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Symmetry of computer-generated figures based on complex dynamics

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Abstract

We discuss the symmetry for computer-generated figures based on complex dynamics. The figures have not continuous lines, and they are plotted on a specific region in the complex plane. They have different properties from classical figures. But we believe that they are variety of figures. The symmetric discussions are necessary because of their properties.

1. Introduction

There are many kinds of figures in nature. Recently, computer graphics are advanced marvelously, and complex numerical chaos can be calculated in reasonable times. We have studied complex iterations as one of the chaos, and plotted complex numbers generated from the iterations on a video display that is a substitution for the complex plane. Sometimes trajectories are found. A trajectory may look like a figure, but it is not drawing one in order. It is consisted of finite points, and is not continuous at everywhere, but such properties are not observed with human eyes. They are only found by computer checks. We call it as a pseudo-figure. In this paper, we discuss their symmetry.

2. Complex dynamics

2.1 Target dynamics

Many complex dynamics are known. In them, there is an interesting complex iteration [1] defined as,

$$Z(n+1)=Z(n)^{**}2-Z(n-1)+A, \quad Z(1)=conj(Z(0)),$$

where Z(i) is a complex series, and A is a complex coefficient. They are also parameters of the iteration. Inner parameter space:

$$|Z(0)| < |(0.21,0.21)|$$

and A=(0.0685,0), the iteration generates many kinds of pseudo-figures that are like to closed curves. They are too many, that suggests there are many bifurcations in generations of the figures. The iterations are chaotic.

Nearby one point of the parameter space: Z(0)=(0.1,0.1), the pseudo-figure [3] is like to an ellipse. As for the shape, the resemblance is too complete, it is examined in details by using computers, so some differences are found at last. Since a generated shape is a closed curve, the first or last points are not defined in the iteration. In the closed curve like to the ellipse, the relation Z(1)=conj(Z(0)) is the relation Z(0)=conj(Z(1)). That means,

$$\{Z(0),Z(1),.....,Z(n)\}=\{Z(1),Z(0),Z(-1),.....Z(-n+1)\}.$$

2.2 Standpoint to study dynamics

The figures have definite shapes within iteration n=10**4 usually. In the computer graphical investigations, properties of the iterations over 10**4 have not been studied. Targets are shapes. However, the researches are not sufficient, if the targets are not the shape but also chaotic movements. Properties of the chaos are that new movements are suddenly arisen. The accidental ones are not predicted. Forecasting for the chaos are only succeed within a short range, and not long ranges. Investigations for the properties are very difficult because of considerations on the infinite. Computer researches arrived at the finite or the limited infinite under the very special treatments. In this paper, we calculated many chaotic properties until limits of computer resources, and inspected movements of them. For example, we calculated the iteration until 10**9, the calculations are 10**5 times more than that of computer graphics. Our investigation-field is huge, but the methodology has a defect yet. We are sure that the methodology is a possible better choice now.

3. Examinations for the shape

3.1 Stability for huge iteration

The chaos includes bifurcations that appear suddenly, and generating shapes (trajectories) are broken. We checked the precision for iterations at first, and got that numerical errors are less than 10**-9 inner n=10**9. Next, we chased the shape likes an ellipse until n=10**8, and could not find such a bifurcation. We plotted the trace on 10000x10000 mesh in order to watch square measure of the trace. The trace is scaled and accorded with the size of mesh. We got a following table.

n	trace ratio
1×10**6	0.03965 %
2×10**6	0.03984 %
4×10**6	0.03990 %
8×10**6	0.03994 %
1.6×10**7	0.03998 %
3.2×10**7	0.03999 %
6.4×10**7	0.03999 %
1.28×10**8	0.04000 %

The trace ratio is the number of plotted pixels divided by all pixels. The upper limit of the ratio is 4*10**-4, i.e., 0.04%. So we get a result that the stability is complete for huge iteration under n=10**8.

