Proceedings of the 14th
KACC, Octorber 1999

A New Method for Identifying Higher Volterra Kernel Having the Same Time Coordinate for Nonlinear System

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Abstract

A lot of researcher have proposed a method of kernel identifying nonlinear system by use of Wiener kernels[6-7] or Volterra kernel[5] and so on. In this research, the authors proposed a method of identifying Volterra kernels for nonlinear system by use of pseudorandom M-sequence in which a crosscorrelation function between input and output of a nonlinear system is taken[4]. we can be applied to an MISO nonlinear system or a system which depends on its input amplitude[2]. But, there exist many systems in which it is difficult to determine a Volterra kernel having the same time coordinate on the crosscorrelation function. In those cases, we have to estimate Volterra kernel by using its neighboring points[4]. In this paper, we propose a new method for not estimating but obtaining Volterra kernel having the same time coordinate using calculation between the neighboring points. Some numerical simulations show that this method is effective for obtaining higher order Volterra kernel of nonlinear control systems.

1. Introduction

The authors[1-4] have already proposed a method of identification of Volterra kernel by use of crosscorrelation function between input and the output. We can not get the kernels which overlap each other and hidden points of the crosscorrelation function. Then, we use the method for separating and estimating the kernels. 1) To estimate the hidden points of Volterra kernel points, we estimate by use of some neighboring points[4]. 2) To separate another orders crosssection of Volterra kernels, we uses plural amplitudes, and by use CVTM (Crosscorrelation Function - Volterra kernel transfer Matrix)[1].

These methods have weak point, respectively. 1)Estimating: Since in our method it is difficult to obtain Volterra kernel having the same time coordinate on the crosscorrelation function. We should estimate linear or square method by use of neighboring points. 2)Separating: We have to use plural amplitudes of input, it can truly determine the Value of crosssection of Volterra kernel. We have to prepare plural amplitudes of input, and calculate plural times.There exists one question, we get the Volterra kernels but how is it amplitude level of input.

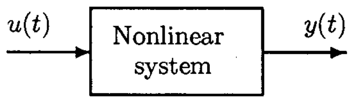

Figure 1: Nonlinear system

In this paper, we propose a new method for identifying higher Volterra kernel having the same time coordinate using calculation between the neighboring points. By calculating the point of Volterra kernel using (we call) factorization of higher order Volterra kernels to lower orders. Thus, we can get those Volterra kernel having coordinate the same time by use of one level of input amplitude. The simulation results show a good agreement with theoretical considerations.

2. Identification of Volterra kernel[4]

Let's consider a nonlinear system having input $u(t)$ and output $y(t)$ as shown in Fig.1. The nonlinear system can usually be written as in the next equation.

$$y(t) = \sum_{i=1}^{\infty} \int_0^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_i(\tau_1, \tau_2, \cdots, \tau_i) \times u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i) d\tau_1 d\tau_2 \cdots d\tau_i \quad (1)$$

Here, $g_i(\tau_1, \tau_2, \cdots, \tau_i)$ is ith order Volterra kernel. We calculate the crosscorrelation function between the input $u(t)$ and the output $y(t)$. When the input is an M-sequence, the crosscorrelation function becomes

$$\phi_{uy}(\tau) \simeq \Delta t g_1(\tau) + \sum_{i=1|odd}^{\infty} F_i(\tau) + \sum_{i=1}^{\infty} i!(\Delta t)^i \times \sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots, \tau - k_{ii}^{(j)}). \quad (2)$$

Here, Δt shows the clock pulse period and 1st term of right hand shows 1st order Volterra kernel (impulse response). 3rd term of it indicates the ith order Volterra kernel as our object. And $k_{ir}^{(j)}$ $(r = 1, 2, \cdots, i)$ are those integers which satisfy the next equation and must be different each other.

$$u(\tau)u(\tau + k_{i1}^{(j)})u(\tau + k_{i2}^{(j)}) \cdots u(\tau + k_{ii-1}^{(j)}) = u(\tau + k_{ii}^{(j)}) \quad (3)$$

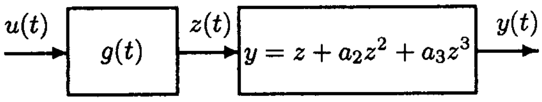

Figure 2: Polynomial type nonlinear system

m_i is the total number of $k_{ir}^{(j)}$. If $k_{ii}^{(j)}$'s are apart from each other sufficiently, we can obtain the Volterra kernels from Eq.(2). 2nd term $F(\tau)$ that is the same time coordinate having higher odd order kernels. Thus, we usually measure impulse response using this method, we have to consider overlap between impulse response and higher odd oder kernels.

