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On Choice of Kautz Functions Pole and its Relation with Accuracy in System Identification

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Abstract

A linear time-invariant model can be described either by a parametric model or by a nonparametric model. Nonparametric models, for which a priori information is not necessary, are basically the response of the dynamic system such as impulse response model and frequency models. Parametric models, such as transfer function models, can be easily described by a small number of parameters. In this paper aiming to take benefit from both types of models, we will use linear-combination of basis fuctions in an impulse response using a few parameters. We will expand and generalize the Kautz functions as basis functions for dynamical system representations and we will consider estimation problem of transfer functions using Kautz function. And so we will present the influences of poles settings of Kautz function on the identification accuracy.

1 Introduction

In order to build a model, we can use either parametric models such as state-space models, and transfer functions models or nonparametric ones such as impulse response model, and frequency models. As the power of computers increases continually, nonparametric models attracted much attention recently. Impulse response models have actually an infinite number of unknown parameters, but these models have to be truncated appropriately in a finite length. However, finite order models still contain too many parameters, since in spite of improvements of computers, a high computational burden is required for simulation. On the contrary, parametric models contain a smaller number of parameters. Unfortunately, an incorrect estimated model order can make the estimated model poor. Recently, a low order impulse response has been proposed using linear-combination of basis [2], [3]. In this paper, we have chosen Kautz functions as basis functions and we will present the influences of poles settings on the identification accuracy. In section 2 we first present the statement of problem, and we formulate the Kautz function in Section 3. In section 4 we introduce an identification of expansion coefficients. A simulation example in Section 5 illustrates the identification method.

Table 1: Characteristic of model identification

Model	System	A priori information	Order of estimation
FIR	non-parametric	×	a large number
Kautz	non-parametric	△	$l > 2n$
TF	paramatic	○	$2n$

2 Statement of Problem

Given the fact that every stable system has a unique series expansion in terms of a pre-chosen basis, a model representation in terms of a finite-length series expansion can serve as an approximate model, where the coefficients of the series expansion can be estimated from input-output data.

A model of a linear stable time-invariant system with additive disturbance is given by:

$$y(t) = G^0(q)u(t) + v(t), \; !\,!$$

$$G^0(q) = \sum_{k=1}^{\infty} g_k q^{-k}. \qquad (1)$$

Where $u(t)$ and $y(t)$ are the input and output signals, respectively. Time shifts are represented by the delay operator $q^{-1}u(t) = u(t-1)$. And $v(t)$ is a unit-variance, zero-mean white noise process. Let $\{f_k(z)\}_{k=0,1,2,\cdots}$ be an orthonormal basis for the set of systems. Then there exists a unique series expansion:

$$G(z) = \sum_{k=1}^{\infty} w_k f_k(z), \qquad (2)$$

with $\{w_k\}_{k=1,2,\cdots}$ the unknown model parameters. A model of the system $G(z)$ can be approximated by a finite-length series expansion:

$$\widehat{G}(z) = \sum_{k=1}^{n} \widehat{w}_k f_k(z) \qquad (3)$$

where the accuracy of the model will be essentially dependent on the choice of basis functions $f_k(z)$. Note that the choice $f_k(z) = z^{-k}$ corresponds to the use of so-called FIR (finite impulse response) models. The accuracy of the models is limited by the basis functions.

3 Kautz Function

! The problem of orthogonalizing a set of continuous time exponential functions has been elegantly solved in [1]. The key idea is to determine the corresponding Laplace transforms, which have very simple structures.

