

A Practical Method for Identification of Nonlinear Chemical Processes by use of Volterra Kernel Model

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Abstract

It is known that Volterra kernel models can represent a wide variety of nonlinear chemical processes. Also, it is necessary for Volterra model identification to excite the process to be identified with a signal having wide range of frequency spectrum and high enough amplitude of input signals. Kashiwagi[4 ~ 7] has recently shown a method for measuring Volterra kernels up to third order using pseudorandom M-sequence signals. However, in practice, since it is not always possible to apply such input sequences to the actual chemical plants. Even when we can apply such a pseudorandom signal to the process, it takes much time to obtain higher order Volterra kernels.

Considering these problems, the authors propose here a new method for practical identification of Volterra kernels by use of approximate open differential equation (ODE) model and simple plant test. Simulation results are shown for verifying the usefulness of our method of identification of nonlinear chemical processes.

1. Introduction

In general, almost all industrial chemical processes exhibit nonlinear behavior. Especially, when we consider the process control strategy over the wide range of operation conditions, such as plant startup, shutdown, batch processes and so on, it is very important to understand their nonlinearities and to identify the nonlinear model.

Volterra kernel model is one of the nonparametric nonlinear model and it can represent a wide variety of nonlinear chemical processes. Also, Kashiwagi[4 ~ 7] has recently shown a measurement method of Volterra kernels up to third order by use of correlation technique with pseudo random M-sequence signals. For the identification, it requires the process to be excited by relatively high order M-sequence signals with high enough amplitude to measure its nonlinearities.

However, when we apply this method directly to actual chemical plants, it is sometimes infeasible because the product quality is disturbed due to the applied pseudorandom signal, and it takes long time to measure Volterra kernels of higher order.

Here, considering the fact that Volterra kernel models consist of a linear part as first order kernel and a nonlinear part as higher order kernels, the authors propose here to identify both linear and nonlinear parts separately using different data sources. The basic idea is that the nonlinear part is measured by use of an approximate ODE model and the linear part is measured by use of the results of the simple actual plant test.

Simulation results of our method applied to a heater process (simple heat exchanger) are shown.

2. Measurement of Volterra kernels up to third order

Volterra kernel model represents a nonlinear system as a sum of an input-output power series. In general, it is defined as follows:

$$\begin{aligned} y(t) = & \int_0^{\infty} g_1(\tau_1)u(t-\tau_1)d\tau_1 \\ & + \int_0^{\infty} \int_0^{\infty} g_2(\tau_1, \tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1d\tau_2 \\ & + \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} g_3(\tau_1, \tau_2, \tau_3)u(t-\tau_1)u(t-\tau_2) \\ & \times u(t-\tau_3)d\tau_1d\tau_2d\tau_3 + \dots + n(t) \end{aligned} \quad (1)$$

where $u(t)$ is the input, $y(t)$ is the output of the nonlinear system, $g_i(\tau_1, \tau_2, \dots)$ is called Volterra kernel of i -th order, and $n(t)$ is a noise.

Since the input $u(t)$ has no correlation with the noise $n(t)$ usually, the crosscorrelation function between the input $u(t)$ and the output $y(t)$ is described as follows:

$$\begin{aligned}
\phi_{uy}(\tau) &= \overline{u(t-\tau)y(t)} \\
&= \int_0^\infty g_1(\tau_1) \overline{u(t-\tau)u(t-\tau_1)} d\tau_1 \\
&+ \int_0^\infty \int_0^\infty g_2(\tau_1, \tau_2) \overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)} d\tau_1 d\tau_2 \\
&+ \int_0^\infty \int_0^\infty \int_0^\infty g_3(\tau_1, \tau_2, \tau_3) \\
&\times \overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)} d\tau_1 d\tau_2 d\tau_3 \\
&+ \dots
\end{aligned} \tag{2}$$

where $\phi_{uy}(\tau)$ is the crosscorrelation function of $u(t)$ and $y(t)$ and $\overline{\quad}$ denotes time average.

