Proceedings of the 14th KACC, Octorber 1999

Identification of 2D Impulse Response by use of M-array with Application to 2D M-transform

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Abstract

In this paper, a new method for identification of two-dimensional(2D) impulse response is presented. As is well known, identification of 2D impulse response is an important and necessary theme for image processing or signal processing. Here, the authors extend M-transform which has been proposed by some of the authors to 2D case where an image is used instead of signal, and M-array is used instead of M-sequence. Firstly, we show that 2D impulse response can be obtained by use of M-array. Next 2D M-transform is defined where any 2D image can be considered to be the output of 2D filter whose input is 2D M-array. Simulation results show the effectiveness of identification of 2D impulse response by either using M-array or by 2D M-transform.

1. Introduction

The authors have recently proposed a new idea of signal transformation by use of pseudorandom M-sequence(which we call M-transform), and its application to identification of impulse response of linear system was shown [7]. In this signal transformation, any time signal can be considered to be the output of a filter, whose input is an M-sequence, which resembles so-called "Prewhitening" of a signal where any time signal is considered to be the output of a filter whose input is white noise. In this paper, the authors extend this M-transform to two-dimensional case where an image is used instead of signal, and M-array is used instead of M-sequence. For this purpose, firstly we show that 2D impulse response can be obtained by use of M-array: M-array is inputted to a 2D filter having 2D impulse response, and the resulting image is 2D cross-correlated with the input M-array, thus obtaining the 2D impulse response. Next 2D M-transform is defined where any 2D image can be considered to be the output of 2D filter whose input is 2D M-array. Simulation results are shown for the identification of 2D impulse response using M-array, and also for 2D M-transform with some applications.

2. M-array and some of its properties

In this chapter, we shall simply introduce the definition of M-array. M-array is a two dimensional arrangement, which can be made by M-sequence, and has almost the same properties as M-sequence with respect to such statistical properties as autocorrelation function, crosscorrelation function and so on.

2.1 The definition of M-array

Let us denote an element $a_i(= 0\ or\ 1)$ of an M-sequence of a period of $N(= 2^n - 1)$.

When the period N is written as $N = N_1 \times N_2$, $N_1 = 2^r - 1$ and $GCD(N_1, N_2) = 1$, M-array $\boldsymbol{B}$ of $N_1 \times N_2$ degree is defined as follows.

$$\boldsymbol{B} = \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,N_2} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,N_2} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N_1,1} & b_{N_1,2} & \cdots & b_{N_1,N_2} \end{bmatrix} \quad (1)$$

Suppose element $b_{i,j}$ of $\boldsymbol{B}$ on the location of i-th row and j-th column of $\boldsymbol{B}$ can be defined as follows by use of a_k of M-sequence:

$$b_{i,j} = a_k \qquad where\ \ 0 \le k \le N - 1 \quad (2)$$

Then we can get the elements $b_{(i+1),j}$ of $\boldsymbol{B}$ and $b_{i,(j+1)}$ of $\boldsymbol{B}$ as follows:

$$b_{i+1,j} = a_{k+s_r} \quad (3)$$
$$b_{i,j+1} = a_{k+s_c} \quad (4)$$

Assume two integer numbers μ and ν have been determined according to the condition: $\mu N_1 + \nu N_2 = 1 \ \ (mod\, N)$ then the two necessary parameters S_c and S_r can be decided by the following equations.

$$s_r = \mu N_1 \quad (5)$$
$$s_c = \nu N_2 \quad (6)$$

2.2 Autocorrelation function of M-array

The 2D autocorrelation function $\phi_{bb}(i,j)$ of M-array $\boldsymbol{B}$ becomes as follows:

$$\phi_{bb}(u,v) = \frac{1}{N}\sum_{i=0}^{N_1-1}\sum_{j=0}^{N_2-1} b_{i,j}b_{i+u,j+v}$$

$$= \begin{cases} 1 & (\ u,v=0) \\ -\dfrac{1}{N} & (\ 0 \le u \le N_1, 0 \le v \le N_2) \end{cases} \qquad (7)$$

This property of M-array resembles "pseudo-orthogonal property of M-sequence". We will use this property of M-array to the identification of 2D impulse response.

