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Robust PID × (*n*-1) Stage PD Controller

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Abstract

This paper presents a design technique based on the root locus method for a class of n^{th} order plants using PID (Proportional-Integral-Derivative) × (n-1) stage PD controller. It is intended to satisfy both transient and steady state response specifications. This controller can be used instead of a conventional PID controller for the higher order plants to obtain better performances. The controlled system is approximated as a stable and robust second order controlled system. Only adjusting the controller gain, the desired performances of the controlled system are satisfied. For the stable plant including the plant with small dead time, the controlled system is made robustly stable. In case of the unstable plant, when the controller gain is adjusted higher than the critical value, the unstable plant can also be made stable. Robustness properties given by this controller proposed in this paper have also been demonstrated by numerical examples.

1. Introduction

The PID (Proportional-Integral-Derivative) controller is widely used by applying the well-known Ziegler-Nichols tuning method [1]. It is clear that the PID controller is properly applied in the typical second order plant. But it is quite difficult to use only the PID controller for the third or higher order plants because the order of the plant is greater than the number of zeros provided by the PID controller [2]. Moreover, the tuning methods sometimes require trial and error procedures, and the Ziegler-Nichols settings do not always produce the best results to meet the transient response requirements because of ¼ decay ratio criterion.

This paper presents a design technique based on the root locus method for a class of n^{th} order plants $G_p(s)$ to satisfy both transient and steady state response specifications. The transfer function of a class of n^{th} order plants should not include zeros, the poles of the plant should have negative real part (stable plant) or at most two poles with positive real part (unstable plant) is allowed. The PID × (n-1) stage PD controller [3] $G_c(s)$ is again proposed. This controller is more widely applied not only for the type 0 and type 1 plant, but it can be used for the type 2 plant, plant with a pair of complex conjugate poles, plant with small dead time and unstable plant with at most two unstable poles as well. The two dominant open-loop poles of $G_c(s)G_p(s)$ may be the simple poles, multiple poles or a pair of complex conjugate poles. The remaining (n-1) poles are considered as non-dominant open-loop poles. Due to the transfer function of the plant $G_p(s)$ usually being determined through testing and physical modeling, linearization of a nonlinear plant, or the uncertain parameters concerned, which cause the location of the poles may not be exact. The (n-1) zeros of the controller are arbitrarily placed near the left-hand side of all non-dominant open-loop poles of $G_c(s)G_p(s)$ in order to reduce the effect of these poles. The desired locations of two dominant closed-loop poles s_d are determined from the transient response specifications. The double zeros of $(s+z_c)^2$ of the controller must contribute the necessary angle to force the root locus to go through s_d. The location of the double zeros of $(s+z_c)^2$ and the gain K_c at s_d can be determined by the root locus method, which is more effective in design than the method provided by pseudo quantitative feedback theory [4]. The other (n-1) closed-loop poles are located near the (n-1) zeros. Hence, the magnitudes of the transient responses of these (n-1) closed-loop poles are very small and negligible though the exact pole-zeros cancellations do not happen [5]. However, the transient response does not completely satisfy the specification because of the effect of the double zeros of $(s+z_c)^2$. By this technique, the dominant root locus (which originated from the two dominant open-loop poles) of the stable plant is located on the left half of the s-plane, and its shape takes a form of a circle or circle-like shape. Then the controller gain can be increased to reduce maximum overshoot to obtain the desired specification. Since the roots of the characteristic equation located on the left half of the s-plane, then the controlled system can be made stable and robust. In case of the unstable plant, the shape of the dominant root locus also takes a form of circle or circle-like shape and encircles the origin in the s-plane. When the controller gain is adjusted higher than the critical value determined in the design processes, the roots of the characteristic equation are forced to locate on the left half of the s-plane. Therefore, the unstable plant can also be made robustly stable (conditionally stable) when the controller gain is adjusted higher than the critical value. Faster responses with a little overshoot could also be achieved by adjusting the controller gain higher than the designed value if desired. Furthermore, the same controller also rapidly eliminates the effect of the disturbances. Consequently, it can be said that the proposed approach gives an effective controller design method for a class of n^{th} order plants.

MATLAB's numerical results of various plants with different controller parameters show the advantages of this technique.