Kashiwagi[4 ~ 7] has shown that if we use an M-sequence signal as an input to the system, the n -th moment of $u(t)$ can be easily obtained by use of so-called "shift and add property" of the M-sequence. So we can obtain the Volterra kernels by the following equation.

$$\begin{aligned}
\phi_{uy}(\tau) &= \Delta t g_1(\tau) + F(\tau) \\
&+ \sum_{i=2}^{\infty} i!(\Delta t)^i \\
&\times \sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \dots, \tau - k_{ii}^{(j)}) \tag{3}
\end{aligned}$$

Here the function $F(\tau)$ is the function of τ and sum of the odd order Volterra kernels when some of its argument are equal. If we consider the Volterra kernels up to the third order, it is described as follows:

$$F(\tau) = -2(\Delta t)^3 g_3(\tau, \tau, \tau) + 3(\Delta t)^3 \sum_{q=1}^N g_3(\tau, q, q)$$

where $k_{ir}(1 \leq r \leq i)$ is the group of the positive integer which satisfy Eqn.(4). m_i is the total number of the group of k_{ir} .

$$\begin{aligned}
u(t)u(t + k_{i1}^{(j)} \Delta t)u(t + k_{i2}^{(j)} \Delta t) \cdots u(t + k_{ii-1}^{(j)} \Delta t) \\
= u(t + k_{ii}^{(j)} \Delta t) \tag{4}
\end{aligned}$$

When we apply a suitable M-sequence, which make the appearance of each crosssection of the Volterra kernels sufficiently apart each other, the Volterra kernels $g_1(\tau_1)$, $g_2(\tau_1, \tau_2)$, $g_3(\tau_1, \tau_2, \tau_3)$ are easily extracted from the crosscorrelation function using Eqn.(3). (See reference 1)

3. Application for actual chemical processes

In order to measure the Volterra kernels of nonlinear system by use of above method, we have to choose, in general, the order of 16th to 18th order

M-sequence. The period of 16th order M-sequence is 65,535 steps, and the typical switching time (or measurement interval) for chemical processes is, at least, the order of minutes. This means that it takes, at least, 65,535 minutes (45.5 days) to complete the identification test. Also, the amplitude of input signal has to be high enough to measure the nonlinearities. Therefore, it seems usually impossible to identify the Volterra kernels of the actual chemical processes, when we consider the product quality, safety and so on.

However, it is not difficult to identify the linear part of the process only. It is well known that a lot of process control applications are successful with identification of the linear model.

The linear part of the Volterra kernel model is described as follows:

$$\begin{aligned}
y_1(t) &= \int_0^\infty g_1(\tau_1)u(t-\tau_1)d\tau_1 \\
&= y(t) - (y_2(t) + y_3(t)) \tag{5}
\end{aligned}$$

where

$$\begin{aligned}
y_2(t) &= \int_0^\infty \int_0^\infty g_2(\tau_1, \tau_2) \\
&\times u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 \\
y_3(t) &= \int_0^\infty \int_0^\infty \int_0^\infty g_3(\tau_1, \tau_2, \tau_3) \\
&\times u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)d\tau_1 d\tau_2 d\tau_3
\end{aligned}$$

$y_1(t)$ is the linear impulse response and $y_2(t), y_3(t)$ are the second and third order kernels showing nonlinear part. If we have an approximate open differential equation (ODE) as an *a priori* information, we can easily extract a linear part and a nonlinear part of the process through the calculation of the Volterra kernels in ODE model. This is very important feature of Volterra kernel model. Volterra kernel model represents a nonlinear system as simply sum of a linear term and nonlinear terms. Usually, the first order Volterra kernel plays a main role for the behaviour of the process, and higher order terms are small, sometimes negligible. However, in order to identify precisely the nonlinear characteristics of the process, we have to obtain the higher order Volterra kernels.

Fig.1 shows a method of linealization of a plant by use of Volterra kernel model. If we have the second and third order kernel calculated by use of the approximate ODE model, we can easily linearize the process by just eliminating the output of higher order kernels from the original process output. Since the relationship between input $u(t)$ and output $y_1(t)$ is linear, we can identify the linear impulse response using any type of linear model such as ARX, ARMAX and so on.