3. Identification of 2D impulse response by use of M-array

We consider here the problem of identifying a 2D impulse response as shown in Figure 1. The sampled input image is $x(i,j)$ and sampled output image is $y(i,j)$.The 2D impulse response of the system to be identified is $g(i,j)$. $n(i,j)$ is an added noise of system.

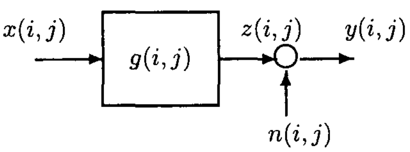

Figure 1: Identification of 2D impulse response

The output of the system is described as follows:

$$y(i,j) = \sum_{\tau_1=0}^{N_1-1}\sum_{\tau_2=0}^{N_2-1} g(\tau_1,\tau_2)x(i-\tau_1, j-\tau_2) + n(i,j) \qquad (8)$$

When the input image $x(i,j)$ is M-array $b(i,j)$, we can get the following equation under the condition that the crosscorrelation function between the M-array $b(i,j)$ and noise $n(i,j)$ is zero,

$$\phi_{by}(i,j) = \sum_{\tau_1=0}^{N_1-1}\sum_{\tau_2=0}^{N_2-1} g(\tau_1,\tau_2)\phi_{bb}(i-\tau_1, j-\tau_2) \qquad (9)$$

Since $\phi_{bb}(\cdot,\cdot)$ becomes a 2D $\delta-$ function as shown in section 2, we can get the following relationship.

$$\phi_{by}(i,j) \simeq g(i,j) \qquad (10)$$

This means that under the condition that we can apply an M-array to an unknown 2D system, the 2D impulse response of this system can be identified. We carried out simulation in this situation and the estimated result is shown in Figure 2.

This method is effective and powerful in the 2D linear system identification in case where we can apply an M-array to the system as an input image. But in many actual situations, we cannot apply any image to the actual system. We can merely know input image $x(i,j)$ and output image $y(i,j)$ of the system to be identified. To overcome this disadvantage, we propose a new method by use of 2D M-transform as an expansion of M-transform in the next section and the effectiveness of the 2D M-transform has been confirmed through simulations.

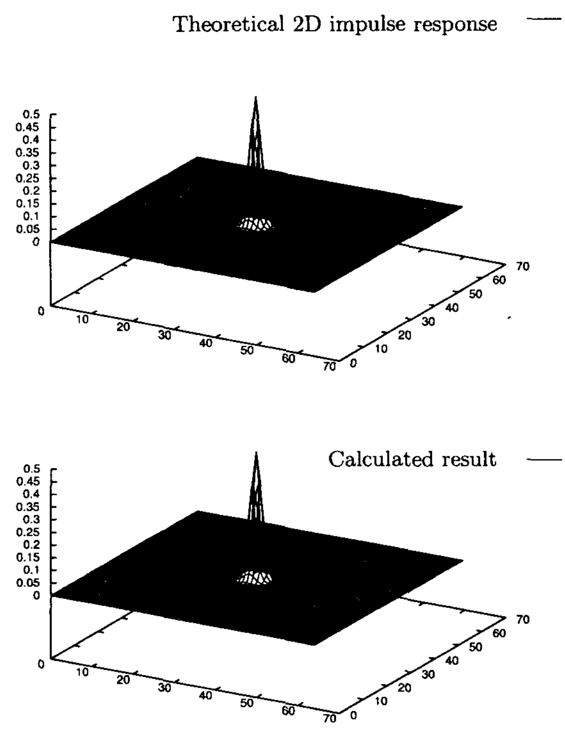

Figure 2: Identification of 2D impulse response when input is M-array

4. 2D M-transform

4.1 The definition of 2D M-transform

The idea of 2D M-transform is shown in Figure 3 which resembles one dimensional M-transform. Just like in case of one dimensional M-transform where any time signal X_i can be considered as the output of a filter whose input is an M-sequence and we call this filter as M-filter, any 2D image $x(i,j)$ can be considered as the output of a 2D filter whose input is an M-array and we call this 2D filter as 2D M-filter. We call this idea as 2D M-transform.