The sequence of functions $\{\Psi_k(z)\}$ is determined as follows:

$$\Psi_{2k-1}(z) = C_1^{(k)}(1 - a_1^{(k)}z)\Gamma^{(k)}(z) \qquad (4)$$

$$\Psi_{2k}(z) = C_2^{(k)}(1 - a_2^{(k)}z)\Gamma^{(k)}(z) \qquad (5)$$

where

$$\Gamma^{(k)}(z) = \frac{\prod_{j=1}^{k-1}(1-\beta_j z)(1-\beta_j^* z)}{\prod_{j=1}^{k}(z-\beta_j)(z-\beta_j^*)}$$

$$C_1^{(k)} = \sqrt{\frac{(1-\beta_k^2)(1-\beta_k^{*2})(1-\beta_k\beta_k^*)}{(1+(a_1^{(k)})^2)(1+\beta_k\beta_k^*) - 2a_1^{(k)}(\beta_k+\beta_k^*)}}$$

$$C_2^{(k)} = \sqrt{\frac{(1-\beta_k^2)(1-\beta_k^{*2})(1-\beta_k\beta_k^*)}{(1+(a_2^{(k)})^2)(1+\beta_k\beta_k^*) - 2a_2^{(k)}(\beta_k+\beta_k^*)}}$$

$$(1 + a_1^{(k)}a_2^{(k)})(1+\beta_k\beta_k^*) - (a_1^{(k)} + a_2^{(k)})(\beta_k+\beta_k^*) = 0$$

Here $\beta_k' s$ are complex numbers such that $|\beta_k| < 1$, and $a_1^{(k)}$, $a_2^{(k)}$ are restricted by the condition (6). The functions $\{\Psi_k(z)\}_{k=1,2,\cdots}$ will be called the discrete Kautz functions. Another special case is for $\beta_k = \beta$. For this case one can take:

$$!\,!!\,!!\, a_1^{(k)} = \frac{1+\beta\beta^*}{\beta+\beta^*}, \quad a_2^{(k)} = 0$$

and thus

$$\Psi_{2k-1}(z) = \frac{\sqrt{1-c^2}(z-b)}{z^2+b(c-1)z-c}\left[\frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}\right]^{k-1} = K_{2k-1}(z)G_b(z)^k$$

$$\Psi_{2k}(z) = \frac{\sqrt{(1-c^2)(1-b^2)}}{z^2+b(c-1)z-c}\left[\frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}\right]^{k-1} = K_{2k}(z)G_b(z)^k$$

where $|b| < 1, \quad |c| < 1$

$$K_{2k-1}(z) = \frac{\sqrt{1-c^2}(z-b)}{z^2+b(c-1)z-c}$$

$$K_{2k}(z) = \frac{\sqrt{(1-c^2)(1-b^2)}}{z^2+b(c-1)z-c}$$

$$G_b(z) = \frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}$$

and $b = (\beta+\beta^*)/(1+\beta\beta^*)$, $c = -\beta\beta^*$. Since $a_1^{(k)}$ and $a_2^{(k)}$ are not unique, several other sets of $\{\Psi_k(z)\}$ are possible.

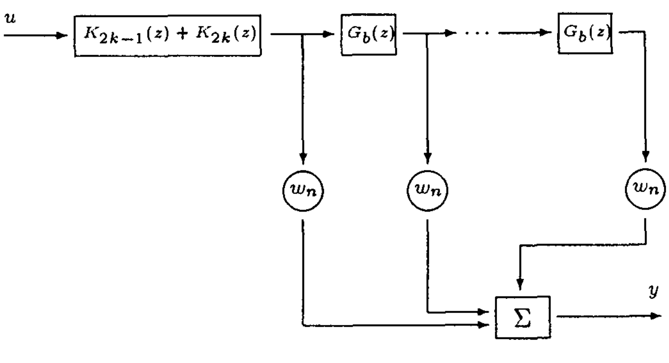

Figure 1: Kautz network for discrete model

4 Identification of Expansion Coefficients

Using Kautz function a practical parameter identification method for linear time-invariant systems is introduced. System identification deals with the problem of finding an estimate of $G(z)$ from observations of $\{y(t), u(t)\}_{t=1\cdots N}$. The identification problem simplifies to a linear regression estimation problem if the model is linear-in-the-parameters, and can be represented by:

$$G(z) = \sum_{k=1}^{n} w_k f_k(z) \qquad (6)$$

where $\{f_k(z)\}$ is a set of given basis functions and $\{w_k\}$ are the unknown model parameters. If $\left\{\begin{bmatrix} f_{2k-1}(z) \\ f_{2k}(z) \end{bmatrix}\right\}$ corresponds to $\{V_k(z)\}$, we call this model a Kautz model. The least squares method can now be applied to estimate the model parameters

