

**A CONSTRUCTION METHOD OF MULTIPLE CONTROL SYSTEMS USING  
PARTIAL KNOWLEDGE UPON SYSTEM DYNAMICS**

□Ikuro Yoshihara\*, Masaaki Inaba\*\*, Tomoo Aoyama\* and Moritoshi Yasunaga\*\*\*

\*Faculty of Engineering, Miyazaki University, Gakuen Kibanadai Nishi, Miyazaki, 889-2155, JAPAN  
(Tel : +81-985-58-7411; Fax : +81-985-58-7398; E-mail : yoshiha@opt.miyazaki-u.ac.jp, aoyama@esl.miyazaki-u.ac.jp)

\*\*Systems Development Laboratory, Hitachi, Ltd., 1099 Ohzenji, Asao-ku, Kawasaki, 215-0013, JAPAN  
(Tel : +81-44-966-9111; Fax : +81-44-959-0851; E-mail : inaba@sdl.hitachi.co.jp)

\*\*\*Institute of Information Science and Electronics, University of Tsukuba, Tennoudai, Tsukuba, 305-8573, JAPAN  
(Tel : +81-298-53-5323; Fax : +81-298-53-5206; E-mail : yasunaga@is.tsukuba.ac.jp)

**Abstract**

This paper presents an effective construction method of adaptive multiple control systems utilizing some knowledge upon the plants.

The adaptive multiple control system operates plants under widely changing environmental conditions. The adaptive multiple control system is composed of a family of candidate controllers together with a supervisor. The system does not require any identification schemes of environmental conditions. Monitoring outputs of the plant, the supervisor switches from one candidate controller to another. The basic ideas of adaptation are as follows: (1) each candidate controller is prepared for each environmental condition in advance; (2) the supervisor applies a sequence of speculative controls to the plant with candidate controllers just after the start of control or just after the detection of a change in the environmental condition. Each candidate controller can keep the system stable during one-step period of the speculative control and the most appropriate candidate controller for the environmental condition to which the system is exposed can be selected before the last trial of speculative control step comes to an end.

We proposed a construction method of adaptive multiple control system without any knowledge of plant dynamics and applied the method to a cart-pole balancing problem and a vehicle anti skid braking system.

In real applications, as we can often easily obtain a piece of knowledge upon plant dynamics beforehand, we intend to extend the method such that multiple control systems can be efficiently designed using the knowledge. We apply the new idea to the cart-pole balancing problem with variable length of the pole. The simulation experiments lead us to the conclusion that the new attempt can reduce the manpower to design the candidate controllers for adaptive multiple control systems.

**1. Introduction**

Increasing attention has been paid to adaptive control systems which are required to operate in a variety of environmental conditions [1]. In recent years, the researches of such systems have been focused on the unexplored area of adaptive multiple control systems [2]-[12]. The typical adaptive multiple control system consists of a family of candidate controllers (CC's) and a supervisor [2]. A supervisor is a logical element capable of selecting in real time which candidate controller should be put in feedback with a plant in order to achieve required control performance.

Approaches using multiple control system can be roughly classified into two categories by type of information which

a supervisor monitors. In the first category, switching from one candidate controller to another is carried out based on monitoring estimated plant parameter values. The work in [3] shows an adaptive fuzzy control approach. A number of fuzzy control laws are obtained for different typical plant models in advance. An appropriate fuzzy control laws are inferred from them based on observing plant parameter values. This approach is successfully implemented in [4] for the cooperation control of several wet pumps. In the second category, switching is carried out based on monitoring observed identification errors between the plant output and multiple identification models. The work in [5] shows an approach using multiple models, where models are switched and tuned based on monitoring identification errors. This approach was extended in [6] to nonlinear systems with successful experimental results for a two-link direct-drive robot arm using eight adaptive models. Despite of much success of these approaches, several drawbacks exist. One of the most serious problems is that their performance may degrade, and what is worse, systems may lose their stability in case when the enough accuracy of plant parameter estimators or multiple identification models can't be achieved.

We presented a construction method of adaptive multiple control systems based on speculative control [7]-[12]. The system does not require any identification schemes of environmental conditions. Monitoring outputs of the plant, the supervisor switches from one candidate controller to another. The basic ideas of adaptation are as follows: (1) each candidate controller is prepared for each environmental condition in advance; (2) the supervisor applies a sequence of speculative controls to the plant with candidate controllers just after the start of control or just after the detection of a change in the environmental condition. Each candidate controller can keep the system stable during one-step period of the speculative control and the most appropriate candidate controller for the environmental condition to which the system is exposed can be selected before the last trial of speculative control step comes to an end. We applied the method to a cart-pole balancing problem [7][8] and a vehicle anti skid braking system [9][10], and also confirmed that multi-modal neural networks as candidate controllers can learn complicated functions effectively in enough accuracy [11][12].