2. Controller Structure

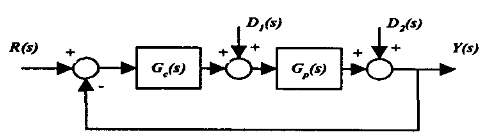

Fig. 1. Structure of the control system.

The structure of the control system is shown in Fig. 1. $R(s)$ is the reference input, $D_1(s)$ and $D_2(s)$ are defined as the process step disturbance and output step disturbance. $G_c(s)$

and $G_p(s)$ are defined as the transfer functions of the controller and the plant, respectively. In general, when the PID controller is applied to the higher order plants with step input, the steady state error is zero, but the transient response does not meet the specifications. The PID × (n-1) stage PD controller is again proposed for a class of n^{th} order plants to meet the specifications and robustness. This controller is more widely applied when compared to the PID × (n-1) stage PD controller in the previous paper [3] in that the controller proposed in this paper can cover the type 2 plant, plant having a pair of complex conjugate poles, plant with small dead time and plant with at most two unstable poles. The transfer function of the PID × (n-1) stage PD controller is again specified as

$$\begin{aligned} G_c(s) &= K_p\left(1+\frac{1}{T_i s}+T_d s\right)\prod_{j=1}^{n-1}K_{pj}\left(1+T_{dj}s\right) \\ &= \frac{K_{c1}}{s}(s+z_c)^2\prod_{j=1}^{n-1}(s+z_j), \end{aligned} \quad (1)$$

Where K_p, T_i and T_d are respectively proportional gain, integral time and derivative time of the PID controller. When $T_i \geq 4T_d$, both zeros are negative real zeros and when $T_i < 4T_d$, the two zeros are a pair of complex conjugate zeros. K_{pj} and T_{dj} are proportional gain and derivative time of the (n-1) stage PD controller. For the proposed controller, K_{c1} is the gain of the controller, $-z_c$ is the negative real double zeros, $-z_j$ (j=1,2,....,n-1) is all negative real zeros or consists of a pair of complex conjugate zeros with negative real part.

3. Plant Structure

The n^{th} order plants that are often found in practice are classified into three types, type 0, type 1 and type 2. Each type consists of a certain number of first order lags. Moreover, the plant that consists of a pair of complex conjugate poles with some first order lags and the plant with dead time is also frequently found in industries. In general, almost of the industrial plants are the stable plants and include none zeros. Therefore, the transfer function of a class of n^{th} order plants is given as

$$G_p(s) = \frac{K}{\prod_{i=1}^{n}(s+p_i)}, \quad (2)$$

where $-p_i$ (i=1,2,.....,n) is the poles at the origin, simple or multiple poles, or consists of a pair of complex conjugate poles, and consists of at most two unstable poles.

For the plant with small dead time, e^{-Ls}, the transfer function of e^{-Ls} can be approximated by the Maclaurin series [5] in the following three terms

$$\begin{aligned} e^{-Ls} &\cong \frac{1}{1+Ls+L^2s^2/2} \cong \frac{1}{\left(s+\frac{1+j}{L}\right)\left(s+\frac{1-j}{L}\right)} \\ &\cong \frac{1}{(s+p_1)(s+\overline{p}_1)}, \end{aligned} \quad (3)$$

where L is the small dead time, $-p_1$ and $-\overline{p}_1$ are a pair of complex conjugate poles on the left half of the s-plane.

In this paper, when the plant with small dead time is analyzed, the transfer function of the dead time e^{-Ls} is considered as a part of the transfer function of the plant. Then the transfer function of the plant with small dead time can also be written in the form of (2).

4. Design Procedures

The design procedures to meet the transient response specifications are as follows:

1) From (1) and (2), the open-loop transfer function is

$$\begin{aligned} G_c(s)G_p(s) &= \frac{K_{c1}K(s+z_c)^2\prod_{j=1}^{n-1}(s+z_j)}{s\prod_{i=1}^{n}(s+p_i)} \\ &= \frac{K_c(s+z_c)^2\prod_{j=1}^{n-1}(s+z_j)}{(s+p_1)(s+p_2)\prod_{i=1}^{n-1}(s+p_i)}, \end{aligned} \quad (4)$$

where $K_c = K_{c1}K$, $-p_1$ and $-p_2$ are the two dominant open-loop poles of $G_c(s)G_p(s)$ (poles with positive or negative real part) which may be the pole at the origin, simple pole, multiple poles ($p_1 = p_2$) or complex poles (p_1 and p_2 are the complex conjugate poles), $-p_i$ (i=1,2,.....,n-1) is the real or complex poles with negative real part.