$$\theta^T = (w_1, w_2, \cdots, w_n) \quad . \qquad (7)$$

The input/output relation can be written in the linear regression form

$$y(t) = z_t^T \theta \qquad (8)$$

where

$$\begin{aligned} z_t^T &= [\bar{u}_1(t), \bar{u}_2(t), \cdots, \bar{u}_n(t)] \\ \bar{u}_k(t) &= f_k(z)u(t), \end{aligned}$$

Let

$$Z^T = [z_{t0}, \cdots, z_N], \quad \mathbf{y}^T = [y(t0), \cdots, y(N)]$$

Then, the least squares estimate of θ minimizes the loss function is such as:

$$\begin{aligned} J &= \frac{1}{N} \sum_{t=t_0}^{N} (y(t) - z_t^T \theta)^2 \\ &= \frac{1}{N} (\mathbf{y} - Z\theta)^T (\mathbf{y} - Z\theta) \end{aligned} \qquad (9)$$

The solution of this quadratic optimization problem is:

$$\hat{\theta}_N = (Z^T Z)^{-1} Z^T \mathbf{y} \qquad (10)$$

where

$$\begin{aligned} Z^T Z &= \frac{1}{N} \sum_{t=t_0}^{N} z_t z_t^T, \\ Z^T \mathbf{y} &= \frac{1}{N} \sum_{t=t_0}^{N} z_t y(t). \end{aligned} \qquad (11)$$

The value of t_0 depends on how the effects of unknown initial conditions are treated. For large N, the effects of t_0 will be negligible.

5 Simulation Example

We give a simple example to illustrate the advantage of using Kautz models for second order resonant systems. Consider a continuous time transfer function

$$G^0(s) = \frac{1}{s^2 + 0.2s + 1} \qquad (12)$$

with resonant frequency $\omega_0 = 1[rad/s]$ and damping $\xi = 0.1$. This system is sampled using a zero-order hold with sampling period $T = 0.5$. Two input signals are generated white noise signal and colored noise signal. The colored noise is determined by:

$$\sin(t) + 0.5\sin(1.5t) + 1.5\sin(3t) + 0.3\sin(4.5t) + 0.3\sin(5t) + 0.2\sin(7t) + 2.5\sin(7.5t) + 5\sin(10.5t)$$

Table 2: Correspondence, in terms of selection of input and slided pole

Input	FIR	Kautz 1	Kautz 2
White noise	Fig3. (n=100)	Fig5. (n=7)	Fig7. (n=7)
colored noise	Fig4. (n=100)	Fig6. (n=7)	Fig8. (n=7)

6 Conclusion

In this paper we considered an estimation method of transfer function $G(z)$ using basis functions expansion. If a colored noise is used as an input signal, the output of a FIR model is more disturbed than the output of a Kautz model. And a Kautz model needs a priori information on two parameters, but even if no accurate a priori information is availble, the output of the Kautz model remains good.

References

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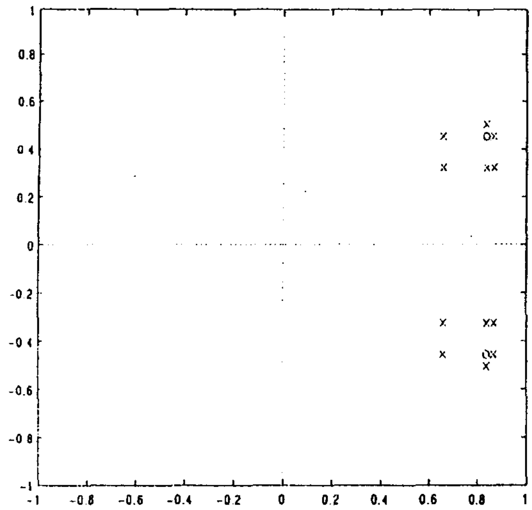