And the same time coordinates of even order kernels does not appear on the crosscorrelation function, but it is necessary to identify the nonlinear system. Especially, nonlinear system has up to third order kernel, $F_3(\tau)$ can be written as

$$F_3(\tau) = -2(\Delta t)^3 g_3(\tau,\tau,\tau) + 3(\Delta t)^3 \sum_{q=1}^{m_3} g_3(\tau, q, q) \quad (4)$$

It is difficult for us to get the same time coordinate kernels and the overlap kernels on the crosscorrelation function. This paper proposes a method for obtaining those kernels.

3. Method of factorization of higher order Volterra kernel to lowers

3.1 Approximated Polynomial[3]

In this paper, let us consider a polynomial type nonlinear system as shown in Fig.2. This nonlinear model is shown in the next equation.

$$z(t) = g(\tau) * u(t) \quad (5)$$
$$y(t) = z + a_2 z^2 + a_3 z^3 \quad (6)$$

Here $*$ denotes the convolution integral. By use of these equations, i-th Volterra kernels can be shown as follows,

$$g_i(\tau_1, \tau_2, \cdots) = a_i \cdot g(\tau_1) g(\tau_2) \cdots \quad (7)$$

Then it is shown in reference[2],

$$g(\tau_1) = \phi_{uy}(\tau) \quad (8)$$
$$g(\tau_1, \tau_2, \cdots) = \phi_{uy}(\tau - k_{ii}^{(j)}). \quad (9)$$

The coefficients a_i's can be written as[3]

$$a_i = \frac{\phi_{uy}(\tau)}{\prod_{q=1}^{i} \phi_{uy}(\tau - k_{q1}^{(j)})}. \quad (10)$$

3.2 Factorization of higher order kernel

Here we propose the factorization of higher order Volterra kernels to lowers by use of this a_i as parametric value. We assume higher order Volterra kernel having the same time coordinate as:

$$I = g_i(\tau_1, \tau_1, \cdots). \quad (11)$$

Table 1: τ_x for 2nd order crosscorrelation function of 36135(oct)

$\tau_2 - \tau_1$	τ_x
1	1567
2	3134
3	763
4	6268
5	7065
6	1526
7	710
8	4345
9	5691
10	5939
11	1704
12	3052
13	6343
14	1420
15	3355
16	499
17	7300
18	3191
19	4402
20	3687
21	5744
22	3408
23	6865

We can not get this kernel in general. But,it can be separated by use of Eq.(6).

$$I = a_i \cdot g(\tau_1) g(\tau_1) \cdots \quad (12)$$

We product another points of i th order kernel $g_i(\tau_2, \tau_3, \cdots)$ to numerator and denominator of right hand.

$$I = \frac{a_i \cdot g(\tau_1) g(\tau_1) \cdots \times a_i \cdot g(\tau_2) g(\tau_3) \cdots}{a_i \cdot g(\tau_2) g(\tau_3) \cdots} \quad (13)$$

The all element of equation has scallor value, then we change combination.

$$I = \frac{g_i(\tau_1, \tau_2, \cdots) \cdot g_i(\tau_1, \tau_3, \cdots) \cdots}{g_i(\tau_2, \tau_3, \cdots) \cdots} \quad (14)$$

Thus, we can calculate same time coordinated value by use of other kernels.

3.3 Used M-sequence

Table 1 shows τ_x which calculated by $u(\tau_x) = u(0) \cdot u(\tau_2 - \tau_1)$ on crosscorrelation function of 13 degrees characteristic polynomial 36135(oct).

Table 2 shows τ_x which calculated by $u(\tau_x) = u(0) \cdot u(\tau_3 - \tau_1) \cdot u(\tau_2 - \tau_1)$ on crosscorrelation function of 13 degrees characteristic polynomial 36135(oct).