This paper presents an effective construction method of multiple control systems utilizing some knowledge upon the plants. In real applications, as we can often easily obtain a piece of knowledge upon plant dynamics beforehand, we intend to extend the method such that multiple control systems can be efficiently designed using the knowledge. We apply the new idea to the cart-pole balancing problem with variable length of the pole. The simulation experiments lead us to the conclusion that the new attempt can reduce the

manpower to design the candidate controllers for multiple control systems.

## 2. Statement of The Problem

### 2.1 Assumptions

We consider a feedback system composed of a set of candidate controllers together with a plant whose input-output characteristics depend on environmental conditions. Four assumptions concerning environmental conditions and a plant are made as follows:

1) The region in which the environmental condition changes during the operation of the system is obtainable in advance.  $\bar{C}$  is a set of an infinite number of environmental conditions which belong to the region.  $\tilde{C}(\subset \bar{C})$  is a set of a finite number of environmental conditions, each of which is representative of each partitioned region. The set  $\tilde{C}$  is given by

$$\tilde{C} = \{c_i | i \in I_C\}, \quad (1)$$

where  $I_C = \{1, 2, \dots, N_C\}$  and  $N_C$  is the number of partitioned regions.

2) The input-output characteristics of the system depend on only environmental conditions. The system in each environmental condition  $c_i \in \tilde{C}$  is described by a pair of differential equations:  $\dot{x}(t) = f_i(x(t), u(t))$  ( $i \in I_C$ ) and  $y(t) = g(x(t))$ .  $x(t)$ ,  $u(t)$  and  $y(t)$  represent respectively the input, state and the output of the system.  $f_i(\cdot)$  and  $g(\cdot)$  are nonlinear functions and a part of  $f_i(\cdot)$  may be unknown. The required output  $y_{d_i}$  ( $i \in I_C$ ) of the system is determined for each environmental condition  $c_i \in \tilde{C}$  in advance.

3) In the design of a candidate controller, it is possible to apply trial controls to a plant from an arbitrary initial state in each environmental condition  $c_i \in \tilde{C}$ .

4) During the control of a plant, the environmental condition to which the system is exposed can not be identified, but the output of the system  $y(t)$  is accessible.

### 2.2 The Required Control Performance

We regard continuously changing environmental conditions as discretely changing ones. The environmental condition is unchangeable, i.e. the environmental condition belongs to the same environmental condition  $c_i \in \tilde{C}$ , for  $nT \leq t < (n+1)T$ . It is assumed that the control is started at  $t = 0$  and is ended before  $t = NT$ . The objective is to construct a multiple control system which achieves the required control performance as follows:

1) An initial output value of the system is between the lower and upper holding bounds,  $y_l$  and  $y_u$  (prespecified constants):

$$y_l \leq y(0) \leq y_u. \quad (2)$$

2) Just after the start of control in an environmental condition  $c_i \in \tilde{C}$ , the output of the system is kept within the holding bounds for a prespecified constant time  $T_s (< T)$ . And then the output of the system is kept between the lower and upper settling bounds,  $y_{d_i} \pm \epsilon$  ( $i \in I_C$ ,  $\epsilon$  is a small prespecified positive constant):

$$y_l \leq y(t) \leq y_u \quad (0 \leq t < T_s), \quad (3)$$

$$|y_{d_i} - y(t)| \leq \epsilon \quad (T_s \leq t < T, i \in I_C). \quad (4)$$

3) Just after the environmental condition changed to another environmental condition  $c_{i'}$  ( $i' \in \tilde{C}$ ) at  $t = nT$  ( $n = 0, 1, \dots, N-1$ ), the output of the system is kept within the

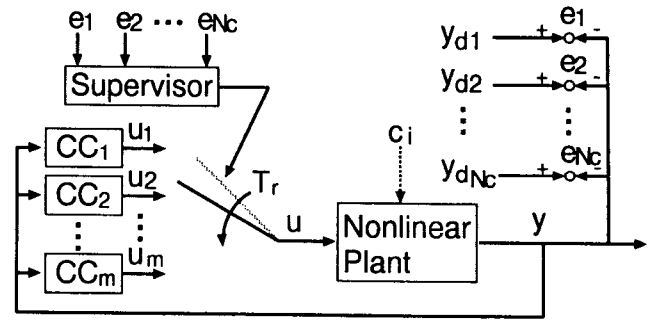


Fig.1. Structure of the multiple control system.

holding bounds for a time  $T_s$ . After a period of time  $T_s$  has passed, the output of the system is kept within the settling bounds  $y_{d_i} \pm \epsilon$  ( $i' \in I_C$ ):

$$y_l \leq y(t) \leq y_u \quad (nT \leq t < nT + T_s), \quad (5)$$

$$|y_{d_{i'}} - y(t)| \leq \epsilon \quad (nT + T_s \leq t < (n+1)T, i' \in I_C). \quad (6)$$

4) For the rest of time, the output of the system is continuously kept within the settling bounds:

$$|y_{d_i} - y(t)| \leq \epsilon \quad (nT \leq t < (n+1)T, i \in I_C). \quad (7)$$

## 3. Speculative Control

In this section, the basic ideas of speculative controls[7]-[12] are presented. A schematic diagram of the structure of the system is shown in Fig.1.

