Proceedings of the 14th KACC, Octorber 1999

Modeling and Synchronizing Motion Control of Twin-servo System

°Bong Keun Kim†, Wan Kyun Chung†, Kyo Beum Lee*, Joong Ho Song*, and Ick Choy*

† Robotics & Bio-Mechatronics Laboratory, POSTECH, Pohang, Korea
(Tel:+82-562-279-2844; Fax:+82-562-279-5899; E-mail:kbk@postech.ac.kr)
* Intelligent System Control Research Center, KIST, Seoul, Korea
(Tel:+82-2-958-5748; Fax:+82-2-958-5749; E-mail:beum@amadeus.kist.re.kr)

Abstract

Twin-servo mechanism is used to increase the payload capacity and speed of high precision motion control system. In this paper, we propose a robust synchronizing motion control algorithm to cancel out the skew motion of twin-servo system caused by different dynamic characteristics of two driving systems and the vibration generated by high accelerating and decelerating motions. This proposed control algorithm consists of separate feedback motion control algorithm of each driving system and skew motion compensation algorithm between two systems. Robust model reference tracking controller is proposed as a separate motion controller and its disturbance attenuation property is shown. For the synchronizing motion, skew motion compensation algorithm is designed, and the stability of whole closed loop system is proved based on passivity theory.

1 Introduction

High precision control systems emphasizing the demand for high performance and high productivity have introduced twin-servo mechanism in many current applications such as semi-conduct devices. Twin-servo mechanism is used to increase the payload capacity and speed of high precision system [1]. This consists of two driving motors controlled independently for one reference input. The difficulties are frequently due to the fact that systems of interests require wide range and high speed motions in which nonlinear effects are significant. Consequently, control algorithm for the high performance systems must address both the synchronizing motion control performance under the dynamic unbalance of twin-servo system and the robustness issue under the nonlinearities and uncertainties.

In this paper, we show a modeling of twin-servo system and propose a robust synchronizing motion control algorithm to cancel out the skew motion caused by different dynamic characteristics of twin-servo system. This proposed synchronizing motion control algorithm consists of a separate feedback controller of each system and a skew motion compensation algorithm between two driving system. We design a model reference position tracking controllers for a separate mechanical system with nonlinear disturbances and the nonlinear dynamic friction. For the synchronizing motion of twin-servo system, skew motion compensation algorithm is designed. The stability of whole closed loop system is analyzed based on passivity based approach.

In the next section, modeling of twin-servo system using network representation is presented. In section 3, we propose a robust motion control algorithm based on the internal-loop compensation. In section 4, a skew motion compensation algorithm of twin-servo system is proposed and the stability analysis of whole closed loop system is represented, and conclusion follows.

2 Twin-Servo Motion Control System

Twin-servo system consists of two motor systems in which the dynamic behaviors of two system are functions of each other. In this section, we deal with modeling and network representation of twin-servo system including general synchronizing motion control algorithm.

Modeling of Twin-Servo System

Most twin-servo systems consist of axes with multiple degree-of-freedoms(DOF). However, a simple one DOF system is considered in order to make the problem simple. It is easy to apply the one DOF algorithm to a multiple DOF system. A twin-servo system consists of the primary and secondary servo system with control loop closed separately around primary and secondary as shown in Fig. 1. The dynamics of two systems are given by the following equations:

$$m_p\ddot{x}_p + b_p\dot{x}_p = \tau_p + f_p \tag{1}$$

$$m_s\ddot{x}_s + b_s\dot{x}_s = \tau_s + f_s \tag{2}$$

where x_p and x_s denote the displacement of the primary and secondary motor, respectively. And m_p and b_p represent mass and viscous coefficient of the primary motor, respectively, whereas m_s and b_s are those of secondary motor. f_p denotes the force that separate feedback controller applies to the primary motor, and f_s denotes the force that separate feedback controller applies to the secondary motor. Driving forces for synchronizing motion of primary and secondary motor are represented by τ_p and τ_s, respectively.

