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Decentralized Active Vibration Control Systems for Multi Degree of Freedom Structures

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Abstract

This paper is concerned with the design method of a decentralized linear control system and its application to vibration control of multi degree of freedom structures. The method is based on the partial model matching on frequency domain by minimizing the relative model error functions between the diagonal elements of the open loop transfer function matrix of the control system and these of the reference model. The method is examined and evaluated by both simulation and experiment of a multi degree of freedom structure(MDFS).

1. Introduction

Active vibration control strategies have been applied to many areas because of their good performance and general applicability. The problem of designing decentralized active vibration control system has been considered by many researchers. Decentralized active vibration control design scheme appears to be a good solution for the multi degree of freedom structures, because of its simple structure of the control system.
Recently, an interesting approach based on partial model matching on frequency domain was applied to the design of decentralized PID controller by Iwai et al. [1]. The special feature of the method is that, it does not need to check the frequency response of the controlled system on all over frequency range, instead, it only requires the data of a few points on Nyquist curve for designing the controller. In this paper, we extend the above-mentioned control system design method based on partial model matching on frequency domain to general decentralized linear controller design. The proposed method is applied to active vibration control system design problem of an MDFS. The proposed method is examined by both simulation and experiment.

2. Control System Design

2.1 Feedback Control System Description

The block diagram of linear feedback active vibration control system of structures is shown in Fig.1. $V(s)$ shows the disturbance acting on the structure to be controlled and $D(s)$ shows the transfer characteristics of $V(s)$. The output $Y(s)$ is given as follows.

$$Y(s)=\{I+G(s)C(s)\}^{-1}(D(s)V(s)+G(s)C(s)r(s)) \quad (1)$$

The purpose of control system design here is to reduce the vibration of the structure caused by disturbance $V(s)$ by using feedback control. Without loss of generality, we assume that $r(s)=0$, then we can consider the design problem as a low sensitivity control system design.
Assume that $G(s)$ be a stable $m \times m$ plant transfer function matrix as follows.

$$G(s)=\begin{bmatrix} g_{11}(s) & \cdots & g_{1m}(s) \\ \vdots & \ddots & \vdots \\ g_{m1}(s) & \cdots & g_{mm}(s) \end{bmatrix} \quad (2)$$

Further, we assume that $g_{ij}(s)$ is proper (or strictly proper). Here, the controller is given in diagonal form shown as

$$C(s)=diag\left\{\frac{n_{ci}(s)}{d_{ci}(s)}\right\},\ i=1,2,\cdots,m \quad (3)$$

where $d_{ci}(s)$ denotes the stable denominator polynomial given by the designer. $n_{ci}(s)$ is the polynomial of s.

$$n_{ci}(s)=\sum_{j=0}^{r_i} b_{ij}s^j \quad (4)$$

Here r_i denotes the order of $n_{c_i}(s)$ and $b_{ij},\ j=1,2,\cdots,r_i$, are the unknown controller parameters to be determined. We assume that the elements of $C(s)$ are proper (or strictly proper).

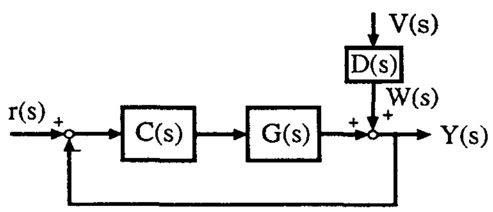


Figure 1: Block diagram of control system

Consider an ideal closed loop system as shown in Fig.2. Here,

$$G_M(s) = diag\{g_{Mi}(s)\}, \;\; i = 1, 2, \cdots, m \qquad (5)$$

is an open loop transfer function matrix with desirable(ideal) frequency characteristics. The control system design can be achieved by determining $C(j\omega)$ so as to match the $G(j\omega)C(j\omega)$ to $G_M(j\omega)$ in some sense.