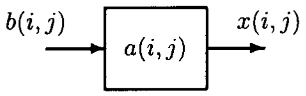

Figure 3: Definition of 2D M-transform

Here, 2D M-transform $a(i,j)$ is defined as:

$$a(i,j) = \frac{N}{N+1}\phi_{bx}(i,j) + \frac{N}{N+1}\sum_{i'=0}^{N_1-1}\sum_{j'=0}^{N_2-1}\phi_{bx}(i',j') \quad (11)$$

4.2 Principle of identification of 2D impulse response by use of 2D M-transform

Now, we will use 2D M-transform to identify 2D impulse response of a linear system which is shown in Figure 1. Here, we suppose that $x(i,j)$ is an output of a 2D filter $a(i,j)$ whose input is M-array $b(i,j)$ (shown in Figure 4). That is, $a(i,j)$ is the 2D M-filter described earlier. $h(i,j)$ is the 2D impulse response of the system from $b(i,j)$ to $y(i,j)$, that is, the cascade system of $a(i,j)$ and $g(i,j)$. $n(i,j)$ is an independent noise signal.

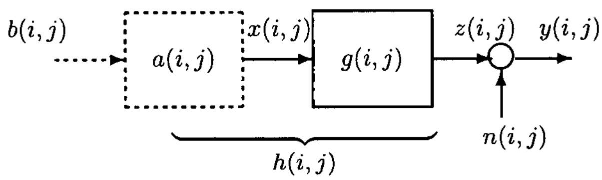

Figure 4: 2D impulse response identification by use of 2D M-transform

Since the impulse response of the cascaded system from $b(i,j)$ to $y(i,j)$ is $h(i,j)$, we have

$$y(i,j) = \sum_{\tau_1=0}^{N_1-1}\sum_{\tau_2=0}^{N_2-1} h(\tau_1,\tau_2)b(i-\tau_1, j-\tau_2) + n(i,j) \quad (12)$$

So from the principle of identification of 2D impulse described in section 3 we get,

$$h(i,j) \simeq \phi_{by}(i,j) \quad (13)$$

Since $h(i,j)$ is the impulse response of the cascaded systems of $a(i,j)$ and $g(i,j)$, we have

$$\boldsymbol{H} = \boldsymbol{A} * \boldsymbol{G} \quad (14)$$

where $\boldsymbol{H}$ is a matrix having element $h(i,j)$ with $N_1 \times N_2$ degree , $\boldsymbol{A}$ is a matrix having element $a(i,j)$ with $N_1 \times N_2$ degree and $\boldsymbol{G}$ is a matrix having element $g(i,j)$ with $N_1 \times N_2$ degree, respectively. * denotes convolution integral operator.

We can use Fourier transform in Equation 14 and we have

$$\boldsymbol{H}(j\omega_1, j\omega_2) = \boldsymbol{A}(j\omega_1, j\omega_2)\boldsymbol{G}(j\omega_1, j\omega_2) \quad (15)$$

where $\boldsymbol{H}(j\omega_1, j\omega_2)$, $\boldsymbol{A}(j\omega_1, j\omega_2)$, $\boldsymbol{G}(j\omega_1, j\omega_2)$ are Fourier transform of $\boldsymbol{H}$, $\boldsymbol{A}$, and $\boldsymbol{G}$, respectively. Thus we have

$$\boldsymbol{G}(j\omega_1, j\omega_2) = \boldsymbol{H}(j\omega_1, j\omega_2)/\boldsymbol{A}(j\omega_1, j\omega_2) \quad (16)$$

5. Simulation

In order to verify the effectiveness of the idea of 2D M-transform, we carried out two kinds of computer simulation tests under the same simulation conditions. The simulation block diagram is just like Figure 1. Any picture shown in Figure 5 is used as an input image($x(i,j)$) of the simulation. It is then added to the system which is needed to be identified and output image($z(i,j)$) of the system is obtained. The output image with an added noise is obtained and it is shown in Figure 6($y(i,j)$). Theoretical result of this identification simulation of 2D impulse response is shown in Figure 7 in order to compare with the calculated one. When we use the conventional method of identification by use of Fourier transform, that is,

$$Y(j\omega_1, j\omega_2) = G(j\omega_1, j\omega_2)X(j\omega_1, j\omega_2) + N(j\omega_1, j\omega_2) \quad (17)$$