4. Simulation of proposed method

Table 2: τ_x for 3rd order cross correlation function of 36135(oct)

$\tau_3-\tau_1,\tau_2-\tau_1$	τ_x	$\tau_3-\tau_1,\tau_2-\tau_1$	τ_x
2,1	7387	10,1	4335
3,1	3655	10,2	5750
3,2	3679	10,3	2158
4,1	2743	10,4	4884
4,2	6583	10,5	4481
4,3	2542	10,6	2615
5,1	2875	10,7	1890
5,2	2442	10,8	5702
5,3	5403	10,9	6039
5,4	2851	11,1	552
6,1	848	11,2	2684
6,2	7310	11,3	3337
6,3	4928	11,4	3337
6,4	7358	11,5	6414
6,5	2194	11,6	7659
7,1	531	11,7	6301
7,2	5971	11,8	2665
7,3	1588	11,9	5949
7,4	4761	11,10	67
7,5	1401	12,1	3060
7,6	2579	12,2	5560
8,1	3440	12,3	1696
8,2	5486	12,4	5931
8,3	2994	12,5	6429
8,4	4975	12,6	6090
8,5	6935	12,7	1665
8,6	5084	12,8	4460
8,7	6242	12,9	6525
9,1	1801	12,10	1548
9,2	5798	12,11	4388
9,3	2950		
9,4	6204		
9,5	4686		
9,6	2031		
9,7	2082		
9,8	2640		

We carried out numerical simulations in order to show the effectiveness of this method of identification of nonlinear system. Fig.2 shows the block diagram of the simulation. We used M-sequence of 36135(oct).

4.1 Identification of 2nd order Volterra kernel

The linear element $g(t)$ is

$$G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}. \quad (15)$$

$$\omega_n = 0.4 \quad (16)$$
$$\zeta = 0.4$$

And nonlinear element of the simulated system is

$$y(t)=z-10\cdot z^2. \quad (17)$$

Fig.3 shows the Impulse response of this system. Fig.4 indicates 2nd order Volterra kernel. Note that there exist no value at which two coordinates become equal (same time coordinate ($\tau_1=\tau_2$)).

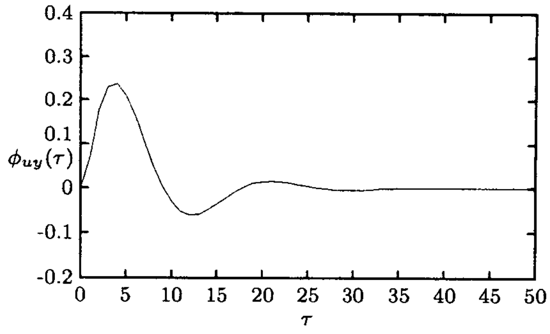

Figure 3: Impulse response of 2nd order system

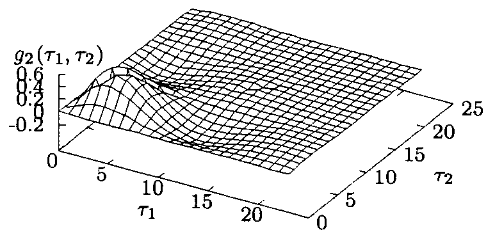

Figure 4: 2nd Volterra kernel

4.1.1 Example of getting same time coordinate

Here, we try to get the points of Volterra kernel having the same time coordinate. From Eq.(13), we can write easily,

$$g_2(0,0) = \frac{g_2(0,1)\cdot g_2(0,2)}{g_2(1,2)} \quad (18)$$
$$g_2(1,1) = \frac{g_2(1,2)\cdot g_2(1,3)}{g_2(2,3)}$$
$$\cdots = \cdots \quad (19)$$
$$\vdots$$

Fig.5 shows 2nd order Volterra kernel having the value of same time coordinate. Fig.6 indicates the simulation result comparing actual system and by use of proposed method. Here, bold line is actual output and o indicates simulated value by use of proposed method, showing good agreement between them.