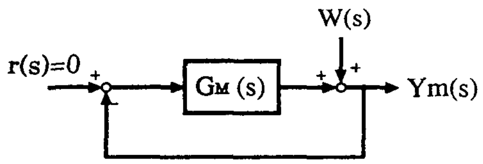


Figure 2: Block diagram of reference model

2.2 Decentralized Controller Design

Without loss of generality, we consider that the controlled plant $G(s)$ have the same resonance frequencies as $D(s)$. Here we assume that the resonance frequencies are known. The design object is of realizing low sensitivity at each peak of resonance frequencies. We select the element $g_{Mi}(s)$ of the diagonal reference model as[3],

$$g_{Mi}(s) = \sum_{j=1}^{n_i-1} \frac{\gamma_{ij}s}{s^2+2\zeta_{ij}\omega_{ij}s+\omega_{ij}^2} + \frac{\gamma_{in_i}s+\beta_i}{s^2+2\zeta_{in_i}\omega_{in_i}s+\omega_{in_i}^2} \qquad (6)$$

where, ω_{ij} and $\omega_{in_i}(> \omega_{ij})$ show the resonance frequencies, γ_{ij}, β_i and ζ_{ij} are positive constant parameters. This equation means that $(1 + g_{Mi}(s))^{-1}$ is chosen to be nearly notch filter at each resonance frequency.

The controller parameters are tuned by matching the diagonal elements of the actual open loop frequency response matrix $G(j\omega)C(j\omega)$ to the ideal system $G_M(j\omega)$ at several points in the frequency domain[1][2][3].

Let us define the relative model error of ith diagonal element between the open loop transfer function GC and G_M on frequency domain as follows.

$$\varepsilon_i(j\omega) = \frac{g_{Mi}(j\omega) - g_{ii}(j\omega)\dfrac{n_{ci}(j\omega)}{d_{ci}(j\omega)}}{g_{Mi}(\omega)} \qquad (7)$$

Further, define the following cost function,

$$J_i = \sum_{k=1}^{M} |\varepsilon_i(j\omega_k)|^2 + \sum_{k=1}^{M} |\varepsilon_i(-j\omega_k)|^2 \qquad (8)$$

where M is the number of frequency points of model matching. The optimal parameter values of ith element of the controller matrix equation (3) are given by the least square solutions of equation (8).

3. Simulation and Experiment

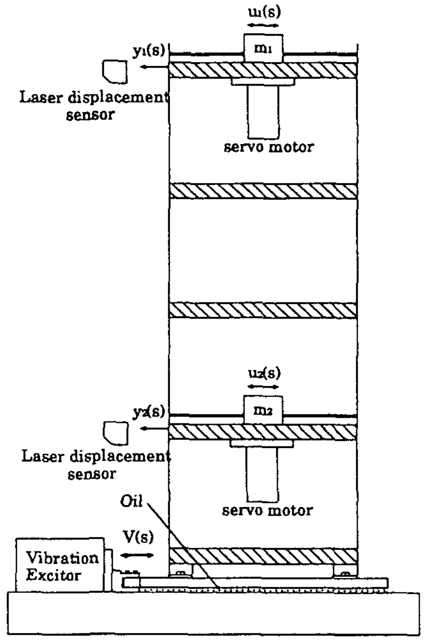


Figure 3: Diagram of the experimental equipment

The proposed design method is applied to the vibration control system design problem of a pilot-scale four layer building structure.

3.1 Experimental Equipment

The active vibration control experiment model of the pilot-scale four layer building structure is shown in Fig.3 The structure is set on a table which is excited by a linear motor. Displacements of the structure are measured by laser sensors. Active mass dampers (actuators) are set at the top layer and the first layer, respectively, to control the vibrations.

The structure is modelled as

$$G(s) = f^T \Delta_1 f \qquad (9)$$

$$D(s) = f^T \Delta_2 f_2 \qquad (10)$$

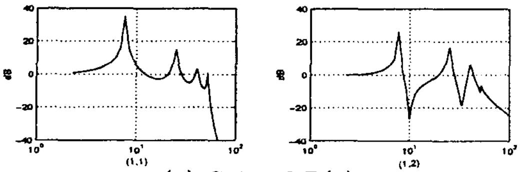
where f is a modal coefficient matrix given as

$$f = \begin{Bmatrix} 0.230 & -0.202 & 0.186 & -0.303 \\ 0.085 & 0.247 & 0.268 & 0.066 \end{Bmatrix}^T \qquad (11)$$

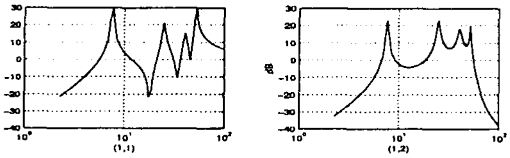
and f_2 denotes the second column vector of f. Δ_1 and Δ_2 are 4×4 diagonal matrix shown as

$$\Delta_1 = \frac{1}{0.0166s+1} diag\left\{\frac{-14.4s^2}{s^2+c_is+\omega_i^2}\right\} \qquad (12)$$