2) The (n-1) zeros of the controller are placed near the left-hand side of the (n-1) non-dominant open-loop poles in order to reduce the effect of these poles. The negative real double zero of $(s+z_c)^2$ of the controller is used to force the root locus to go through s_d. Hence,

$$G_c(s)G_p(s) = \frac{K_c(s+z_c)^2\prod_{i=1}^{n-1}(s+p_i+\varepsilon_i)}{(s+p_1)(s+p_2)\prod_{i=1}^{n-1}(s+p_i)}, \quad (5)$$

where $-z_j = -(p_i + \varepsilon_i)$ (j=1,2,....,n-1; i=1,2,....,n-1), ε_i is a small real number.

3) The damping ratio ζ, undamped natural frequency ω_n and s_d are determined from the transient response specifications in (6).

$$\begin{aligned} &P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}\,\%, \quad t_s = 4/\zeta\omega_n \ (\pm 2\%), \\ &s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}, \end{aligned} \quad (6)$$

where $P.O.$ is the percent overshoot, t_s is the settling time and s_d are the dominant closed-loop poles.

4) Find the sum of the angles at s_d with all of the open-loop poles and (n-1) zeros of $G_c(s)G_p(s)$, then determine the necessary angle of $2\times\angle(s_d+z_c)$ to be added so that the total sum of the angles satisfies (7).

$$\begin{aligned} &\left[2\times\angle(s_d+z_c)+\sum_{i=1}^{n-1}\angle(s_d+p_i+\varepsilon_i)\right] \\ &-\left[\angle(s_d+p_1)+\angle(s_d+p_2)+\sum_{i=1}^{n-1}\angle(s_d+p_i)\right] \\ &= \pm(2k+1)\pi, \qquad k = 0,1,2,... \end{aligned} \quad (7)$$

5) Determine the location of the double zeros of $(s+z_c)^2$ using the angle of $\angle(s_d+z_c)$ found in (7).

6) Determine the gain K_c at s_d from

$$K_c = \frac{|(s_d + p_1)||(s_d + p_2)|\prod_{i=1}^{n-1}|(s_d + p_i)|}{|(s_d + z_c)|^2 \prod_{i=1}^{n-1}|(s_d + p_i + \varepsilon_i)|}. \quad (8)$$

7) The closed-loop transfer function is

$$\frac{Y(s)}{R(s)} = \frac{K_c(s+z_c)^2 \prod_{i=1}^{n-1}(s+p_i+\varepsilon_i)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)\prod_{i=1}^{n-1}(s+p_i+\delta_i)}, \quad (9)$$

where $\prod_{i=1}^{n-1}(s+p_i+\delta_i)$ are real or complex closed-loop poles located near the open-loop zeros $\prod_{i=1}^{n-1}(s+p_i+\varepsilon_i)$, δ_i is a small real or complex number.

Since all of the (n-1) closed-loop poles located near the open–loop zeros, it can be shown that the coefficients of these closed-loop poles are proportional to (ε_i -δ_i), which is a very small number. This implies that though the poles at -p_i can not be cancelled, the resulting transient responses due to these closed-loop poles have insignificant magnitudes, and their effect can be neglected [5].

Since (ε_i -δ_i) ≅ 0, then (9) can be written as

$$\frac{Y(s)}{R(s)} \cong \frac{K_c(s+z_c)^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}. \quad (10)$$

It is evident that the transient response does not completely satisfy the desired specifications because greater overshoot occurs due to the effect from the double zeros. Since the dominant root locus is a circle shape located on the left half of the s-plane, then the controller gain can be adjusted to reduce overshoot to obtain the desired specification. Moreover, faster responses with a little overshoot are achieved by further increase the controller gains while the controlled system still stable.