Figure2. Pole of Kautz model
o : Actual pole × : Slided pole

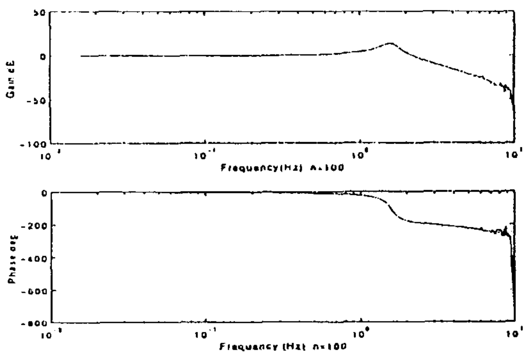

Figure3. FIR model (n=100)

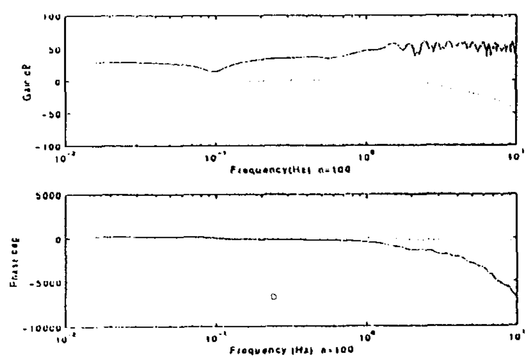

Figure4. FIR model (n=100)

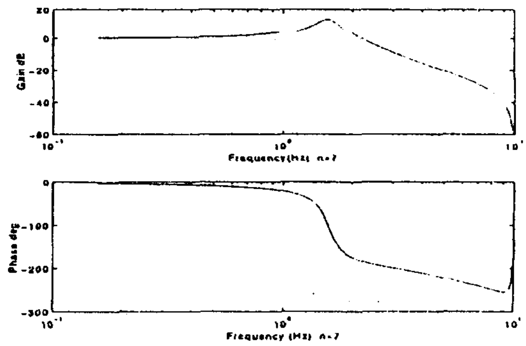

Figure5. Kautz model (n=7)

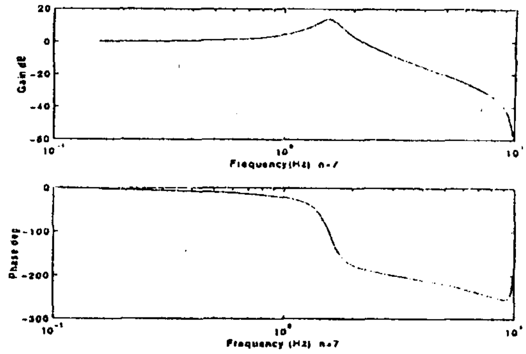

Figure6. Kautz model (n=7)

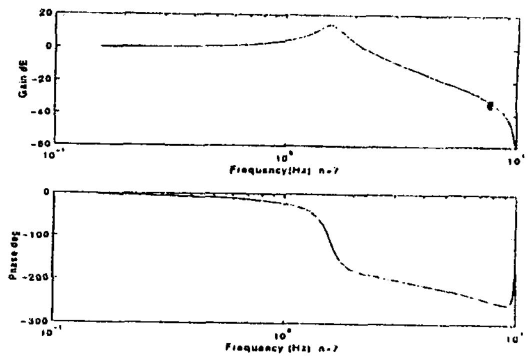

Figure7. Kautz model (n=7)

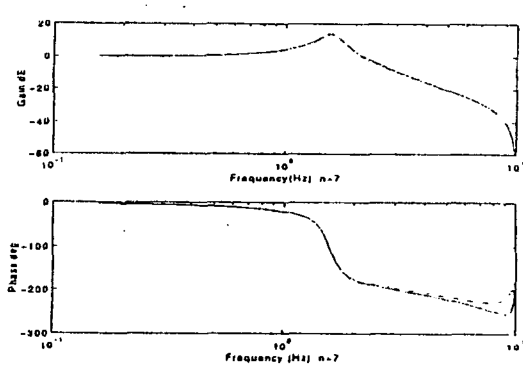

Figure8. Kautz model (n=7)