In order to achieve the required control performance in different operating environmental conditions, we need to address, in general, the design of a multiple control system which can rapidly regulate the output of the system to the corresponding required output. Since we do not use any identification schemes of environmental conditions, the supervisor must determine in real time which candidate controller should be put in feedback with a plant based on monitoring the output of the system. We consider a multiple control system in which switching from one candidate controller to another is carried out based on monitoring some of tracking errors  $e_i(t) = y_{d_i} - y(t)$ .

For each environmental condition candidate controllers are constructed in advance, so that the output of the system can be kept within either pair of bounds from an arbitrary initial state,

$$y_l \leq y(t) \leq y_u \quad (0 \leq t < T_r), \quad (8)$$

$$|y_{d_i} - y(t)| \leq \epsilon \quad (T_r \leq t < NT, i \in I_C), \quad (9)$$

or

$$y_l \leq y(t) \leq y_u \quad (0 \leq t < T_r), \quad (10)$$

$$y_l \leq y(t) \leq y_u \text{ and } |y_{d_i} - y(t)| > \epsilon \quad (T_r \leq t < NT, \forall i \in I_C), \quad (11)$$

where  $T_r$  is a prespecified constant satisfying  $0 < T_r \leq T_s$ , and we call such  $T_r$  a regulation time. Because of this property, the supervisor can judge whether the candidate controller is appropriate or inappropriate for the operating environmental condition, and the candidate controller can at least keep the system stable even if inappropriate.  $m (\leq N_C)$  candidate controllers are selected for the construction of a multiple control system, so that a sequence of speculative controls can be ended in less than  $T_s$ , and an appropriate candidate

controller for the operating environmental condition can be selected before the last speculative control is ended.

Let  $CC_k$  be the  $k$ th candidate controller in a multiple control system. Just after the start of control, the supervisor simply repeats to switch from  $CC_1$  to  $CC_2, \dots, CC_m$  at intervals of a constant time  $T_r$ , but stops switching when an appropriate candidate controller  $CC_k$  is selected. Similarly, just after the detection of a change in the environmental condition, the supervisor repeats to switch from  $CC_k$  to  $CC_{k+1}, \dots, CC_m, CC_1, \dots, CC_{k-1}$ . Whether the switched candidate controller is appropriate or inappropriate is judged as follows. Tracking errors are measured at the time when a constant time  $T_r$  has passed since the start of a speculative control. If a tracking error is smaller than  $\epsilon$ , such candidate controller is appropriate for the operating environmental condition. Else if every tracking error is larger than  $\epsilon$ , such candidate controller is inappropriate. On the other hand, the change of the environmental condition is detected as follows. The tracking error which has been kept smaller than  $\epsilon$  is measured at all times after the candidate controller had been judged appropriate. If the tracking error has become larger than  $\epsilon$  again, it is judged that the environmental condition has already changed.

#### 4. Control Performance Criteria

In order to construct candidate controllers, a holding set  $C_H$  for stability, a settling set  $C_S$  and non-settling set  $C_N$  for responsiveness are introduced. These sets are defined by a set of holding criteria  $\tau_H^i$  ( $\forall i \in I_C$ ), settling criteria  $\tau_S^i$  ( $\forall i \in I_C$ ) and non-settling criteria  $\tau_N^i$  ( $\forall i \in I_C$ ) respectively.

##### 4.1 Criteria of Stability and Responsiveness

We consider stability and responsiveness of the system. In an environmental condition  $c_i \in \tilde{C}$ , stability and responsiveness for each candidate controller are evaluated for the same set  $\tilde{X}_0$  of initial states given by

$$X_0 = \{x(0) | x(0) = g^{-1}(y(0)), y_l \leq y(0) \leq y_u\}, \quad (12)$$

$$\tilde{X}_0 = \{x_0^j | x_0^j \in X_0, j \in I_{X_0}\}, \quad (13)$$

where  $I_{X_0} = \{1, 2, \dots, N_{X_0}\}$  and  $N_{X_0}$  is the number of partitioned state regions.

