Proceedings of the 14th
KACC, Octorber 1999

A NEW METHOD OF LQ INTEGRAL CONTROL FOR NONMINIMUM PHASE SYSTEMS

°Byung-Moon Kwon* and Oh-Kyu Kwon**

* Dept. of Elec. Eng., Inha Univ., Inchon, 402–751, Korea
(Tel : 82–032–860–7395; Fax : 82–032–863–5822; E-mail : g1983661@inhavision.inha.ac.kr)
** Dept. of Elec. Eng., Inha Univ., Inchon, 402–751, Korea
(Tel : 82–032–860–7395; Fax : 82–032–863–5822; E-mail : okkwon@inha.ac.kr)

Abstract : The right half plane (RHP) zeros may cause severe problems, such as undershoots, oscillations and time delay in the transient response of the systems. In this paper, we formulate a linear quadratic type problem to deal with the effects of the RHP zeros in the nonminimum phase systems. Based on the LQ formulation, this paper shows the trade-off relation between undershoot and rising time performances in nonminimum phase systems by using a new performance index which consists of new state and tracking error. And performances of the proposed method are shown via computer simulations.

Keywords : integral control, linear quadratic regulator (LQR), nonminimum phase system, right half plane (RHP) zero

1. Introduction

Much works have been done to clarify the influence of the zeros on the transient response of the nonminimum phase systems to the step type reference input. Specially, RHP zeros which are nearer to the imaginary axis than poles cause severer transients, like undershoots, oscillations and time delay, in the step response. The occurrence of these phenomenon on the step type reference input is usually undesirable in the controlled outputs of the systems. But, it may be impossible to acquire response without these phenomenon because there exist fundamental limitations on the achievable transient response of the system with RHP zeros[4, 6, 7, 9]. Generally, this limitation can be characterized completely by the number and location of the RHP zeros[9].

It is well known that continuous time systems with an odd number of real open RHP zeros have the initial undershoot on the step type reference input, i.e., the initial response is in the opposite direction from the steady state response[3, 5, 6, 8, 10]. Besides, when the conjugate complex zeros have even positive real parts, and absolute values of these zeros are not so large as compared with absolute values of the poles, the output has so-called Type B undershoot– the initial response is in the same direction from the steady state response, but mid-term period of transient response is in the opposite direction from the steady state response[8]. However, the global characterization of the effects for the RHP zeros upon the step response of system, both real and complex, still remains as an open problem[6]. The amount of the undershoot is related to the rising time, the location and the number of the zeros on the nonminimum phase systems[7]. Thus, we can design controller which has trade-off relation between undershoot and rising time on the nonminimum phase systems.

In this paper, a controller design method is proposed to solve the trade-off problem between undershoot and rising time. It uses the linear quadratic integral control weighting matrix with new state and tracking error in controller design procedure. The new state is used to represent the amount of the undershoot. Here, we consider only the SISO rational system characterized by the continuous time strictly proper transfer function $G(s)$. And, it is assumed that $G(s)$ is both detectable and stabilizable, with no zeros at the origin of the complex plane.

The remainder of this paper is organized as follows : Section 2 induces new state x_{m_1} which represents the amount of undershoot in case of the system with one RHP real zero. In Section 3, we consider extensive systems with p RHP real zeros and $2q$ RHP complex zeros. In Section 4, we show trade-off relation between undershoot and rising time using two examples via computer simulations. The concluding remarks are given in Section 5.

2. System with One RHP Real Zero

Consider the system with one RHP real zero as follows :

$$G(s) = \frac{B_1(s)}{A_1(s)}(-z_1 s + 1), \tag{1}$$

where $A_1(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$,
$B_1(s) = b_{n-2}s^{n-2} + b_{n-3}s^{n-3} + \cdots + b_1 s + b_0$
and $z_1 > 0$. Note that the one RHP real zero of the system is $1/z_1$. Let $G_{m_1}(s) = B_1(s)/A_1(s)$, then $G_{m_1}(s)$ is a minimum phase system because $B_1(s)$ is a stable polynomial. Hence Eq. (1) is then

$$\begin{aligned} G(s) &= G_{m_1}(s)(-z_1 s + 1) \\ &= G_{m_1}(s) - z_1 s\, G_{m_1}(s). \end{aligned} \tag{2}$$

Let us define the nonminimum phase system output $Y(s) = G(s)U(s)$ and the minimum phase system output $Y_{m_1}(s) = G_{m_1}(s)U(s)$. From Eq.(2), we have

$$Y(s) = Y_{m_1}(s) - z_1 s Y_{m_1}(s). \tag{3}$$

Thus the time response of the nonminimum phase system can be represented by the minimum phase system output and its first derivative as follows[2] :

$$y(t) = y_{m_1}(t) - z_1 \dot{y}_{m_1}(t). \tag{4}$$

Note that $y_{m_1}(t)$ itself does not represent undershoot phenomenon since it is a minimum phase system output. Hence the maximum undershoot can be reduced, if we make a $\dot{y}_{m_1}(t)$ zero, since $z_1\dot{y}_{m_1}(t)$ cause undershoot phenomenon. Note that $z_1\dot{y}_{m_1}$ become large as z_1 is large, therefore $G(s)$ has a large maximum undershoot.

