coarse stage tracking error(e_1) as the reference input with the feedback of relative position(y_2). This means that the fine stage naturally tracks the coarse stage error and it maintains the neutral position in average(i.e., $y_2 \approx 0$), since e, $e_1 \to 0$ means $y_2 \to 0$. However, an independent null motion controller is necessary to increase the bandwidth of the restoring action.

Perturbation observer design

First of all, we'd like to define the "perturbation" as the total internal and external uncertainties which perturbs a plant from the *nominal* model.

Recently, as a robust control approach, there were much reports associated with *signal-based* perturbation observers. For example, "disturbance observer(DOB)"[10], "time delay control(TDC)"[7], "perturbation observer" [6] belong to this family. These are similar approaches from the viewpoint that they estimates the *effective* perturbations using plant input/output signals without relying on the perturbation model.

To derive the perturbation estimation signal, let's rewrite the dual-stage system model of Eq. (3)-(5) in the form:

$$\ddot{y}_j + a_i \dot{y}_j = b_i \left(u_j(t) + \psi_j(t) \right), \quad (j = 1, 2) \tag{20}$$

where u_j is the *normalized* control input such that $|u_j| \leq 1$ and ψ_j is the normalized effective perturbation to nominal dynamics, which includes the modeling errors, dynamic couplings, external disturbances, and nonlinear frictions.

Basically, all perturbation estimation algorithm is implemented in the following form where $f_j = -a_i \dot{y}_j$, (j=1,2), and L is usually the control frequency.

$$\hat{\psi}_j(t) = \ddot{y}_j(t - L) - f_j(t - L) - b_j u_j(t - L) \tag{21}$$

As shown, the concept of perturbation observer is physically intuitive and easy to implement. A notable fact is that it does not require any disturbance model. Under the assumption that the rate of change of perturbation is not so sharp, which is the most cases, the algorithm works well.

The perturbation estimation signal is phisically an acceleration. So, when only output measure is available, the estimation signals must be smoothened through a low pass filter(LPF) as $\hat{\Psi}_j(s) = Q(s)\hat{\psi}_j(s)$. This signal *adaptively* compensates the perturbations in real time and the closed loop approximately behaves as the nominal model and, as a result, the performance robustness increases. Therefore, this approach can be interpreted as a *variable gain* integral control or a signal-based adaptive control.

Design of sliding mode controller with no variable structure

First of all, the nominal dynamics in Eq. (20) can be arranged as

$$\begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_1 + f_2 \end{Bmatrix} + \begin{bmatrix} b_1 & 0 \\ b_1 & b_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{22}$$

$$\to \ddot{\boldsymbol{y}} = \boldsymbol{f} + \boldsymbol{B}\boldsymbol{u} \tag{23}$$

where $f_1 = -a_1 \dot{y}_1$ and $f_2 = -a_2 \dot{y}_2$.

Next, the sliding surface, $\boldsymbol{s}(t) = \{s_1(t),\ s(t)\}$ for coarse(y_1) and coarse/fine end point(y) motion are defined as proper systems, $\boldsymbol{s}(t) = \dot{e} + \boldsymbol{\Lambda} \boldsymbol{e}$, where $\boldsymbol{y} = \{y_1,\ y\}$, $\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{y}_r$, and $\boldsymbol{\Lambda} = \text{diag}\{\lambda_1,\ \lambda\}$ denotes the *desired bandwidth* of coarse and end point motion respectively, on the sliding surface. Considering the actuator bandwidth and structural limit, it should be $\lambda \gg \lambda_1$.

The conventional sliding control law is composed of continuous *equivalent control* and discrete *switching control*[5]. For the nominal model of Eq.(23), the equivalent control input is determined in the following manner.

$$\dot{\boldsymbol{s}}(t) = \ddot{e} + \boldsymbol{\Lambda}\dot{e} = \hat{\boldsymbol{f}} + \boldsymbol{B}\boldsymbol{u} - \ddot{\boldsymbol{y}}_r + \boldsymbol{\Lambda}\dot{e} = 0 \tag{24}$$

$$\to \boldsymbol{u}_{eq}(t) = \boldsymbol{B}^{-1}\left(-\hat{\boldsymbol{f}} + \ddot{\boldsymbol{y}}_r - \boldsymbol{\Lambda}\dot{e}\right) \tag{25}$$

On the other hand, the switching control action gives rise to the *approaching mode* to the sliding surface for any initial conditions and it gives robustness property to the system by keeping $\boldsymbol{s}(t) \approx 0$ in spite of model uncertainty and external disturbances.

The *discrete* switching input is of *variable structure* type such that $\boldsymbol{u}_{sw} = -\boldsymbol{k}\text{SGN}(\boldsymbol{s})$. Usually, the switching gain($\boldsymbol{k}$) is designed based on Lyapunov design approach so that it *sufficiently* guarantees the asymptotic stability for the upper bound of perturbations. However, the *sufficiency* requires the fixed high control gain and also the chattering problem due to the high frequency switching has been a critical issue.