A Holding criterion  $\tau_H^i$  ( $\forall i \in I_C$ ) is introduced as a measure of stability for a candidate controller in an environmental condition  $c_i \in \tilde{C}$ , and a settling criterion  $\tau_S^i$  ( $\forall i \in I_C$ ) and a non-settling criterion  $\tau_N^i$  ( $\forall i \in I_C$ ) are introduced as measures of responsiveness for a candidate controller in an environmental condition  $c_i \in \tilde{C}$ . These criteria,  $\tau_H^i$ ,  $\tau_S^i$  and  $\tau_N^i$ , are respectively defined by

$$\tau_H^i = \min_{j \in I_{X_0}} t_H^{i,j}, \quad (14)$$

$$\tau_S^i = \max_{j \in I_{X_0}} t_S^{i,j}, \quad (15)$$

$$\tau_N^i = \max_{j \in I_{X_0}} t_N^{i,j}, \quad (16)$$

where  $t_H^{i,j}$ ,  $t_S^{i,j}$  and  $t_N^{i,j}$  are a holding time, a settling time and non-settling time respectively given by

$$t_H^{i,j} = \sup_{t \in [0, NT)} \{t | c_i \in \tilde{C}, x_0^j \in \tilde{X}_0, y_l \leq y(t') \leq y_u, \forall t' \in [0, t)\}, \quad (17)$$

$$t_S^{i,j} = \inf_{t \in [0, NT)} \{t | c_i \in \tilde{C}, x_0^j \in \tilde{X}_0, y_l \leq y(t') \leq y_u, \forall t' \in [0, t)\},$$

$$t_N^{i,j} = \inf_{t \in [0, NT)} \{t | c_i \in \tilde{C}, x_0^j \in \tilde{X}_0, y_l \leq y(t') \leq y_u, \forall t' \in [0, t), y_l \leq y(t') \leq y_u \text{ and } |y_{d_i} - y(t')| > \epsilon, \forall i' \in I_C, \forall t' \in [t, NT)\}. \quad (18)$$

We set a large value to the number  $N_{X_0}$  of partitioned state regions as to guarantee stability and responsiveness for each candidate controller.

##### 4.2 Sets for Stability and Responsiveness

We consider robustness of the system for changes of the environmental condition. A holding set  $C_H$  is introduced as a set of environmental conditions in which the candidate controller can keep stability of the system, and a settling set  $C_S$  and non-settling set  $C_N$  are also introduced as sets of environmental conditions in which the candidate controller can keep responsiveness of the system. These sets,  $C_H$ ,  $C_S$  and  $C_N$ , are respectively defined by

$$C_H = \{c_i | c_i \in \tilde{C}, \tau_H^i + \delta = NT, \delta > 0\}, \quad (20)$$

$$C_S = \{c_i | c_i \in C_H, \tau_S^i < T_r\}, \quad (21)$$

$$C_N = \{c_i | c_i \in C_H, \tau_N^i < T_r\}, \quad (22)$$

where  $\delta$  is a small prespecified positive constant. we also define a subset  $C_H \cap (\overline{C_S \cup C_N})$  as  $C_{SN}$ .

#### 5. Multiple Control System

In order to construct a multiple control system, we select  $m$  numbers of candidate controllers. Let  $C_H^k$ ,  $C_S^k$  and  $C_N^k$  be the holding set, the settling set and the non-settling set for the  $k$ th candidate controller, and let  $C_{SN}^k$  be  $C_H^k \cap (\overline{C_S^k \cup C_N^k})$ . The selected candidate controllers must satisfy the following conditions.

1) Detect the change of the environmental condition:

$$C_H^1 = C_S^1 \cup C_N^1 \quad (C_{SN}^1 = \phi), \quad (23)$$

$$C_H^2 = C_S^2 \cup C_N^2 \quad (C_{SN}^2 = \phi), \quad (24)$$

⋮

$$C_H^m = C_S^m \cup C_N^m \quad (C_{SN}^m = \phi). \quad (25)$$

2) Terminate a sequence of speculative controls in less than  $T_s$ :

$$(m+1) \times T_r \leq T_s. \quad (26)$$

3) Keep the system stable during a sequence of speculative controls:

$$C_H^1 = C_H^2 = \dots = C_H^m = \tilde{C}. \quad (27)$$

4) Select an appropriate candidate controller before the last speculative control is ended:

$$C_S^1 \cup C_S^2 \cup \dots \cup C_S^m = \tilde{C}. \quad (28)$$

#### 6. Construction Method of Multiple Control System

In real applications, we can often easily obtain a piece of knowledge upon plant dynamics from experiment results or simulation results beforehand. In such a case, we can apply the following construction method of a multiple control system in order to reduce the manpower to design the candidate controllers.

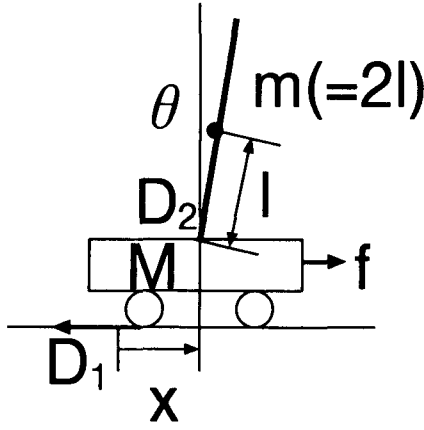


Fig.2. Cart-pole balancing system.