3.2 Stability for initial conditions

We put negligible deformations to the initial values as, Z(0)+delta, Z(1)+epsilon. The test is for the initial sensitivity, which makes traces to deformed ones completely in large iterations. We represent a deformation measure by the iteration number, when Euclid distance among them is greater than 0.001. On practical calculation, we set epsilon=0, and changed imaginary part of delta. We got following table.

delta	iteration number
1.953125×10**-5	8646
9.765625×10**-6	17412
4.882812×10**-6	34940

Even if the deformation is vary small (|O(delta)|<10**-5, epsilon=0), the shape is broken over n=10**4. But, if Z(1) is set a conjugation of Z(0), the shape is stabilized. The conjugate condition is necessary, because of the condition makes shapes stable, and suppress the initial sensitivity. The suppression effect is useful to draw the pseudo-figures on specified position in the complex plain [2].

3.3 continuity of the line

The nearest neighbor for any element Z(i) in the calculated set {Z(0),Z(1),...,Z(n)} is not Z(i+1) or Z(i-1). Its location is unknown. If you wish to detect it, you must calculate all distances from Z(i), and compare them. We get the element that is written as Z(j). The nearest neighbor for the Z(j) is searched similarly. The periphery of a curve is evaluated from a chain of the calculations. The calculation spends very huge computer resources.

n	periphery	distance
1×10**4	0.790016	0.00007900
2×10**4	0.790016	0.00003950
4×10**4	0.790016	0.00001975
8×10**4	0.790016	0.00000987
1.6×10**5	0.790016	0.00000493
3.2×10**5	0.790016	0.00000246
6.4×10**5	0.790016	0.00000123
1.28×10**6	0.790016	0.00000062

From above table, we find that the periphery is constant for increasing iterations, and that a distance between any plotting converges in zero. So we judge that the curve of the figure has continuous character.

3.4 existence of the tangential line

After some plotting, a location of point reaches almost equal to initial Z(0). The approach is not once but repeatable. This is expected from Poincare's theorem. We find one regression is almost n=94, and consider them to be typical points of the figure. Around these points, we investigate tangential lines. Averages of their gradients represented as |sin(x)| is listed.

n	3.16×10**-4	3.16×10**-5
1×10**5	4.41×10**-7	4.92×10**-9

2×10**5	4.38×10**-7	4.69×10**-9
4×10**5	4.39×10**-7	4.42×10**-9
8×10**5	4.40×10**-7	4.46×10**-9
1.6×10**6	4.40×10**-7	4.42×10**-9
3.2×10**6	4.40×10**-7	4.44×10**-9
6.4×10**6	4.40×10**-7	5.28×10**-9

The gradients are variables linking with infinitesimal radii around the points. We calculate the gradients by using two kinds of radii 3.16×10**-4 and 3.16×10**-5. Under a radius, the calculated gradients are nearly equal to the constant that is independent on the numbers of iteration. If the radii converge into zero, the gradients are decrease in square proportion. We are sure that the tangential lines of the figure are existed on any point. Above mentioned properties indicate that the figure has similar characters to the ellipse.

3.5 mass and geometrical centers

We calculated the mass center of set {Z(0),Z(1),...,Z(n)}. The center was not swayed in numbers of iterations.

n	x-coordinate	y-coordinate
1×10**6	0.0320529	0.0000001
2×10**6	0.0320529	0.0000001
4×10**6	0.0320529	0.0000000
8×10**6	0.0320529	0.0000000
1.6×10**7	0.0320529	0.0000000

We calculated the geometric center (G) of the shape. The center is defined as,

Gx=(xmax+xmin)/2, Gy=(ymax+ymin)/2,

where xmax is the maximum coordinate of the shape. Gx means the x-coordinate of the G-point. The center was not swayed in numbers of iterations.

n	Gx	Gy
1×10**6	0.0252713	0.0000000
2×10**6	0.0252713	0.0000000
4×10**6	0.0252713	0.0000000
8×10**6	0.0252713	0.0000000
1.6×10**7	0.0252713	0.0000000

The figure has two symmetry axes, and the shape is very similar to the ellipse.

But the mass center is not geometrical one. It is amazing fact, because it means breaking of the symmetry. It shows there are two kinds of symmetry in the figure. We didn't know such a strange shape.