It is assumed that the dynamics of the separate feedback controllers can be approximately represented as a simple spring-damper system:

$$f_{pr} - f_p = b_{pc}\dot{x}_p + k_{pc}x_p \tag{3}$$

$$f_{sr} - f_s = b_{sc}\dot{x}_s + k_{sc}x_s \tag{4}$$

where b_{pc} and k_{pc} are viscous coefficient and stiffness of the separate primary feedback controllers, respectively. b_{sc} and k_{sc} are those of the separate secondary feedback controller, and f_{pr} and f_{sr} denote primary and secondary reference command determined by the desired trajectory.

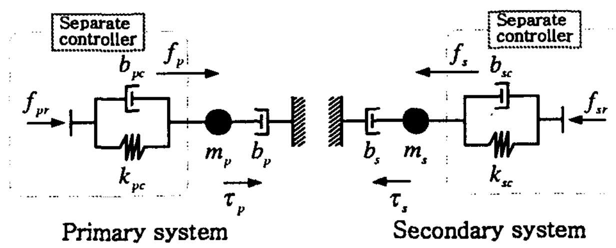


Fig. 1: Twin-servo motion control system

Skew Motion Compensating Control

Different dynamic characteristics of two motors cause the skew motion of twin-servo system. Moreover, high accelerating and decelerating motions may generate the undesirable mechanical vibration. Under such conditions, the synchronization performance may seriously get worse if these are not properly attenuated.

Synchronizing motion controller is used to synchronize the motion of two motors by cancelling out the skew motion. Hence, this has to recognize skew motion in real time and compensate dynamic difference during high-speed motion. The separate robust feedback controller compensates different dynamic characteristics of two motors and the skew motion compensating controller is appended to this.

Consider the following control schemes for primary and secondary motor as a general expression which determines compensating forces to synchronize motions:

$$\tau_p = \left(K_{pp} + K'_{pp}\frac{d}{dt} + K''_{pp}\frac{d^2}{dt^2}\right)x_p - \left(K_{ps} + K'_{ps}\frac{d}{dt} + K''_{ps}\frac{d^2}{dt^2}\right)x_s \quad (5)$$

$$\tau_s = \left(K_{sp} + K'_{sp}\frac{d}{dt} + K''_{sp}\frac{d^2}{dt^2}\right)x_p - \left(K_{ss} + K'_{ss}\frac{d}{dt} + K''_{ss}\frac{d^2}{dt^2}\right)x_s \quad (6)$$

where K_{pp}, K'_{pp}, and K''_{pp} are feedback gains of the primary motor position, velocity, and acceleration, whereas K_{ps}, K'_{ps}, and K''_{ps} are gains of the secondary motor, respectively.

In (5) and (6), we assume an ideal situation where time delay due to the data transmission between two systems is negligible. We also assume that the scales of position are identical for the primary and secondary sites.

Network Representation

Two-terminal-pair network [2, 3] shown in Fig. 2 is used in the analysis of twin-servo control system. Impedance matrix $\boldsymbol{Z}$ is defined from the relations between current and voltage of a two-terminal-pair network.

$$V_1 = z_{11}I_1 + z_{12}I_2 \quad (7)$$

$$V_2 = z_{21}I_1 + z_{22}I_2 \quad (8)$$

$$\boldsymbol{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \quad (9)$$

where I_1 and I_2 denote current, and V_1 and V_2 denote voltage at each terminal pair.