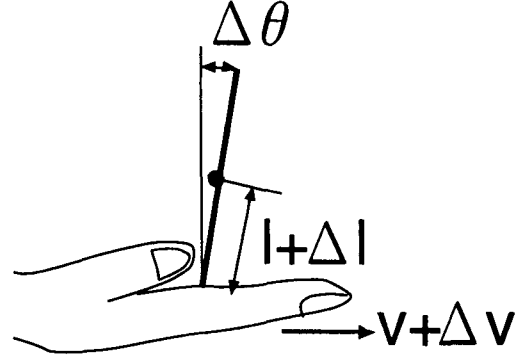


Fig.3. Pole controlled by the hand.

(Step1) Obtain a piece of knowledge upon sensitivity to environmental condition changes. Quality and quantity of the knowledge depend on applications.

(Step2) Partition the region of the set  $C$  into  $N_C$  numbers of regions in the following way: regions where the system is sensitive to environmental condition changes are smaller than those where the system is not. After that, obtain the set  $\tilde{C}$  from the partitioned regions. Size of the partitioned regions depends on applications.

(Step3) Partition the region of the set  $\tilde{C}$  into  $m$  numbers of subsets,  $C_A^1, C_A^2, \dots, C_A^m$  where  $m$  and  $T_r$  satisfy the condition  $(m+1) \times T_r \leq T_s$ .

(Step4) Decide  $m$  numbers of settling set  $C_S^1, C_S^2, \dots, C_S^m$ , no-settling set  $C_N^1, C_N^2, \dots, C_N^m$ , holding set  $C_H^1, C_H^2, \dots, C_H^m$  respectively, where settling set  $C_S^i$ , non-settling set  $C_N^i$  and holding set  $C_H^i$  satisfy the condition  $C_S^i = C_A^i$ ,  $C_N^i = \{\tilde{C} \cap \overline{C_A^i}\}$  and  $C_H^i = \tilde{C}$  respectively.

(Step5) Construct  $m$  numbers of candidate controllers  $CC^i$  whose settling set, non-settling set and holding set are identical the above  $C_S^i$ ,  $C_N^i$  and  $C_H^i$  respectively by reinforcement learning of neural networks.

## 7. Cart-Pole Balancing Problem

A cart is free to move along a one-dimensional track while a pole is only free to rotate in the vertical plane of the cart and track. A schema of the physical system is shown in Fig.2. The cart-pole system is simulated by following equations:

$$\begin{bmatrix} 4/3mL^2 & mL \cos \theta \\ mL \cos \theta & m + M \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{x} \end{bmatrix} - \begin{bmatrix} mLg \sin \theta \\ mL\dot{\theta}^2 \sin \theta \end{bmatrix} = \begin{bmatrix} 0 \\ f \end{bmatrix} \quad (29)$$

where  $L$ :half length of pole,  $m$ :mass of pole(=  $2L$ ),  $M$ :mass of cart,  $D_1$ :coefficient of friction of cart on track,  $D_2$ :coefficient of friction of pole hinge,  $\theta$ :angle of pole from vertical,  $x$ :position of cart on track,  $f$ :force applied to cart and  $g$ :gravitational acceleration. The fourth-order Runge-Kutta method with a step size of 0.01s is used to calculate  $\theta$ ,  $\dot{\theta}$ ,  $x$  and  $\dot{x}$ .

The objective of the control is to make the system have the following stability and responsiveness just after the start of control:

$$\theta_i \leq \theta(t) \leq \theta_u \quad (0 \leq t < T_s), \quad (30)$$

$$|\theta(t)| \leq \epsilon \quad (T_s \leq t < NT), \quad (31)$$

where  $\theta_l = -15.0$  (deg),  $\theta_u = 15.0$  (deg),  $\epsilon = 1.0 \times 10^{-6}$  (deg),  $T_s = 50.0$ ,  $N = 100$ ,  $T = 1.0$ . We pose the pole length  $l$  changes from 0.1 to 1.0 (m) by 0.1 (m). It is assumed that dynamics of the system and length of the pole are unknown for controllers. Initial states are  $\theta(0) = 0, \pm 1, \pm 2, \pm 3$  (deg),  $\dot{\theta}(0) = 0, \pm 3, \pm 6, \pm 9$  (deg/s),  $x(0) = 0.0$  (m), and  $\dot{x}(0) = 0.0$  (m/s).

Let's imagine the situation shown in Fig.3., that is, moving the hand at a constant velocity of  $v$ , we bring back the angle of the pole from  $\Delta\theta$  to zero.