4. Symmetry

4.1 Approximate symmetry

In traditional symmetry discussion for figures, lines were continuous. Then, a new point that is generated as a result of symmetrical operation is existence surely on the lines. The pseudo-figure has not any continuous line. It is uncertain that the operated point is existence. Therefore, the figure is not symmetric mathematically, however the shape is symmetric visually. We didn't regard that the visual impression was an illusion. It is an interesting phenomenon, and gives us an opportunity, that extends traditional symmetry to new symmetry as fuzzy idea.

Even if the figure is consisted of finite points, where there is a point on the location corresponding with the result of symmetry operation, the symmetry can be defined. Such symmetry is found in crystal groups. In the pseudo-figure, there is not the location, but very close points exist. This is the testify fact. We considered Euclid distance between the location and the most closed point to be a kind of indexes of the symmetry. It means a traditional symmetry that the distance is zero strictly. It means rough or approximate symmetry that the distance is an infinitesimal. This is a continuous extension for symmetric conceptions. We defined the approximate symmetry in finite sets as following.

It means a traditional symmetry that the distance converges into infinitesimal on limit of iterations.

It means the approximate symmetry that the distance keeps a small value for increasing iterations.

We test the hypothesis in the pseudo-figure, which has two symmetric axes. One is a parallel axis of y-coordinate axis. For the axis, following Euclid distances are given.

n	max	average

1000	1.7×10**-3	8.8×10**-4
2000	1.6×10**-3	8.4×10**-4
4000	1.6×10**-3	8.0×10**-4
8000	1.6×10**-3	7.9×10**-4
16000	1.6×10**-3	7.9×10**-4
32000	1.6×10**-3	7.9×10**-4
64000	1.6×10**-3	7.9×10**-4

In the case, the distance converges a value near 7.9×10**-4. The value is small certainly, but is not infinitesimal. The other is a parallel axis of x-coordinate axis. For the axis, following Euclid distances are given.

n	max	average
1000	1.0×10**-3	1.7×10**-4
2000	6.0×10**-4	2.5×10**-4
4000	3.4×10**-4	6.5×10**-5
8000	1.4×10**-4	2.9×10**-5
16000	6.5×10**-5	3.3×10**-5
32000	3.2×10**-5	1.0×10**-5
64000	1.7×10**-5	1.9×10**-6

In the case, the distance converges a value less than 1×10**-6. We calculated it until limit of our computer resource, and got {n=768000, max=9.8×10**-7, average=3.7×10**-7}. The average value is less than 1/1000 of former ones. The minimum value is 0.0 exactly. Therefore, we believe the symmetry is similar to the traditional.

In the pseudo-figure, two different symmetries are coexistence. A similar phenomenon was found in mathematical discussions for breaking of the symmetry. It was inquired into discussions for new shape generations of biological phenomena. In the discussion, the shape was consisted of continuous lines. We find the symmetry-breaking in a finite plotted figure.

5. Conclusion

We discussed the symmetry for computer-generated figures, and found a new figure. The visual shape of the figure is very similar to the ellipse, however its properties are difference. It has two kinds of the symmetry. We believe that one of them is equivalent to the symmetry-breaking on the finite set. We are sure that many kinds of figures whose shapes are similar to the ellipse, in finite set.

References

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[2]T.Aoyama, "Properties of Images Generated by using Complex Dynamics", IPSJ SIG Notes, 98-HPC-73(1998).

[3]

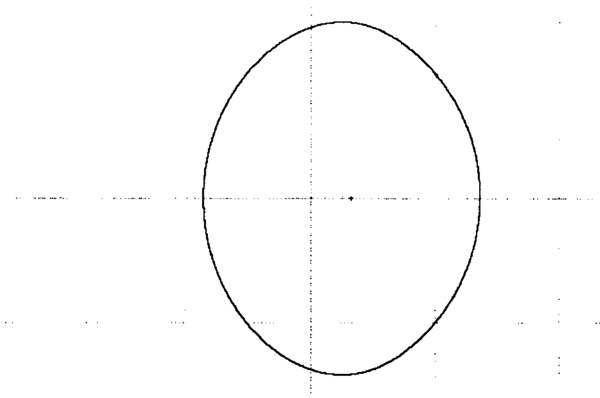

Figure 1. A pseudo-figure generated from complex dynamics