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A Study on the Generalization of the Manabe Standard Forms with the Genetic Algorithm

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Abstract

The step response of the Manabe standard form[1] has little overshoot and shows almost same waveforms regardless of the order of the characteristic polynomials. In some situations it is difficult to control the rise time and settling time simultaneously of the step response of the Manabe standard form. To control its rise time and settling time efficiently, we develop the generalization of the Manabe standard form: we try to find out the SRFS(Slow Rise time & Fast Settling time) form which has the slower rise time and faster settling time than those of the Manabe standard form. we also consider the other three forms: FRSS(Fast Rise time & Slow Settling time), SRFS(Slow Rise time & Fast Settling time) and SRSS(Slow Rise time & Slow Settling time) forms. In this paper, by using the genetic algorithm, we obtain all the coefficient of the four forms we mention above. Finally, we design a controller for a given plant so that the overall system has the performance that the rise time is faster, the settling time is slower than those of the Manabe standard form.

1. Introduction

The stability of a closed loop polynomial can be expressed by the inequalities with the coefficients of the polynomial in terms of the Hurwitz stability condition. Even though these conditions are complicated, Lipatov & Sokolov[2] gives a sufficient condition for the stability of a polynomial with the stability indices. This sufficient condition is described by the second order inequalities with the stability indices associated with the coefficient of the polynomial. Kessler[3] makes a transfer function whose denominator has all the stability indices equal to 2. The step response of the Kessler form has some overshoot. Manabe designs a transfer function having little overshoot by changing the index $\gamma_1 = 2.5$ Manabe standard form has almost equal waveforms irrespective of the order of the characteristic polynomial. Kim [4] modified the Manabe standard form by using the delay time, 10-99% rise time and 10% rise time. The objective of this paper is to control the rise time and settling time simultaneously of the step response of the Manabe standard form. In this paper, the rise time is defined as the elapsed time for the 10% to 90% of the final steady state value of the step responses. The settling time is defined as the time required for the system to settle within a 1% of the final steady state value of the step responses. We develop the generalization of the Manabe standard form: we try to find out the SRFS(Slow Rise time & Fast Settling time) form which has the slower rise time and faster settling time than those of the Manabe standard form. we also consider the other three forms: FRSS(Fast Rise time & Slow Settling time), SRFS(Slow Rise time & Fast Settling time) and SRSS(Slow Rise time & Slow Settling time) forms. Even though the change of the equivalent time constant of the standard Manabe form leads to the FRFS(Fast Rise time & Fast Settling time) and SRSS (Slow Rise time & Slow Settling time), we try to find out the coefficients of FRFS(Fast Rise time & Fast Settling time) and SRSS (Slow Rise time & Slow Settling time) without changing the equivalent time constant. In this paper, by using the genetic algorithm, we obtain the four forms we mentioned above. Finally, we design a controller for a given plant so that the overall system has the performance that the rise time is faster, the settling time is slower than those of the

Manabe standard form.

2. The Manabe Standard Form

With the stability indices associated with the coefficients a polynomial, the sufficient condition for the stability of the polynomial is described. For the characteristic polynomial $\sum_{i=0}^{n} a_i s^i (a_i > 0, (i=0,1,2,\cdots,n))$ of a system, the stability index γ_i is defined as $\gamma_i = \dfrac{a_i^2}{a_{i-1}a_{i+1}}$

Now we define a Manabe standard form using the stability index. Before the definition of the Manabe standard form we need a Lemma.

Lemma 1[2]: Suppose that a polynomial satisfies the inequalities $\gamma_i > 1.4656\ (i=1,2,\cdots,n-1)$. Then the polynomial is left half plane stable.

Definition 1[1]: The characteristic polynomial $\sum_{i=0}^{n} a_i s^i$ of the Manabe standard form has the stability indices $\gamma_1 = 2.5,\ \gamma_2 = \cdots = \gamma_{n-1} = 2$. The equivalent time constant is $\tau = a_1/a_0$. Its coefficient is expressible as $a_i = \dfrac{a_0 \tau^i}{\gamma_{i-1}\gamma_{i-2}^2 \cdots \gamma_2^{i-2}\gamma_1^{i-1}}$ $(i=1,2,\cdots,n)$.