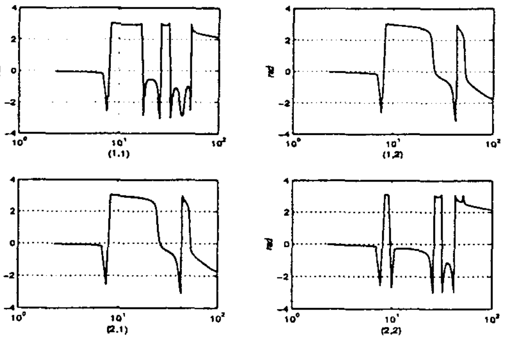
$$\Delta_2 = diag\left\{\frac{2.9s+3518}{s^2+c_is+\omega_i^2}\right\} \qquad (13)$$



(a) Gain of $D(s)$



(b) Gain of $G(s)$



(c) Phase of $G(s)$

Figure 4: Frequency responses of the modelled plant

The values of parameter ω_i and c_i are shown in Table 1.

3.2 Simulation

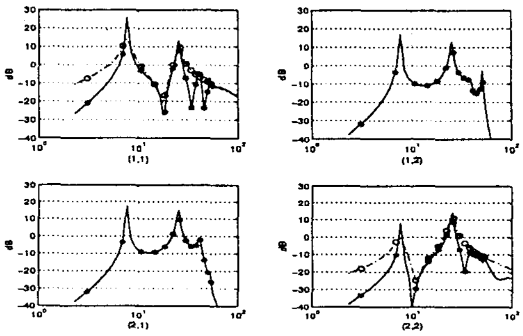
The frequency characteristics of the plant are shown in Fig.4. Considering that the resonant peaks of first and second resonance frequencies of $D(s)$ are larger than those of the third and the fourth, we take the first and the second resonance frequencies into consideration in the vibration control system design. The reference model is chosen as

$$g_{Mi}(s) = \frac{\gamma_{i1} s}{s^2 + 2\zeta_1\omega_1 s + \omega_1^2} + \frac{\gamma_{i2} s + \beta_i}{s^2 + 2\zeta_2\omega_2 s + \omega_2^2} \quad (14)$$

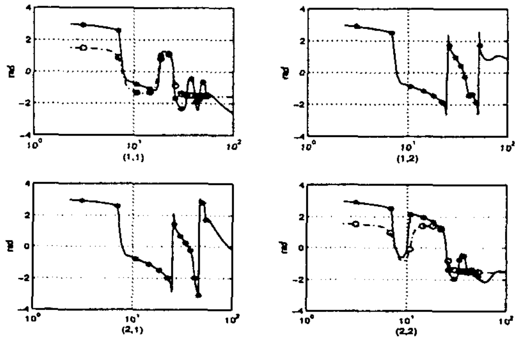
where the parameters are given as follows.

Table 1: Values of ω_i and c_i

i	1	2	3	4
ω_i	7.6	25.0	40.8	52.0
c_i	0.1002	1.104	3.016	1.17



(a) Gain



(b) Phase

$*$: $G(j\omega)C(j\omega)$ at matching frequency points
o : $G_m(j\omega)$ at matching frequency points

Figure 5: Model matching results of simulation

$$\zeta_1 = 0.08, \zeta_2 = 0.04, \gamma_{11} = 2\zeta_1\omega_1 \times 5, \gamma_{12} = 2\zeta_2\omega_2 \times 4,$$

$$\gamma_{21} = 2\zeta_1\omega_1 \times 1,\ \gamma_{22} = 2\zeta_2\omega_2 \times 5,\ \beta_1 = \beta_2 = 0.01$$

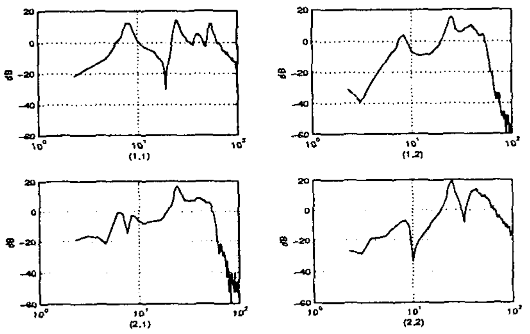
Both $d_{c1}(s)$ and $d_{c2}(s)$ are given in the form of Butterworth filter. The orders and the cut-off frequencies are selected to be 7 and 40(rad/sec), respectively. The orders of the numerators of the controllers are chosen to be 6.