Let us consider a two-terminal-pair-network connected to a power source at each terminal pair as shown in Fig. 3. By regarding the power source as a reference command and two-terminal-pair network as a twin-servo system, the whole system can be replaced by the electrical circuit in Fig. 3. The correspondence between the modeling in the previous subsection and the circuit representation in Fig. 3 is given as

$\dot{x}_p, \dot{x}_s$	$\longleftrightarrow$	current	I_p, I_s
f_{pr}, f_{sr}	$\longleftrightarrow$	current	V_{pr}, V_{sr}
f_p, f_s	$\longleftrightarrow$	voltage	V_p, V_s
τ_p, τ_s	$\longleftrightarrow$	voltage	T_p, T_s.

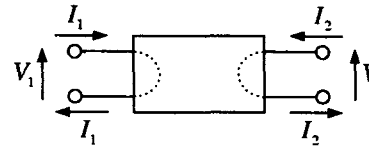


Fig. 2: Two-terminal-pair network

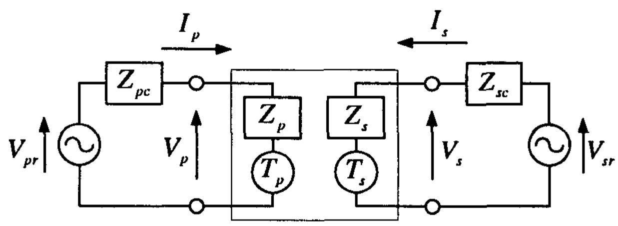


Fig. 3: Circuit representation of twin-servo system

(1), (2), (5), and (6) can be transformed from time domain into s domain as follows:

$$T_p + V_p = (m_p s + b_p)I_p \triangleq Z_p I_p \quad (10)$$

$$T_s + V_s = (m_s s + b_s)I_s \triangleq Z_s I_s \quad (11)$$

$$T_p = \left(K''_{pp}s + K'_{pp} + K_{pp}\frac{1}{s}\right)I_p - \left(K''_{ps}s + K'_{ps} + K_{ps}\frac{1}{s}\right)I_s \triangleq P_p I_p - R_p I_s \quad (12)$$

$$T_s = \left(K''_{sp}s + K'_{sp} + K_{sp}\frac{1}{s}\right)I_p - \left(K''_{ss}s + K'_{ss} + K_{ss}\frac{1}{s}\right)I_s \triangleq P_s I_p - R_s I_s. \quad (13)$$

By eliminating T_p and T_s from (10), (11), (12), and (13), the impedance matrix of the twin-servo system is obtained as follows:

$$\boldsymbol{Z} = \begin{bmatrix} Z_p - P_p & R_p \\ -P_s & Z_s + R_s \end{bmatrix}. \quad (14)$$

Noting that I_1, I_2, V_1, and V_2 in Fig. 2 correspond to I_p, I_s, V_p, and V_s in Fig. 3, respectively. The determinant, $|\boldsymbol{Z}|$, is given by

$$|\boldsymbol{Z}| = (Z_p - P_p)(Z_s + R_s) + P_s R_p \triangleq D. \quad (15)$$

Dynamics of the separate primary and secondary feedback controller can also be represented in a form of impedance.

$$Z_{pc} = b_{pc} + k_{pc}\frac{1}{s} \quad (16)$$

$$Z_{sc} = b_{sc} + k_{sc}\frac{1}{s} \quad (17)$$

Equations (16) and (17) are obtained from the simple modeling of the separate primary and secondary feedback controller in (3) and (4). Of course, one can design more appropriate impedance models for Z_{pc} and Z_{sc} if it is necessary.

3 Separate Model Reference Control

In this section, a robust model reference trajectory tracking controller based on internal-loop compensation scheme with 2-DOF control structure is presented [4]. Disturbance attenuation property of the proposed controller is shown in frequency domain using disturbance observer filter Q.

The dynamics of primary or secondary system actuated by a separate feedback controller is described as follows:

$$m\ddot{x} + b\dot{x} = f \tag{18}$$

where f is a separate feedback controller to follow the desired trajectory. Here, note that the driving force for synchronizing motion control is not concerned because only the separate feedback controller is dealt with in this section. Tracking error is defined as

$$e = x_d - x \tag{19}$$

where x_d is a desired trajectory. We assume that x_d, its first, and second derivatives are all bounded as function of time.