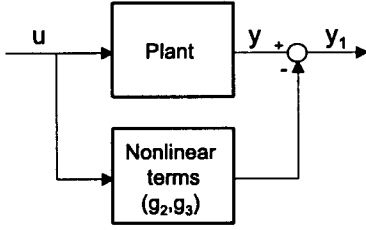


Figure 1: Linearization method

In case where we choose ARX as the linear model, we have

$$y_1(k) = \sum_{j=1}^p a_j y_1(k-j) + \sum_{j=1}^q b_j u(k-j) \quad (6)$$

$$y(k) = y_1(k) + \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} g_2(l, m) u(k-l) \times u(k-m) + \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} g_3(l, m, n) u(k-l) \times u(k-m) u(k-n) \quad (7)$$

4. Simulation results

Here we consider the identification of a heat exchanger having nonlinear characteristics, as shown in Fig.2. The purpose of this heat exchanger is increasing temperature of liquid flow f_1 using hot oil f_2 . Input $u(t)$ is the hot oil flow rate f_2 , and output $y(t)$ is the outlet temperature of liquid flow. The energy balance equations are written as follows:

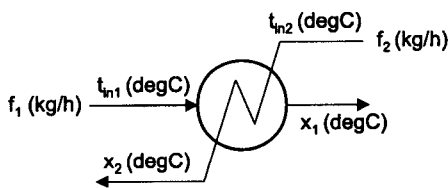


Figure 2: Heat exchanger

$$Hc_1 \frac{dx_1}{dt} = f_1 C_{p1} (t_{in1} - x_1) + Q_{ex} \quad (8)$$

$$Hc_2 \frac{dx_2}{dt} = f_2 C_{p2} (t_{in2} - x_2) - Q_{ex} \quad (9)$$

$$y = x_1$$

$$u = f_2$$

where the heat exchange rate Q_{ex} is,

$$Q_{ex} = UA \frac{(t_{in2} - x_1) - (x_2 - t_{in1})}{\log(t_{in2} - x_1)/(x_2 - t_{in1})}$$

The state variables x_1 and x_2 represent the outlet temperature of liquid flow and hot oil flow. The flow rate f_1 and the inlet temperature t_{in1}, t_{in2} are the fixed values $\{2,000\text{kg/h}, 150\text{degC}, 250\text{degC}\}$. The parameters $\{Hc_1, Hc_2, Cp_1, Cp_2\}$ have nominal values $\{50.3 \text{ kJ/kg}, 251.4 \text{ kJ/kg}, 3.35 \text{ kJ/kgdegC}, 2.43 \text{ kJ/kgdegC}\}$. The values of heat exchange rate constant U and heat exchange area A are $\{5,028 \text{ kJ/degCm}^2\text{h}, 10 \text{ m}^2\}$. The initial value for the input $u(0)$ is $2,500 \text{ kg/h}$. At the time, $x_1(0)$ and $x_2(0)$ are given $\{234 \text{ degC}, 157.47 \text{ degC}\}$ as the steady-state solution of Eqn.(8) and Eqn.(9).

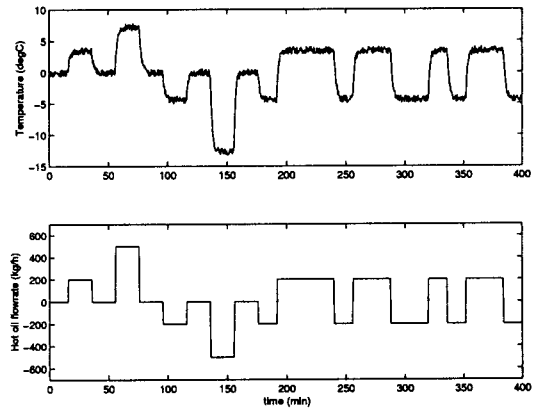


Figure 3: Generated step test data

Firstly, we calculate the Volterra kernels up to third order applying the 16th order and 100kg/h amplitude of M-sequence inputs signal to this ODE model. Secondly, we generate the step test data including 17 step changes and this signal is applied to the actual (simulated) heat exchanger to get linear model $y_1(t)$. The generated data is shown in Fig.3. The number of 15 - 17 changes is commonly used when we design the model predictive controller for actual chemical plants. Also, we selected ARX model as the linear model with the order of $p = q = 4$ based on Eqn.(6). The orders are decided by AIC. The estimated step responses are shown in Fig.4. In Fig.4,