Since a period of vibration of a pole whose length is  $2l$  is given by

$$T = 2\pi \sqrt{\frac{2l}{g}}, \quad (32)$$

we can obtain the relationship between velocity  $v$  and pole length  $l$  as

$$v = A\sqrt{l}, \quad (33)$$

where  $A$  is constant. Differentiating logarithms of both sides of equation(33), we can obtain the following equation:

$$\frac{\Delta l}{\Delta v} = B\sqrt{l}, \quad (34)$$

where  $B$  is constant.

We consider the hand as an actuator which can generate velocity of  $v$ . We partition an interval of velocity  $[v_1, v_m]$  at by  $v_2, v_3, \dots, v_{m-1}$  whose intervals are even and prepare  $m$  numbers of actuators,  $a_1, a_2, \dots, a_m$ . The actuators can generate velocity in the interval of  $[v_1 - \Delta v, v_1 + \Delta v]$ ,  $[v_2 - \Delta v, v_2 + \Delta v]$ ,  $\dots$ ,  $[v_m - \Delta v, v_m + \Delta v]$  respectively, where  $\Delta v$  is constant.

From the relationship between  $\Delta l$ ,  $\Delta v$  and  $l$  given by equation (34),  $\Delta l$  is proportional to  $l$  because  $\Delta v$  is constant. This means that an actuator  $a_i$  can be applied for larger fluctuation  $\Delta l$  in case pole length  $l$  is larger while an actuator  $a_i$  can be applied for smaller fluctuation  $\Delta l$  in case pole length  $l$  is smaller. The followings are examples for different pole length:

$$\Delta l = B\sqrt{0.10}\Delta v \simeq 0.31 \times B\Delta v \quad \text{in case } l = 0.10,$$

$$\Delta l = B\sqrt{0.50}\Delta v \simeq 0.71 \times B\Delta v \quad \text{in case } l = 0.50,$$

$$\Delta l = B\sqrt{1.00}\Delta v \simeq 1.00 \times B\Delta v \quad \text{in case } l = 1.00.$$

As shown in Fig.4., we partition the region of the pole length  $[L_0 (= 0.10), L_m (= 1.00)]$  into  $N_C$  numbers of regions  $[L_0, L_1], [L_1, L_2], \dots, [L_{N_C-1}, L_{N_C}]$ .  $l_1, l_2, \dots, l_M$  are the middle values of partitioned regions, respectively. From

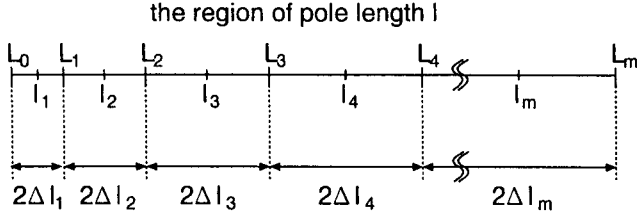


Fig.4. Partition of region of pole length.

Fig.4., the following simultaneous equations can be obtained:

$$l_1 - L_0 = \Delta l_1, \quad (35)$$

$$l_2 - L_0 = 2\Delta l_1 + \Delta l_2, \quad (36)$$

$$l_3 - L_0 = 2(\Delta l_1 + \Delta l_2) + \Delta l_3, \quad (37)$$

$$l_4 - L_0 = 2(\Delta l_1 + \Delta l_2 + \Delta l_3) + \Delta l_4, \quad (38)$$

⋮

$$l_m - L_0 = 2(\Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4 + \dots + \Delta l_{m-1}) + \Delta l_m, \quad (39)$$

$$L_m - L_0 = 2(\Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4 + \dots + \Delta l_{m-1} + \Delta l_m) \quad (40)$$

Solving equations (34) and (35)–(40), concerning  $\Delta l_1, \Delta l_2, \dots, \Delta l_i, \dots$  and  $\Delta l_m$ , we can find the values:

$$\Delta l_i = \frac{L_m - L_0 + (2i - m - 1)mC}{2m} \quad (41)$$

$$(i = 1, 2, \dots, m),$$

where a constant  $C$  is given by

$$C = \frac{m(L_0 + L_m) - \sqrt{L_0^2 + L_m^2 + 2(2m^2 - 1)L_0 L_m}}{(m - 1)m(m + 1)}. \quad (42)$$

From equation (41), the bounds of  $L_1, L_2, L_3, L_4, \dots, L_{M-1}$  are given by

$$L_1 = L_0 + 2\Delta l_1, \quad (43)$$

$$L_2 = L_0 + 2(\Delta l_1 + \Delta l_2), \quad (44)$$

⋮

$$L_i = L_0 + 2(\Delta l_1 + \Delta l_2 + \dots + \Delta l_i), \quad (45)$$

⋮

$$L_{m-1} = L_0 + 2(\Delta l_1 + \Delta l_2 + \dots + \Delta l_i + \dots + \Delta l_{m-1}). \quad (46)$$

Fig.5. shows partitioned regions and how to assign partitioned regions to five(=  $m$ ) actuators in case the number of partitioned regions  $N_C$  is 15 and one actuator covers three partitioned regions.