we cannot avoid the effect of noise $N(j\omega_1, j\omega_2)$. The simulation result in this case is shown in Figure 8. It is easily seen that conventional Fourier transform method is not effective when noise exist. The 2D M-transform method(Figure 9) is much better than the conventional method(Figure 8). Then, under the same simulation condition we try another computer simulation test, using M-sequence a_i of degree 14 with a characteristic polynomial $f(x)$ ($f(x) = 40053$ in octal notation). Here $N_1 = 127$ and $N_2 = 129$ in this test. Figure 9 show the simulation result by the method proposed in this paper, that is, 2D M-transform. The results show a good agreement with the theoretical considerations and it is clearly seen 2D M-transform is a very effective way for 2D impulse response identification.

6. Conclusion

In this paper, identification of 2D impulse response by use of M-array is first proposed, and the extension of one-dimensional M-transform to 2D case is proposed. We call this idea as 2D M-transform. The application of 2D M-transform to identification of 2D impulse response is shown. In this 2D-signal transformation, any image can

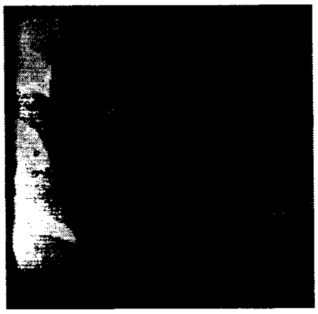

Figure 5: Input image

Figure 6: Output image with an added noise

be considered to be the output of a 2D filter, whose input is an M-array. Simulation results are shown for the identification of 2D impulse response using M-array, and also for use of 2D M-transform. The effectiveness of 2D M-transform is obviously seen from the simulation test. The other applications of 2D M-transform are now under development.

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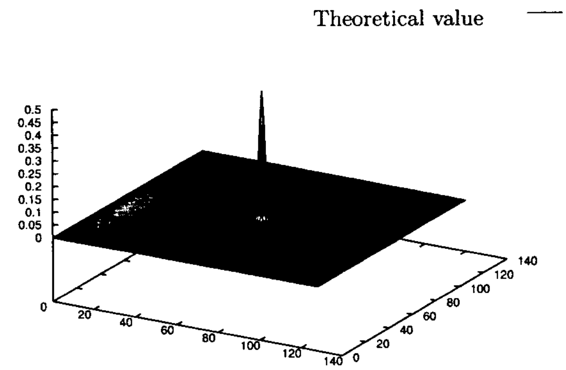

Figure 7: Theoretical value of 2D impulse response

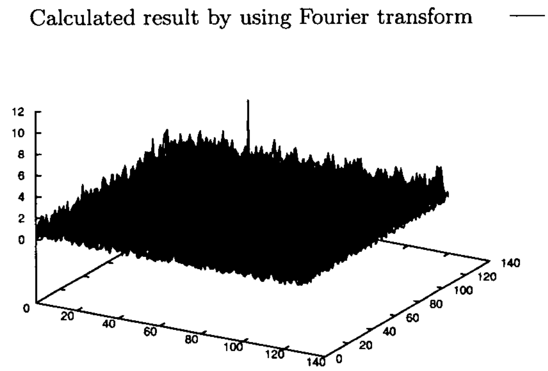

Figure 8: Simulation result by using conventional Fourier transform

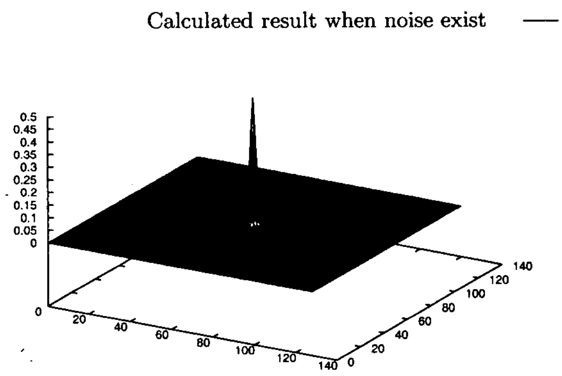

Figure 9: Simulation result by using 2D M-transform