- Square : $+50\text{kg/h}$ step change output
- Circle : $+20\text{kg/h}$ step change output
- Diamond : -20kg/h step change output
- Triangle : -20kg/h step change output
- Solid line : Volterra model up to 3rd order
- Dotted line : Linear model
- Dashed line : ARX + Volterra(2nd, 3rd) model

The results show its nonlinearities and good agreement between the actual output and the output calculated by the proposed method. In the range of 20kg/h step changes, it does not exhibit obvious nonlinear behavior, so in this region, the simplest linear

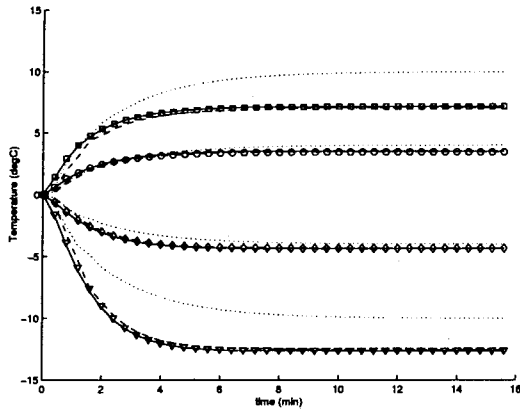


Figure 4: Step responses (perfect model)

model can be applied. But outside of them, the nonlinearities, \pm gains are very different, and it shows the usefulness of the Volterra type model. However, this is case where the used model is correct and precise. Next, we investigated the effect of model error. In the model equations, the heat exchange rate constant U is mostly uncertain because of falling matter in the process fluid. The simulation results of 10% and 20% down of U are shown in Fig.5 and Fig.6. We can see the output of the proposed method (dashed line : ARX+2nd and 3rd order Volterra kernel) follows the actual output more closely than that of linear model.

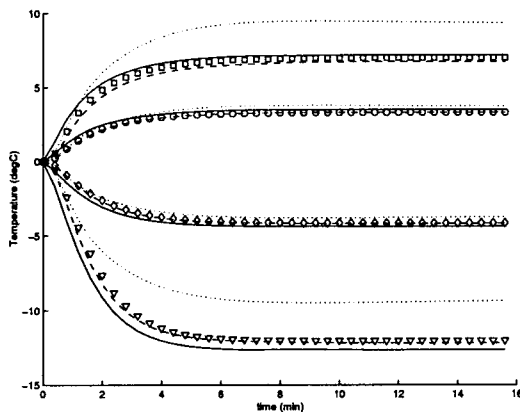


Figure 5: Step responses (10% U down)

5. Conclusion

In this paper, the authors propose a new method for practical identification of Volterra kernels by use of approximate ODE model and simple actual plant test. Firstly, the second and third order Volterra kernels are calculated based on approximate ODE models by Kashiwagi's method. And secondly, the

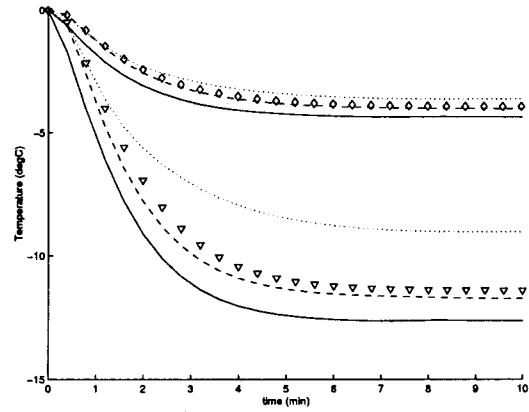


Figure 6: Step responses (20% U down)

linear model is identified using the data of simple actual plant test, where the plant is linearized by subtracting the output of the second and third order Volterra kernels from plant output. Since the estimation error is shifted into the linear model, it does not require perfect ODE model. Simulation results by use of a nonlinear heat exchanger show that the proposed method is widely applicable to the identification of nonlinear processes.

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