From Eq. (4), $\dot{y}_m$ has the form

$$\dot{y}_{m_1}(t) = \frac{1}{b}\left\{y_{m_1}(t) - y(t)\right\}. \tag{5}$$

Thus a new state $x_{m_1}(t)$ is defined as difference between $x_{m_1}(t)$ and $y(t)$ as follows :

$$\begin{aligned} \dot{x}_{m_1}(t) &= y(t) - x_{m_1}(t) \\ &= Cx(t) - x_{m_1}(t). \end{aligned} \qquad (6)$$

Note that $x_{m_1}(t)$ is not observable and that it should be constructed as stable. And the integral of the error between the reference input and output is generated by the following equation :

$$\begin{aligned} \dot{e}_I(t) &= r - y(t) \\ &= r - Cx(t). \end{aligned} \qquad (7)$$

Provided that the system state equation for the nonminimum phase system (1) is given as follows :

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t). \end{cases} \qquad (8)$$

The augmented state equation is then derived from the new state equation (6) and the error equation (7) as follows :

$$\begin{cases} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_{m_1}(t) \\ \dot{e}_I(t) \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ C & -1 & 0 \\ -C & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_{m_1}(t) \\ e_I(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r \\ y(t) = \begin{bmatrix} C & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_{m_1}(t) \\ e_I(t) \end{bmatrix}. \end{cases} \qquad (9)$$

Now LQR theory can be used to generate a state feedback for the augmented plant[1]. e_I is included in the augmented system to solve the tracking problem. The cost function that penalizes the new state and the integral of the error is as follows :

$$J = \int_0^{t_f} \left\{ x_a^T(t) Q x_a(t) + u^T(t) R u(t) \right\} dt, \qquad (10)$$

where $x_a = \left[x^T(t)\ x_{m_1}^T(t)\ e_I^T(t)\right]^T$, $Q = \begin{bmatrix} Q_x & 0 & 0 \\ 0 & Q_{x_{m_1}} & 0 \\ 0 & 0 & Q_{e_I} \end{bmatrix} \geq 0$ and $R > 0$. The control weighting matrix is usually generated by trial and error, but if $Q_{x_{m_1}}$ is larger than Q_{e_I}, the nonminimum phase system output shows small amount of undershoot at the price of long rising time, and *vice versa*. The optimal control is then

$$\begin{aligned} u(t) &= -K(t) \begin{bmatrix} x(t) \\ x_{m_1}(t) \\ e_I(t) \end{bmatrix} \\ &= -\begin{bmatrix} K_x(t) & K_{x_{m_1}}(t) & K_{e_I}(t) \end{bmatrix} \begin{bmatrix} x(t) \\ x_{m_1}(t) \\ e_I(t) \end{bmatrix}, \end{aligned} \qquad (11)$$

where $K(t) = \left[K_x(t)\ \ K_{x_{m_1}}(t)\ \ K_{e_I}(t)\right]$ is the optimal feedback gain matrix. This control can be written explicitly in terms of the new state and tracking error as follows :

$$\begin{aligned} u(t) = &-K_x(t)x(t) \\ &-K_{x_{m_1}}(t) \int_0^t \{y(t) - x_{m_1}(t)\} dt \\ &-K_{e_I}(t) \int_0^t \{r - y(t)\} dt, \end{aligned} \qquad (12)$$

which shows that the control includes the tracking error and the difference between $y(t)$ and x_{m_1} with an integral feedback.