5. Effect of the Disturbances

If the process step disturbance is applied, the transfer function from the process step disturbance to the output is

$$\frac{Y(s)}{D_1(s)} = \frac{G_p(s)}{1 + G_c(s)G_p(s)}. \quad (11)$$

The effect of the process step disturbance on the step response depends on the characteristics of the process, process gain K, controller gain K_{c1}. From (11), increasing of the controller gain, the effect of the process step disturbance is decreased.

If the output step disturbance is applied, the transfer function from the output step disturbance to the output is

$$\frac{Y(s)}{D_2(s)} = \frac{1}{1 + G_c(s)G_p(s)}. \quad (12)$$

The output step disturbance has an important effect on the step response at the initial state. However, the controller rapidly eliminates the effect of the output step disturbance.

6. Special Case

For the type 0-second order plant that the two poles are a pair of complex conjugate poles locate on the left half of the s-plane. The PID controller with T_i<4T_d is used instead of the proposed controller. The complex conjugate zeros of the PID controller are placed near the open-loop complex conjugate poles. Hence, the open-loop transfer function is

$$G_c(s)G_p(s) = \frac{K_c(s+z_{c1})(s+\bar{z}_{c1})}{s(s+p_1)(s+\bar{p}_1)}. \quad (13)$$

The closed-loop transfer function can be approximated as

$$\frac{Y(s)}{R(s)} \cong \frac{K_c}{s+K_c}. \quad (14)$$

7. Numerical Examples

Example of the type 0 and type 1, fourth order plant

The example of the type 0 and type 1, fourth order plants with the proposed controller had been presented [3].

Example of type 2-fourth order plant, 2 unstable poles

$$G_p(s) = \frac{1}{s^2(s-1)(s-2)}.$$

The desired specifications for step input are

$P.O. \le 5$ %, $t_s(\pm 2\%) \le 1$ sec, $e_{ss}(t) = 0$.

From the desired specifications,

$\zeta = 0.690$, $\omega_n = 5.796$ rad/sec, s_d = -4 ± j 4.195.

With the design procedures, the transfer function of the proposed controller is

$$G_c(s) = \frac{1.764}{s}(s+7.566)^2(s+0.1)^3.$$

Therefore, the open-loop transfer function is

$$G_c(s)G_p(s) = \frac{1.764(s+7.566)^2(s+0.1)^3}{s^3(s-1)(s-2)}.$$

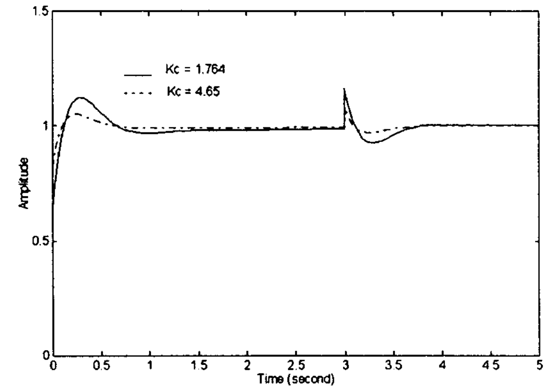

Fig. 2. Step responses of the controlled system (type 2).

When the gain K_c > 0.212 (critical value), the type 2 unstable plant can be made stable. Fig. 2 shows that the step response (with the effect of 50% output step disturbance at t = 3 sec.) of the controlled system does not satisfy all the specifications for the designed K_c = 1.764 ($P.O.$ = 12.5 %, t_s= 1.5 sec, $e_{ss}(t)$ = 0). When K_c is increased to 4.65, the step response then satisfy all the specifications ($P.O.$ = 5 %, t_s= 0.425 sec, $e_{ss}(t)$ = 0).

Example of the plant with complex conjugate poles

The proposed controller is designed for the simplified position control of an AC induction motor model that has been implemented [6]. The step responses are also compared to the step responses designed by the PIDA controller in [2]. From (16) in [2],

$$G_p(s) = \frac{168.0436}{s\left(s^2 + 25.921s + 168.0436\right)} = \frac{168.0436}{s(s+12.96+j0.263)(s+12.96-j0.263)}.$$

Again, when the same specifications are desired,

$$G_c(s)G_p(s) = \frac{0.863(s+8.208)^2(s+13.5+j0.35)(s+13.5-j0.35)}{s^2(s+12.96+j0.263)(s+12.96-j0.263)}.$$