3. The Genetic Algorithm

The genetic algorithm is a kind of search algorithm based on the evolution, the natural selection and genetics. The genetic algorithms use the binary number associated with the given parameter and then evaluate the fitness function. Two operators, called crossover and mutation, determine the next generation from the former generation with the selection procedure such as the roulette wheel, tournament and tournament with elitism.

The mathematical foundation for the GA[4] is as follows: The fixed population is N and its individual is represented by the binary string of length ℓ. The population space is denoted as S^N and $X \in S^N$ is described by

$$\boldsymbol{X} = (X1, X2, \ldots, X_N)^T = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1l} \\ x_{21} & x_{22} & \cdots & x_{2l} \\ \cdots & \cdots & \cdots & \cdots \\ x_{N1} & x_{N2} & \cdots & x_{Nl} \end{pmatrix}$$

where X_i is the i-th individual of $\boldsymbol{X}$ and x_{ij} is the j-th component of X_i GA can be given as follows:

Step 1) Set $k=0$ and generate initial population $\boldsymbol{X}(0)$

Step 2) Select N pairs of individuals from the current population for reproduction.

Step 3) Perform crossover to the N pairs of individuals to generate N new intermediate individuals.

Step 4) Mutate the N intermediate individuals to get the next generation

$$\boldsymbol{X}(k+1) = [X_1(k+1), \cdots, X_N(k+1)].$$

Step 5) Stop if some stopping criteria is met. Else, set $k=k+1$ and go to Step 2).

Simulation

At first, if the order of a polynomial is fixed as a sixth order, the value of the stability indices assumes to lie in the interval [1.47 10.0]. We assume that all the stability indices have 10 bits and the crossover probability is 0.25 and the mutation probability is 0,01, We use the proportional selection rule and the fitness function as $1/[(t_r - t_{rd})^2 + (t_s - t_{sd})^2]$ where t_r, t_{rd}, t_s, t_{sd} are rise time, desired rise time, settling time and desired settling time, respectively. It is allow that the overshoot is less than 0.1%. For the seventh and eighth order polynomial, we increase the number of the stability indices and use GA like the procedure for the sixth order polynomial case. For the fifth order case the stability index γ_5 is eliminated. For the fourth order case, two different methods are used. For the SRFS and SRSS forms, two higher stability indices are deleted and for the FRFS and FRSS form, the heuristic method is used. During simulations we find the fact that as γ_1 and γ_2 make smaller, the rise time and settling time is faster. We also find that as γ_1 makes smaller, the overshoot is larger and as γ_2 makes larger, the overshoot is smaller and the undershoot is larger. The final observation is that as γ_3 is larger, the rise time is slower and the settling time is faster. From the figures from 1 to 4, the step responses are plotted for the four cases: FRSS, FRSS, SRFS and SRSS forms.