Though the above consideration of for the partition is performed for actuators which move a cart at a constant velocity, it is also effective for the design of actuators which apply force to a cart.

Fig.6. shows holding and settling sets of constructed five candidate controllers. Arrows shaded with black color mean holding sets while arrows shaded with gray color mean settling sets.

Fig.7. shows the simulation results. Each response shows  $\theta$  for the case where pole length is 1.00, 0.90,  $\dots$ , 0.10 respectively. A settling time is shown in each response

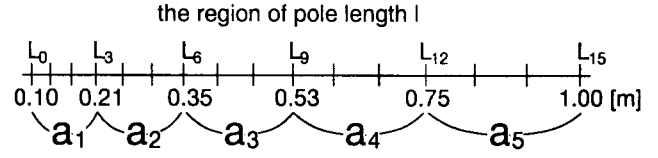


Fig.5. Partition of fifteen regions of pole length.

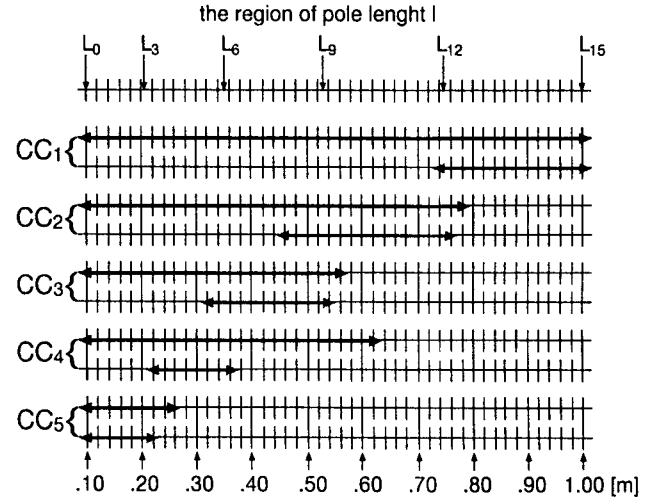


Fig.6. Holding and settling sets of candidate controllers.

graph. Though in each case small oscillations are occurred in switching,  $\theta$  is kept at all times between  $\pm 15.0$  (deg) during the control and  $\theta_u$  is kept between  $1.0 \times 10^{-6}$  (deg) at the time when 50.0 (s) has passed after the start of control. This implies that the constructed multiple control system can achieve the objective of control.

## 8. Conclusions

An effective construction method of multiple control systems based on speculative control is proposed. In real applications, as we can often easily obtain a piece of knowledge upon plant dynamics beforehand, we intend to develop the method with which we can efficiently design multiple control systems using the knowledge.

We apply the new idea to the cart-pole balancing problem with variable length of the pole. The region of pole length values was partitioned in such a way as regions where the system is sensitive to changes of the pole length are smaller than those where the system is not. Candidate controllers were efficiently constructed for these partitioned regions.

The simulation results show the new attempt can reduce the manpower to design the candidate controllers for multiple control systems and the constructed multiple control system can achieve the required control performances.

This research is partly supported by Japan Science and Technology Cooperation.

## References

- [1] H. E. Rauch, "Autonomous Control Reconfiguration", *IEEE Control Systems Magazine*, 15–6, pp. 37–48, 1995.