The proposed model reference feedback controller is formulated as follows:

$$f = f_m + K(x_m - x) \tag{20}$$

where K is the controller of internal-loop compensator, and f_m is reference control input written as

$$f_m = m_m\ddot{x}_m + b_m\dot{x}_m. \tag{21}$$

This control input is used to generate internal model state. Since m_m and b_m are designed value, x_m becomes the state of implicit internal model (21).

Suppose that we now want to design a model following controller for the system given in (18). To begin, we select the reference trajectory

$$x_m = x_d + \lambda \int_0^t e\, dt. \tag{22}$$

The difference between the state of system (18) and the state of internal model (21) is represented as follows:

$$x_m - x = e + \lambda \int_0^t e\, dt. \tag{23}$$

Differentiating both side of (23) with respect to time yields

$$\dot{x}_m - \dot{x} = \dot{e} + \lambda e \triangleq r, \tag{24}$$

leading to the following dynamic controller based on internal model compensation as shown in Fig. 4.

$$f = m_m\ddot{x}_m + b_m\dot{x}_m + K_r r \tag{25}$$

Remark 1 *From (20) and (25), reference model and controller of internal-loop compensator are represented as follows:*

$$P_m = \frac{1}{m_m s^2 + b_m s}, \qquad K = K_r s. \tag{26}$$

Using (26) and (27), we get the disturbance observer filter Q which is represented as follows [4]:

$$Q \triangleq \frac{P_m K}{1 + P_m K} = \frac{K_r/m_m}{s + (b_m + K_r)/m_m}. \tag{27}$$

Since b_m is much less than K_r in general, $\frac{K_r}{m_m}$ becomes the cutoff frequency of first order low pass filter.

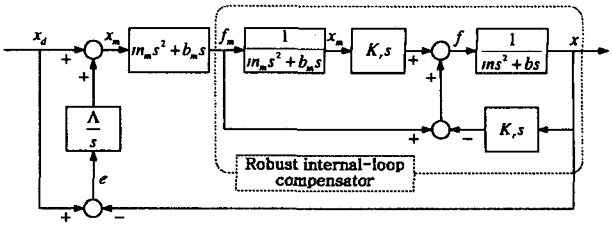


Fig. 4: Robust model reference control structure

Remark 2 *To compensate nonlinear disturbances, we can append additional control algorithm to (25) such that*

$$f = m_m\ddot{x}_m + b_m\dot{x}_m + K_r r + F(x, x_m) \tag{28}$$

where $F(x, x_m)$ is either robust or adaptive algorithm to attenuate nonlinear disturbances.

4 Stability Analysis

In this section, we discuss the stability of proposed synchronizing controller including separate primary and secondary feedback controller. Synchronizing motion controller makes the system reciprocal so that the necessary and sufficient condition can be calculated analytically.

Passivity Based Approach

The motion of primary motor results from two control inputs: f_p, the separate control command of the primary and τ_p, the skew motion compensating command of twin-servo mechanism. Control inputs of secondary motor are similar to those of the primary. Primary and secondary motor are interconnected in a feedback loop, and the dynamics of the whole closed loop system must also be considered.

From electric circuit representation of section 2, the twin-servo system can be represented as follows:

$$\boldsymbol{b} = \boldsymbol{S}\boldsymbol{a} \tag{29}$$

where the matrix $\boldsymbol{S}$ is called scattering matrix, $\boldsymbol{a}$ and $\boldsymbol{b}$ are input and output wave defined as follows:

$$\boldsymbol{a} = [a_1, a_2]^T \triangleq \frac{\boldsymbol{V} + \boldsymbol{I}}{2}, \qquad \boldsymbol{b} = [b_1, b_2]^T \triangleq \frac{\boldsymbol{V} - \boldsymbol{I}}{2} \tag{30}$$