To avoid this problem, in PCSMC, the approaching mode to the sliding surface is attained by the input, $\boldsymbol{u}_{sw} = -\boldsymbol{K}\boldsymbol{s}$, which can be regarded as a *continuous switching* input. By applying this input, due to the dynamic property of sliding surface($\boldsymbol{s}$), a *low frequency switching* will be occurred around the sliding surface. This approach is equivalent to the case that the "boundary layer"[5] of sliding surface is expanded to the maximum. Then, the sliding control law with no variable structure is

$$\boldsymbol{u}_{smc} = \boldsymbol{u}_{eq} + \boldsymbol{u}_{sw} = \boldsymbol{B}^{-1}\left(-\hat{\boldsymbol{f}} + \ddot{\boldsymbol{y}}_r - \boldsymbol{\Lambda}\dot{e} - \boldsymbol{P}\boldsymbol{s}\right) \tag{26}$$

After all, it is a PD control type with feedforward terms. However, the merits of the SMC frame are fully utilized in controller design step. That is, the closed loop bandwidth for coarse motion and coarse/fine end point motion can be considered explicitly through the sliding mode gains, $\Lambda = \{\lambda_1,\ \lambda\}$ and reaching phase gains, $\boldsymbol{P} = \{P_1,\ P\}$. In simple PD control, this is not the case. □

Now, the PCSMC law is obtained by combining the perturbation compensation signal and the SMC law as

$$\boldsymbol{u}_{pcsmc}(t) = \boldsymbol{u}_{smc} - \hat{\boldsymbol{\Psi}}(t) \tag{27}$$

Then, from Eq.'s, (23),(26),and (27), the closed loop error dynamics under the PCSMC input is

$$\ddot{e} + (\boldsymbol{P} + \Lambda)\dot{e} + \boldsymbol{P}\Lambda\dot{e} = \tilde{\boldsymbol{\Psi}}(t) \tag{28}$$

where $\tilde{\boldsymbol{\Psi}} = \boldsymbol{\Psi} - \hat{\boldsymbol{\Psi}}$. As shown, the perturbation signal to the nominal error dynamics is reduced from $\boldsymbol{\Psi}$ to $\tilde{\boldsymbol{\Psi}}$ with the action of perturbation observer, which directly means the performance robustness has been increased.

From the error dynamics of Eq.(28), the closed loop bandwidths are approximately $\omega_{BW_1} = \sqrt{P_1 \lambda_1}$ for coarse motion and $\omega_{BW} = \sqrt{P\lambda}$ for coarse/fine motion. If the PCSMC gains are selected as $P = \lambda$, under the assumption that the perturbation observer loop works well, the closed loop bandwidth is equal to the sling mode bandwidth and reaching phase bandwidth, $\omega_{BW} \approx P = \lambda$ and the response is critically damped. This relationships make the gain tuning process clear since the closed loop performance is expected in advance, which explains the merit of PCSMC for dual-stage servo design.

Finally, the overall control input for dual-stage system including the null motion control vector of Eq. (19) is

$$\boldsymbol{u}(t) = \boldsymbol{u}_{smc} - \boldsymbol{\Psi}(t) + \boldsymbol{u}_n \tag{29}$$

With the time-optimal reference trajectory, this input would achieve the robust time-optimal performance of dual-stage systems.

6 Simulations

The plant parameters in Eq.'s (3) and (20) used in simulations are: for coarse stage, $a_1 = B_{e1}/J_{e1} = 2.146$, $b_1 = pT_1/J_{e1} = 10.93$, and, for fine stage, $a_2 = B_{e2}/J_{e2} = 2.695$, $b_2 = pT_2/J_{e2} = 40.97$, where T_1 and T_2 are the rate torques of coarse and fine actuators.
The selected gains of PCSMC law are: $\lambda_1 = P_1 = 100$, $\lambda = P = 500$. So, the closed loop bandwidth is expected as: for coarse motion, $\omega_{BW_1} \approx 100$ rad/s (16 Hz) and for coarse/fine end point motion, $\omega_{BW} \approx 500$ rad/s (80 Hz). That is, the fine stage is about five times faster than the coarse stage.
As mentioned before, the fundamental role of fine stage is to compensate the coarse stage tracking errors and so, increases the overall positioning bandwidth. The step response in Fig. 3 proves this fact.
To verify the *performance robustness* of PCSMC with the null motion controller, the effective friction torques and the arbitrary large external *input* disturbances have been applied as the perturbations to both stages (*normalized* values in Fig. 4). Fig. 5 shows the performance variations, when the time-optimal trajectory is applied with 5 *mm* moving range, according as the perturbation observer(PO) is working or not. It demonstrates the adaptive performances of perturbation observer.
When comparing the 'with PO case' of Fig. 5 with Fig. 6, with the action of the null motion controller, the relative motion of fine stage has nearly kept the neutral position while the compound motion error(e) and the total control input level almost not affected.