4.2 Identification of impulse response of nonlinear system(up to 3rd order)

In Fig.2, a linear element $g(t)$ is

$$G(s)=\frac{1}{s+5}. \quad (20)$$

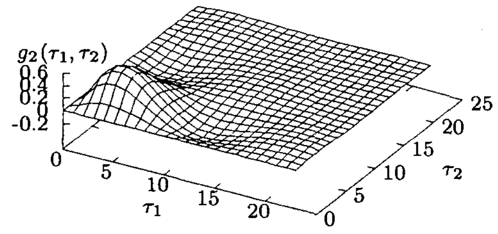

Figure 5: 2nd Volterra kernel having same time coordinate

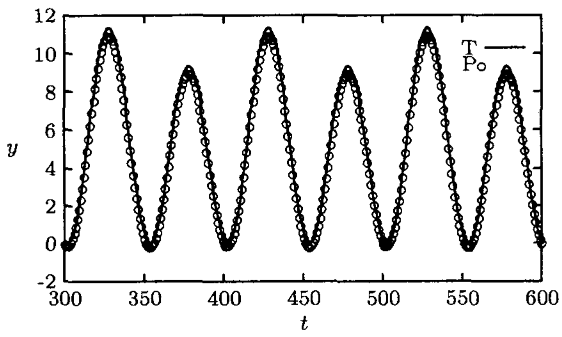

Figure 6: Simulation result

And nonlinear element of the simulated system is

$$y(t) = z - 2 \cdot z^3. \tag{21}$$

Fig.7 shows the impulse response of this system. Here, solid lines shows the simulation result without same time coordinate. We would usually measure the correlation method, the result has to be one. I has not agreement with theoretical value (bold line). o shows proposed value with same time coordinate, showing good agreement between theoretical value and proposed value.

6. Conclusion

We proposed here a new method of identification of higher Volterra kernel having the same time coordinate.

By calculating the same time coordinate of higher order Volterra kernel using factorization to lower order, we may not use estimating method, then we can identify more accuracy. The simulation results show a good agreement between the theoretical considerations and the proposed method.

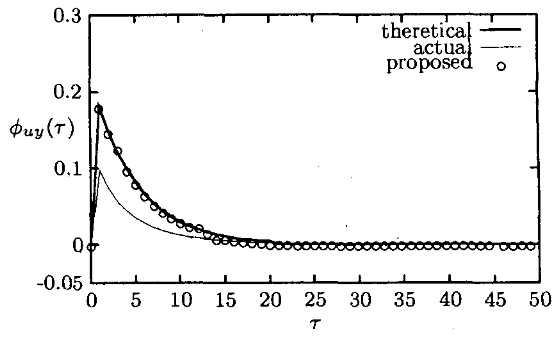

Figure 7: Impulse response of up to third order

References

[1] H.Harada, E.Nishiyama, H.Kashiwagi and T.Yamaguchi : A Method for Separating Volterra Kernels of Nonlinear Systems by Use of Different Amplitude M-Sequence *Proc. '98 KACC* held in Pusan, Korea pp.271-274, 1998.

[2] Nishiyama,E. : Linearization and Polynomial Modeling for Nonlinear System by use of Volterra Kernels: *Ph.D Thesis*, Graduate School of Science and Technology, Kumamoto University, 1998.

[3] Harada,H., E.Nishiyama, and H.Kashiwagi : A Method for Identifying Voltterra Kernels of Nonlinear System Using Low-degree M-sequence *Trans. SICE*, Vol.34, No.1, pp.9-12, 1998.

[4] Kashiwagi.H. and Y.Sun : Identification of Volterra Kernels of Nonlinear System : *Trans. SICE*, Vol.31, No.8, pp.1054-1060, 1995.

[5] Wakamatsu,H. : *New Methods of Controlling Nonlinear System Represented by Discrete Volterra Functional Series*, Proc. IFAC Symposium on Theory and Application of Digital Control, pp.223-228, 1982.

[6] Schetzen,M. : *The Volterra and Wiener Theories of Nonlinear Systems*, A Wiley Intersci., Pub., 1980.

[7] Barker,H.A. and S.N.Obidegwu : Combined Cross-correlation Method for the Measurement of 2nd-order Volterra Kernel. *Proc. IEE*, Vol.120-1, pp.144-118, 1973.