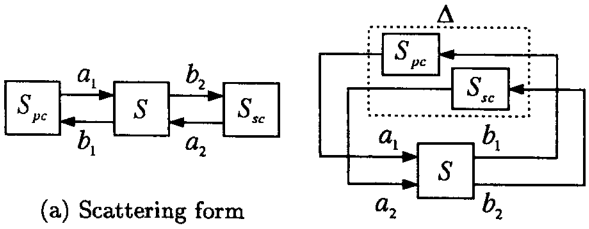
where $\boldsymbol{V} = [V_p, V_s]^T$ and $\boldsymbol{I} = [I_p, I_s]^T$. The system is passive when the power consumed in the system satisfies the following equation:

$$\begin{aligned} P &= \mathrm{Re}(V_p^* I_p - V_s^* I_s) \\ &= \boldsymbol{a}^*\boldsymbol{a} - \boldsymbol{b}^*\boldsymbol{b} \\ &= \boldsymbol{a}^*(\boldsymbol{E}_2 - \boldsymbol{S}^*\boldsymbol{S})\boldsymbol{a} \geq 0 \end{aligned} \tag{31}$$

where superscript $*$ denotes conjugate transpose, and $\boldsymbol{E}_2$ is 2×2 identity matrix. From the above inequality, the system is passive if the following inequality is satisfied.

$$\begin{aligned} \|\boldsymbol{S}\|_\infty &= \bar{\sigma}(\boldsymbol{S}(j\omega)) \\ &= \sup_\omega \lambda^{1/2} \left(\boldsymbol{S}(j\omega)^* \boldsymbol{S}(j\omega)\right) \leq 1 \end{aligned} \tag{32}$$

The scattering form of the twin-servo system is shown in Fig. 5(a). This can be described as Fig. 5(b). Here, the



(a) Scattering form

(b) Structured uncertainty

Fig. 5: Scattering form of the twin-servo system

separate primary and secondary feedback controller can be regarded as a virtual environment which is interacting with the twin-servo system. This is given as follows:

$$\boldsymbol{\Delta} = \begin{bmatrix} S_{pc} & 0 \\ 0 & S_{sc} \end{bmatrix}. \quad (33)$$

Since $|S_{pc}| \le 1$ and $|S_{sc}| \le 1$ from (3) and (4), $\|\boldsymbol{\Delta}\|_\infty \le 1$ is achieved. The necessary and sufficient condition of the stability of twin-servo system is given as follows [5, 6]:

Theorem 1 *Necessary and sufficient condition of the twin-servo system is that the scattering matrix of the twin-servo system $\boldsymbol{S}$ is analytic in $\mathrm{Re}s \ge 0$ and the structured singular value of $\boldsymbol{S}$ against the block structure $\underline{\boldsymbol{\Delta}}$ should be less than or equal to one, that is*

$$\mu_{\underline{\Delta}}(\boldsymbol{S}) \le 1, \qquad \forall \omega \quad (34)$$

where $\underline{\boldsymbol{\Delta}}$ is the following block structure corresponding to $\boldsymbol{\Delta}$ and $\mathcal{C}$ means complex number

$$\underline{\boldsymbol{\Delta}} = \{\mathrm{diag}[\Delta_1, \Delta_2] : \Delta_i \in \mathcal{C}\}. \quad (35)$$

If the system is reciprocal, that is, $\boldsymbol{S}$ is symmetric, following equation can be satisfied.

$$\mu_{\underline{\Delta}}(\boldsymbol{S}) = \|\boldsymbol{S}\|_\infty \quad (36)$$

The scattering matrix $\boldsymbol{S}$ of the system is given by

$$\boldsymbol{S} = \frac{1}{D + z_{11} + z_{22} + 1} \times \begin{bmatrix} D + z_{11} - z_{22} - 1 & 2z_{12} \\ 2z_{21} & D - z_{11} + z_{22} - 1 \end{bmatrix}. \quad (37)$$

Hence we can analyze the stability of the system by (32).