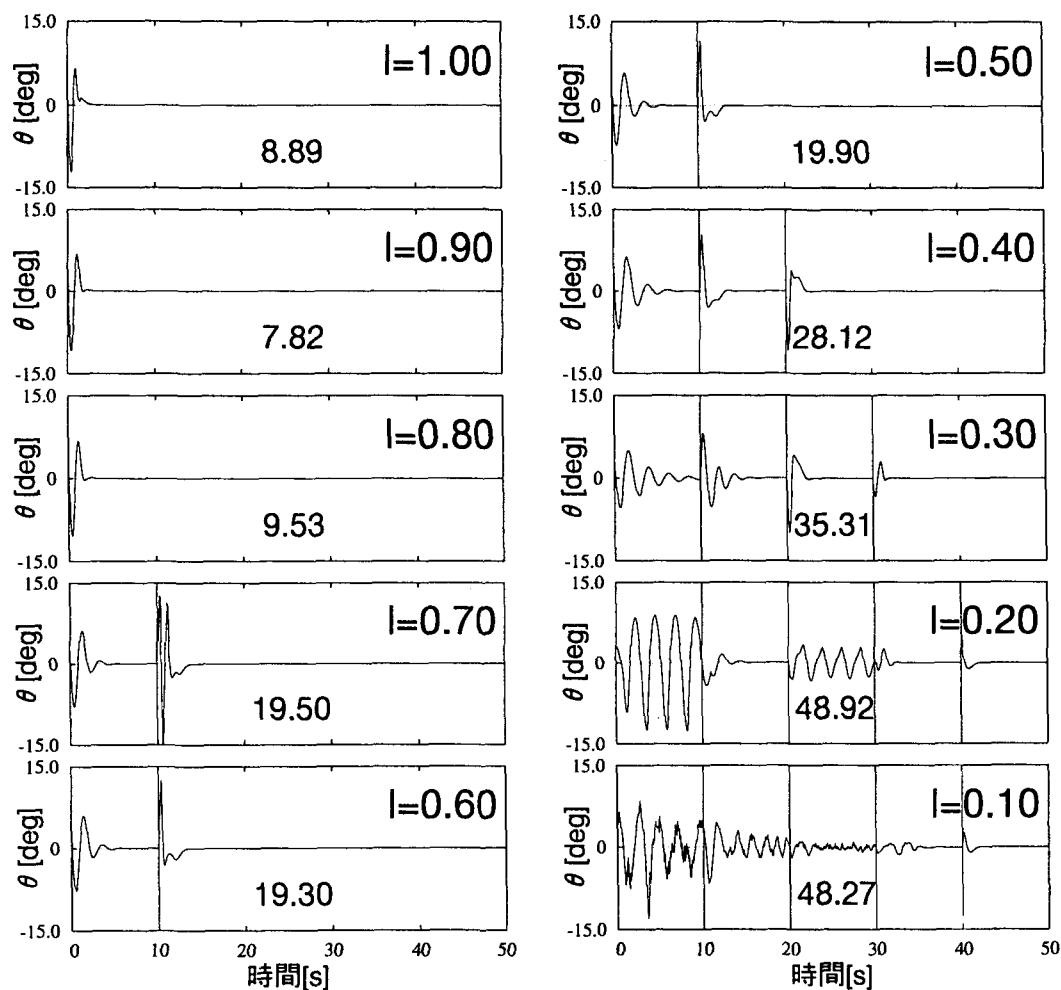


Fig.7. Simulation results for each pole length.

- [2] A. S. Morse, "Control Using Logic-Based Switching", Alberto Isidori, editor, *Trends in Control*, Springer-Verlag, pp. 69-113, 1995.
- [3] M. Kosuge, *Fuzzy Control*, NikkankogyoSinbunsha, Tokyo; 1991, pp. 113-122(in Japanese).
- [4] N. Satake, O. Ito, K. Kobayashi and O. Yagishita, "Cooperation Control of Wet Pump by Fuzzy Adaptive Controller", *Trans. IEE Japan*, 109-C, no.5, pp. 361-366, 1989(in Japanese).
- [5] K. S. Narendra and J. Balakrishnan, "Adaptive Control Using Switching and Tuning", *Proc. of the Eighth Yale Workshop on Adaptive And Learning Systems*, pp. 13-15, 1994.
- [6] K. S. Narendra, J. Balakrishnan and M. K. Cliz, "Adaptation and Learning Using Multiple Models, Switching, and Tuning", *IEEE Control Systems Magazine*, 15-3, pp. 37-51, 1995.
- [7] M. Inaba, H. J. Guo, K. Nakao and K. Abe, "Adaptive Control Systems Switched by Control and Robust Performance Criteria", *Proc. of 1996 IEEE Conference on Emerging Technologies and Factory Automation*, pp. 690-696, 1996.
- [8] M. Inaba, H. J. Guo, K. Nakao and K. Abe, "Modelless Multiple Control Systems Switched by Robust Performance Criteria Concerned with Stability and Responsiveness", *Trans. IEE Japan*, 117-C, no.12, pp. 1818-1826, 1997(in Japanese).
- [9] M. Inaba, I. Yoshihara, H. J. Guo, K. Nakao and K. Abe, "A Proposal of Switching Control System based on Speculative Control and its Application to Antiskid Braking System", *Proceedings of the 12th KACC*, pp. 585-588, 1997.
- [10] M. Inaba, I. Yoshihara, H. J. Guo, K. Abe and K. Nakao, "A Proposal of Multiple Control System based on Real-Time Trial Control and Its Application to Antiskid Braking", *Trans. IEE Japan*, 118-C, no.6, pp. 897-908, 1998(in Japanese).
- [11] T. Nakagawa, M. Inaba, K. Sugawara, I. Yoshihara and K. Abe, "A Proposal of Enhanced Neural Network Controllers for Multiple Control Systems", *Proceedings of the 13th KACC*, pp. 201-204, 1998.
- [12] T. Nakagawa, M. Inaba, K. Sugawara, I. Yoshihara and K. Abe, "An Application of Multi-modal Neural Network to Multiple Control System", *Proceedings of the 4th International Symposium on Artificial Life and Robotics*, pp. 737-740, 1999.