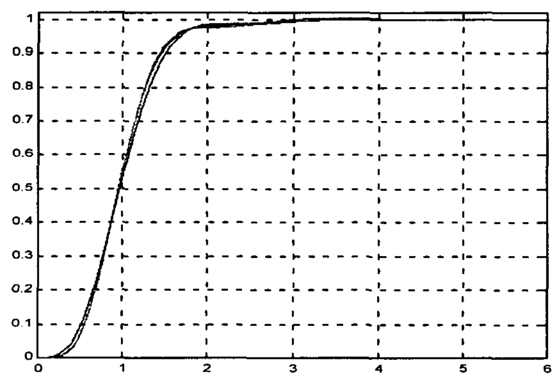

Fig 1. FRSS Forms of order 4,5,6,7.8

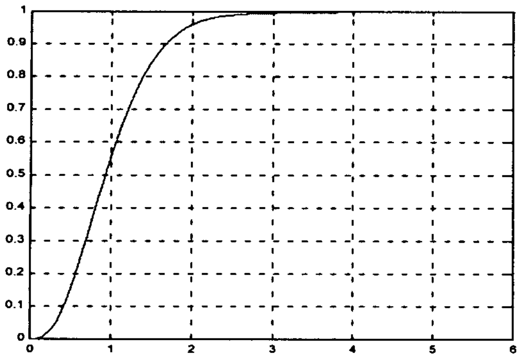

Fig 2. SRSS Forms of order 4,5,6,7.8

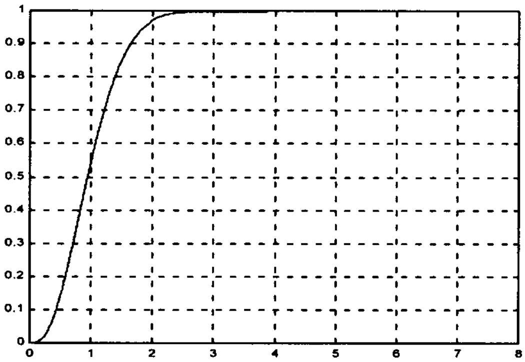

Fig 3. SRFS Forms of order 4,5,6,7.8

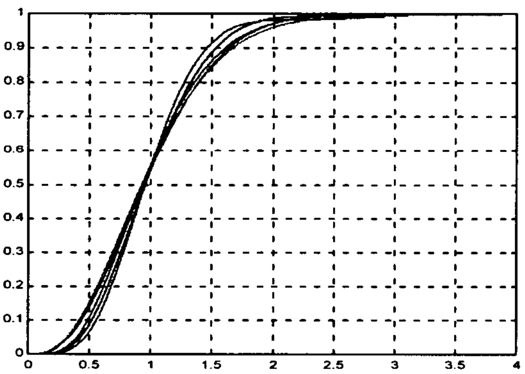

Fig 4. FRFS Forms of order 4,5,6,7.8

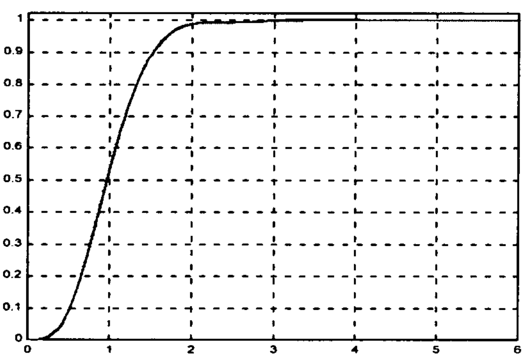

Fig 5. Comparison of FRFS, FRSS, SRFS, SRSS and Manabe forms

From Table 1 to table 4, performance criteria: the rise time, the settling time and overshoot are described. In Fig. 5 the step responses of the sixth order transfer function for the four forms are compared with that of the Manabe standard form.

Table 1. The characteristics of FRFS form

type	Stability index						
	γ_7	γ_6	γ_5	γ_4	γ_3	γ_2	γ_1
FRFS					1.7000	1.9200	2.3800
				2.1071	2.0299	1.7874	2.4315
			2.1126	2.1071	2.0299	1.7874	2.4315
		3.2127	2.1126	2.1071	2.0299	1.7874	2.4315
	6.5632	3.2127	2.1126	2.1071	2.0299	1.7874	2.4315

order	Manabe		FRFS	
	Tr	Ts	Tr	Ts
4	1.0960	2.3826	0.9976	2.1650
5	1.1159	2.3095	1.0025	2.1025
6	1.1165	2.3074	1.0059	2.0830
7	1.1166	2.3076	1.0059	2.0832
8	1.1166	2.3076	1.0059	2.0830

Table 2. The characteristics of FRSS form

type	Stability index						
	γ_7	γ_6	γ_5	γ_4	γ_3	γ_2	γ_1
FRSS					1.6500	1.9100	2.3900
				1.6493	1.7887	1.6493	2.4262
			2.7450	1.6493	1.7887	1.6493	2.4262
		8.9911	2.7450	1.6493	1.7887	1.6493	2.4262
	4.2123	8.9911	2.7450	1.6493	1.7887	1.6493	2.4262

차수	Manabe		FRSS	
	Tr	Ts	Tr	Ts
4	1.0960	2.3826	0.9985	2.5883
5	1.1159	2.3095	0.8768	2.6857
6	1.1165	2.3074	0.8947	2.7681
7	1.1166	2.3076	0.8947	2.7658
8	1.1166	2.3076	0.8947	2.7658