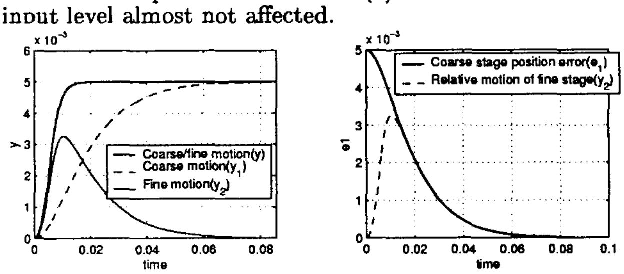


Fig. 3: Step response

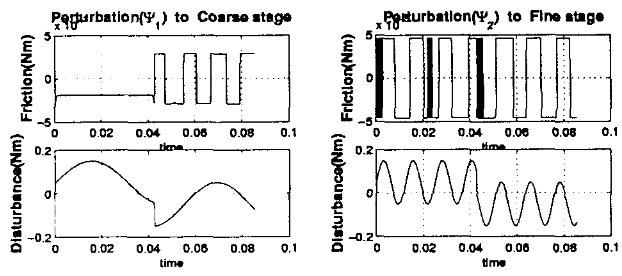
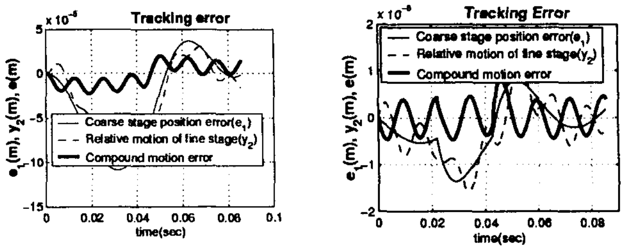


Fig. 4: Normalized perturbation inputs($\Psi_j (j = 1, 2)$): effective friction and external disturbance

7 Concluding Remarks

We have proposed the "perturbation compensated sliding mode control(PCSMC) law" as a robust control strategy. Within this control frame, the merit of SMC is available and the perturbation observer guarantees the robust performance. Also, the null motion controller satisfying the dynamic consistency has been designed to avoid the range saturation of fine stage. In addition, the time optimal solution has been determined with respect to the coarse stage(mass-damper model) and applied as a reference trajectory so that the overall closed loop system achieves the "robust time optimal performance".



Without PO action　　With PO action

Fig. 5: Performance of PCSMC (without null motion control).

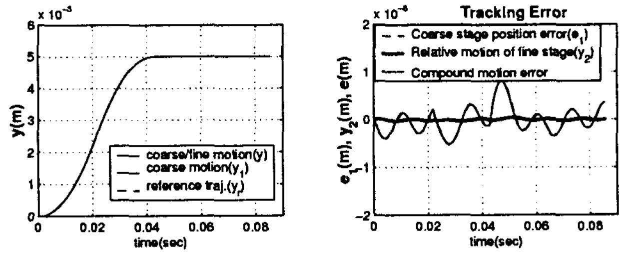


Fig. 6: Performance of PCSMC with PO and null motion control.

Through simulations, the performance robustness has been demonstrated and the action of null motion controller made it possible for the fine stage to maintain its neutral position in spite of its compensating action for the coarse stage errors. Conclusively, with the addition of time optimal reference generator and null motion controller, the PCSMC framework provides an efficient control environment for coarse/fine dual-stage servo systems.

REFERENCES

[1] Yen, J.-Y., Hallamasek, K., and Horowitz, R., 1990, "Track Following controller Desing for a Compound Disk Drive Actuator," *ASME J. of Dyn. Sys.Meas. and Cont.*, Vol. 112, Sep., pp. 391-402.

[2] Sharon, A., Hogan, N., and Hardt, D. E., 1993, "The macro/micro manipulator: An improved architecture for robot control," *Robotics and computer integrated manufacturing*, Vol. 10, No. 3, pp. 209-222.

[3] Khatib, O. , 1995, "Inertial properties in robotic manipulation: An object-level framework," *Int. J. of Robotics Research*, Vol. 13, No. 1, Feb. 1995, pp. 19-36.

[4] Staroselsky, S. and Stelson, K. A., 1988, "Two-Stage Actuation for Improved Accuracy of Contouring," *1988 American Contr. Conf.*, pp. 127-132.

[5] Slotine, J.-J. E. and Li, W., 1991, "Applied Nonlinear Control," *Prentice Hall.*

[6] Moura, J. T., Elmali, H., and Olgac, N., 1997, "Sliding Mode Control with Sliding Perturbation Observer," *ASME J. Dyn. Sys., Meas., and Contr.*, Vol. 119, Dec., pp. 657-665.

[7] Youcef-Toumi, K. and Ito, O., 1990, "A Time Delay Controller for Systems With Unknown Dynamics," *ASME J. Dyn. Sys., Meas., and Contr.*, Vol. 112, Mar., pp. 133-142.

[8] Kirk, D. E., 1970, "Optimal control theory," *Prentice Hall.*

[9] Park, J. H. and Kim, Y. M., 1999, "Time-Delay Sliding Mode Control for a Servo," *ASME J. Dyn. Sys., Meas., and Contr.*, Vol. 121, Mar., pp. 143-148.

[10] Ohnishi, K., Shibata, M., and Murakami, T., 1996, "Motion Control for Advanced Mechatronics," *IEEE/ASME Trans. on Mechatronics*, Vol. 1, No. 1, Mar. pp. 56-67.