Stability Analysis

Synchronizing motion control scheme is a symmetric type proportional- and-derivative(PD) control by which one motor follows the position of the other. The control algorithm is given as follows:

$$\tau_p = k_p(x_s - x_p) + k_d(\dot{x}_s - \dot{x}_p) \quad (38)$$

$$\tau_s = k_p(x_p - x_s) + k_d(\dot{x}_p - \dot{x}_s) \quad (39)$$

where k_p and k_d are PD gain, respectively. The scattering matrix is symmetric when the system is reciprocal, so that it is easier to analyze stability. Substituting the parameter of (38) and (39) into (37), we get

$$\boldsymbol{S} = \frac{1}{(1 + ms + b)\left(1 + ms + b + 2(k_d + \frac{k_p}{s})\right)} \times \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \quad (40)$$

where

$$\alpha = \left(ms + b + \frac{k_p}{s} + k_d\right)^2 - \left(\frac{k_p}{s} + k_d\right)^2 - 1 \quad (41)$$

$$\beta = -2\left(\frac{k_p}{s} + k_d\right). \quad (42)$$

The singular values of $\boldsymbol{S}$ are given as follows:

$$\sigma_1 = \frac{|ms + b - 1|}{|ms + b + 1|} \le 1 \quad (43)$$

$$\sigma_2 = \frac{\left|ms + b + 2k_d + \frac{2k_p}{s} - 1\right|}{\left|ms + b + 2k_d + \frac{2k_p}{s} + 1\right|} \le 1. \quad (44)$$

Both of them never violate the inequality (32). Therefore, the stability of twin-servo system when the proposed control algorithm is applied has been guaranteed under the condition such that both of the separate primary and secondary feedback controller's dynamics are passive, and reference commands, f_{pr} and f_{sr} are independent of the state variables. But note that there is no time delay in the two communication channel between primary and secondary motor system.

5 Conclusions

We proposed modeling and network representation of twin-servo system including general synchronizing motion control algorithm which consists of separate feedback controllers and skew motion compensator. Model reference tracking controller based on internal-loop compensator is proposed as the separate feedback controller and symmetric type PD controller which make the system reciprocal is designed as the skew motion compensator. The stability analysis of proposed synchronizing motion control algorithm is shown based on passivity based approach.

REFERENCES

[1] S. Ahn, M. Choi, Y. Park, and J. Kim, "Synchronized control of 2-driving axes in large scale gantry robot system," *Proc. 1998 Korean Conf. on Precision Eng.*, pp. 436–439, 1998.

[2] G. J. Raju, G. C. Verghese, and T. B. Sheridan, "Design issues in 2-port network models of bilateral remote manipulation," *Proc. 1989 IEEE Int. Conf. on Robotics and Automation*, pp. 1316–1321, 1989.

[3] Y. Yokokohji and T. Yoshikawa, "Bilateral control of master-slave manipulators for ideal kinesthetic coupling - formulation and experiment," *IEEE Trans. on Robotics and Automation*, vol. 10, pp. 605–620, October 1994.

[4] B. K. Kim, W. K. Chung, H. T. Choi, I. H. Suh, and Y. H. Chang, "Robust optimal internal loop compensator design for motion control of precision linear motor," *Proc. 1999 IEEE Int. Symposium on Industrial Electronics*, pp. 1045–1050, 1999.

[5] J. E. Colgate, "Power and impedance scaling in bilateral manipulation," *Proc. 1991 IEEE Int. Conf. on Robotics and Automation*, pp. 2292–2297, 1991.

[6] T. Yoshikawa and J. Ueda, "Analysis and control of master-slave systems with time delay," *Proc. 1996 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pp. 1366–1373, 1996.