Table 3. The characteristics of SRFS form

type	Stability index						
	γ_7	γ_6	γ_5	γ_4	γ_3	γ_2	γ_1
SRFS					6.7398	2.0787	2.6207
				4.5635	6.7398	2.0787	2.6207
			9.8666	4.5635	6.7398	2.0787	2.6207
		3.2127	9.8666	4.5635	6.7398	2.0787	2.6207
	3.1592	3.2127	9.8666	4.5635	6.7398	2.0787	2.6207

차수	Manabe		SRFS	
	Tr	Ts	Tr	Ts
4	1.0960	2.3826	1.1999	2.2783
5	1.1159	2.3095	1.2003	2.2780
6	1.1165	2.3074	1.2003	2.2780
7	1.1166	2.3076	1.2003	2.2780
8	1.1166	2.3076	1.2003	2.2780

Table 4. The characteristics of SRSS form

type	Stability index						
	γ_7	γ_6	γ_5	γ_4	γ_3	γ_2	γ_1
SRSS					9.8499	2.1871	2.7374
				9.9083	9.8499	2.1871	2.7374
			3.1793	9.9083	9.8499	2.1871	2.7374
		8.9911	3.1793	9.9083	9.8499	2.1871	2.7374
	3.1210	8.9911	3.1793	9.9083	9.8499	2.1871	2.7374

order	Manabe		SRSS	
	Tr	Ts	Tr	Ts
4	1.0960	2.3826	1.2511	2.5353
5	1.1159	2.3095	1.2512	2.5351
6	1.1165	2.3074	1.2512	2.5351
7	1.1166	2.3076	1.2512	2.5351
8	1.1166	2.3076	1.2512	2.5351

4. An Example

To find out the usefulness of the FSSS form, we consider the plant transfer function

$$G(s) = \frac{2}{s(s^2 + 0.25s + 6.25)}$$

and the feedback structure is a two parameter structure given in [5. p.565]. We want to design a controller and a gain so that the overall closed loop system satisfies the following specifications. 1) The settling time is faster than that of Manabe form and the rise time is slower than that of the Manabe form.
2) The steady state error is equal to zero for the step response.
3) The denominator and the numerator of a controller are of fixed 2nd order.

Solution
We start to solve this problem by the FSSS type. The equivalent time constant fixed to 1 and the closed loop characteristic polynomial is equal to the 4th order FRSS type. The controller is

$$\frac{B(s)}{A(s)} = \frac{123.5246s^2 + 174.6709s + 410.1569}{s^2 + 11.5549s + 75.3555}$$

and a gain is $F(s)$=410.1569. The resulting step response is shown in Fig. 6 and the settling time is

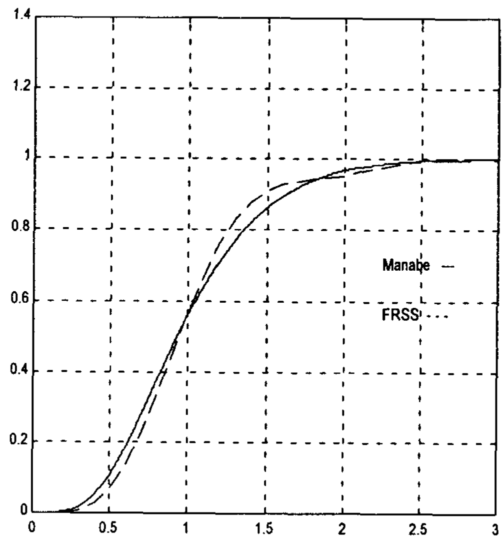

Fig.6 A Step response for an Example

2.6857 sec and the rise time is 0.8768 sec which satisfy the specifications.

5. Conclusion

We determined the coefficient of the transfer function whose characteristic has slower settling time and slower rise time than that of the Manabe standard form. In addition we obtain the other three transfer function of which characteristics have slower and/or faster than those of the Manabe standard form. We called these four forms: FRFS (faster rise time and faster settling time) form, FRSS (faster rise time and slower settling time) form SRFS (slower rise time and faster settling time) form and SRSS (slower rise time and slower settling time) form. We found that these four forms have a very little overshoot and have nearly the same waveforms regardless of the order of the denominator of the transfer function. In the future, if we use adaptive GA, we may obtain the transfer function having arbitrary settling time and arbitrary rise time with allowing overshoot or not.

6. References

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