The matching frequency points are chosen from 3.068 (rad/sec) to 52.92(rad/sec) with an interval of 3.835 (rad/sec). The model matching results are shown in Fig.5. It is obvious from Fig.5 that the frequency characteristics of each diagonal elements match those of the reference model well at matching frequency points. The stability of the feedback system is checked by Nyquist stability theory.

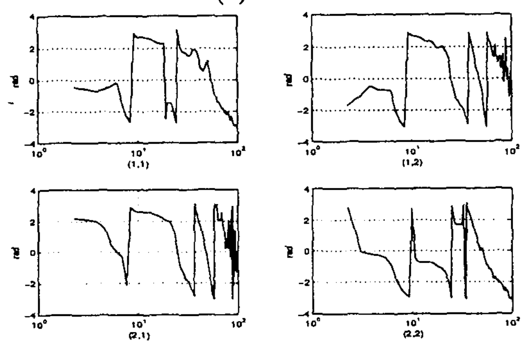
The time responses of the simulation are omitted here.

3.3 Vibration Control Experiment

Instead of transfer function, here, we use the real spectrum information on the plant to design the control system. Random signals are introduced into the actuator to get responses of the top layer and the first layer from inputs. The results are shown in Fig.6. Compare Fig.6 with Fig.4, it was confirmed that the resonance frequencies of the actual plant matched well to simulation case.

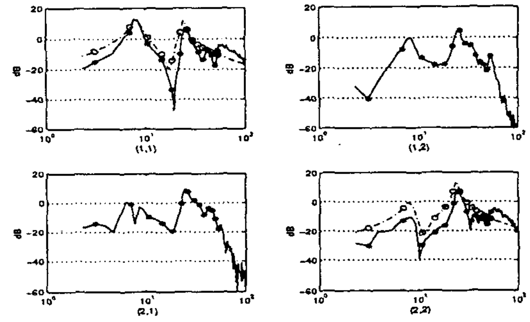


(a) Gain

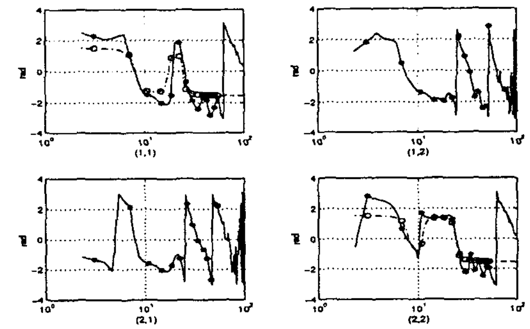


(b) Phase

Figure 6: Frequency responses of actual plant



(a) Gain



(b) Phase

$*$: $G(j\omega)C(j\omega)$ at matching frequency points
o : $G_m(j\omega)$ at matching frequency points

Figure 7: Model matching results of actual plant

We use the same design parameters as used in simulation. The matching results are shown in Fig.7. It can confirm that the frequency characteristics of the diagonal elements of the control system match the reference model well from Fig.7. In this case, the feedback system is stable.
The structure is excited using sinusoidal waves with the first and the second resonance frequencies respectively. Time responses of y_1 and y_2 are shown in Fig.8. From Fig.8, it is obvious that the vibration displacements of y_1 and y_2 are suppressed effectively.

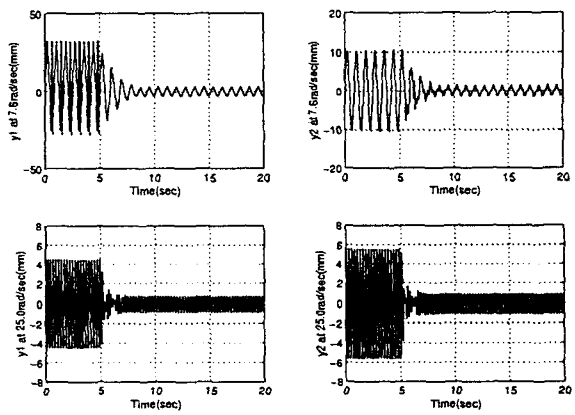


Figure 8: Time responses
Control started at 5 second

4. Conclusions

In this paper, a decentralized controller design method on frequency domain was applied to active vibration control system design of multi degree of freedom structure. The effectiveness was examined by both simulation and experiment of a four layer building structure.

References

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