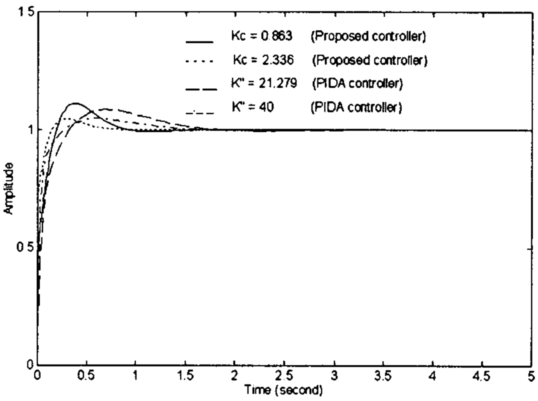

Fig. 3. Step responses of an AC induction motor model.

Figure 3 shows the step response of an AC induction motor model for the designed K_c = 0.863 ($P.O.$ = 11.5 %, t_s= 0.8 sec, $e_{ss}(t)$ = 0). When K_c = 2.336, the step response that satisfy all specifications ($P.O.$ = 5 %, t_s= 0.6 sec, $e_{ss}(t)$ = 0) is also shown. These step responses are compared to the step responses using the PIDA controller in [2]. From [2], when gain K' = 21.279, $P.O.$ = 8.95 %, and when K' = 40, $P.O.$ = 4.9 % and t_s= 1.18 sec. It can be concluded that these two design technigues give almost the same results but this proposed design technique give fast settling time and low controller gain compared to the PIDA controller.

Example of the plant with small dead time

Consider the following plant with (2) and (3),

$$G_p(s) = \frac{Ke^{-5s}}{s+2} = \frac{K}{(s+2)(s+0.2+j0.2)(s+0.2-j0.2)}.$$

With the same specifications desired, then

$$G_c(s)G_p(s) = \frac{0.51(s+10.0)^2(s+0.25+j0.25)(s+0.25-j0.25)}{s(s+2)(s+0.2+j0.2)(s+0.2-j0.2)}.$$

It is shown in Fig. 4 that the step response of the above example almost satisfies the desired specifications for the designed K_c = 0.51 ($P.O.$ = 9.1 %, t_s= 0.8 sec, $e_{ss}(t)$ = 0). When K_c is adjusted to 1.3, the step response then satisfies all the desired specifications ($P.O.$ = 5 %, t_s= 0.6 sec, $e_{ss}(t)$ = 0). However, this pair of complex conjugate closed-loop poles and zeros located near the imaginary axis produces a long tail of small amplitude [7].

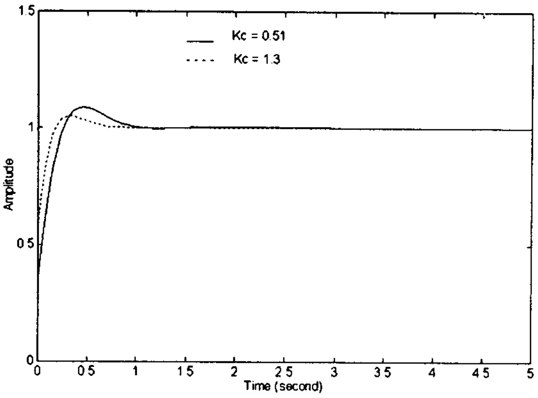

Fig. 4. Step responses of the controlled system (type 0) with small dead time.

Example of the plant with uncertain parameters

The type 0-fourth order plant with uncertain parameters while the parameters of the proposed controller remain unchanged is also discussed in the foregoing paper [3].

8. Conclusions

The PID × (n-1) stage PD controller designed by the root locus method has been again proposed in this paper. This controller can be applied to the higher order plants instead of the PID controller to obtain better performances. Moreover, this controller can also be applied to the plant with small dead time, plant with a pair of complex conjugate poles and unstable plant with at most two unstable poles. When this controller is applied to the stable plant, only adjusting the controller gain, the desired performances of the controlled system are satisfied and the system can be made robustly stable. The unstable plant could also be made stable when the controller gain is adjusted higher than the critical value. Faster responses with a little overshoot can also be obtained. Moreover, the controller rapidly eliminates the effect of the disturbances.

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