3. General Nonminimum Phase System

Consider the system with p RHP real zeros and $2q$ RHP complex zeros as follows :

$$G(s) = \frac{B(s)}{A(s)} \prod_{i=1}^{p} (-z_i s + 1) \prod_{j=1}^{q} (-z_j s + 1)(-\bar{z}_j s + 1), \qquad (13)$$

where $A(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$, $B(s) = b_{n_b-1} s^{n_b-1} + b_{n_b-2} s^{n_b-2} + \cdots + b_1 s + b_0$, $n_b = n - p - 2q$, $z_i > 0$ and $real(z_j) > 0$. Note that the p RHP real zeros of the system are $1/z_1, 1/z_2, \cdots, 1/z_p$ and $2q$ RHP complex zeros of the system are $\{1/z_1, 1/\bar{z}_1\}$, $\{1/z_2, 1/\bar{z}_2\}$, $\cdots$, $\{1/z_q, 1/\bar{z}_q\}$. Let $G_m(s) = B(s)/A(s)$, then $G_m(s)$ is a minimum phase system because $B(s)$ is a stable polynomial. Hence Eq. (13) is then

$$G(s) = G_m(s) \prod_{i=1}^{p} (-z_i s + 1) \prod_{j=1}^{q} (-z_j s + 1)(-\bar{z}_j s + 1). \qquad (14)$$

Let us define the nonminimum phase system output $Y(s) = G(s)U(s)$ and the minimum phase system output $Y_m(s) = G_m(s)U(s)$. From Eq. (14), we can see that

$$\begin{aligned} Y(s) = &\ Y_m(s) + \alpha_1 s Y_m(s) + \cdots \\ &+ \alpha_{p+2q-1} s^{p+2q-1} Y_m + \alpha_{p+2q} s^{p+2q} Y_m(s), \end{aligned} \qquad (15)$$

where the coefficient $\alpha_1, \cdots, \alpha_{p+2q-1}, \alpha_{p+2q}$ can compute from Eq. (14). Thus time response of the nonminimum phase system represented by the linear combination of minimum phase system output and its $(p+2q)$th derivative as follows :

$$\begin{aligned} y(t) = &\ y_m(t) + \alpha_1 \dot{y}_m(t) + \cdots \\ &+ \alpha_{p+2q-1} y_m^{(p+2q-1)} + \alpha_{p+2q} y_m^{(p+2q)}(t). \end{aligned} \qquad (16)$$

Similarly as the system with one RHP real zero, we can make a new state $x_m(t) = \left[x_{m_1}(t)\ x_{m_2}(t)\ \cdots\ x_{m_{p+2q}} \right]^T$ as follows :

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m y(t) \\ &= A_m x_m(t) + B_m C x(t), \end{aligned} \qquad (17)$$

where $A_m = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ -1 & -\binom{p+2q}{p+2q-1} & -\binom{p+2q}{p+2q-2} & \cdots & -\binom{p+2q}{2} & -\binom{p+2q}{1} \end{bmatrix}$

and $B_m = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix}^T$. Note that the matrix A_m has repeated poles at $s = -1$ in the complex plane. And the integral of the error between the reference input and output is the same as the case of the system with one RHP real zero. Therefore the augmented state model is the combination of the system state equation, the new state equation and the state equation for the integral of the error as follows :

$$\begin{cases} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_m(t) \\ \dot{e}_I(t) \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ B_m C & A_m & 0 \\ -C & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_m(t) \\ e_I(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r \\ y(t) = \begin{bmatrix} C & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_m(t) \\ e_I(t) \end{bmatrix}. \end{cases} \qquad (18)$$

The cost function is

$$J = \int_0^{t_f} \left\{ x_a^T(t) Q x_a(t) + u^T(t) R u(t) \right\} dt, \qquad (19)$$

where $x_a(t) = \left[x^T(t)\ x_m^T(t)\ e_I^T(t)\right]^T$, $Q = \begin{bmatrix} Q_x & 0 & 0 \\ 0 & Q_{x_m} & 0 \\ 0 & 0 & Q_{e_I} \end{bmatrix} \geq 0$ and $R > 0$. The optimal control is then

$$\begin{aligned} u(t) &= -K(t) \begin{bmatrix} x(t) \\ x_m(t) \\ e_I(t) \end{bmatrix} \\ &= -\left[K_x(t)\ \ K_{x_m}(t)\ \ K_{e_I}(t) \right] \begin{bmatrix} x(t) \\ x_m(t) \\ e_I(t) \end{bmatrix} \\ &= -K_x(t)x(t) \\ &\quad -K_{x_m}(t) \textstyle\int_0^t \{A_m x_m(t) + B_m y(t)\}\, dt \\ &\quad -K_{e_I}(t) \textstyle\int_0^t \{r - y(t)\}\, dt, \end{aligned} \qquad (20)$$

where $K(t) = \left[K_x(t)\ \ K_{x_m}(t)\ \ K_{e_I}(t) \right]$ is the optimal feedback gain matrix.

4. Simulations

In this section, we deal with two examples to exemplify the usefulness of the proposed method. One is the case for the system with one RHP real zero, and another is the case for system with one RHP real zero and two RHP complex zeros.

4.1 System with One RHP Real Zero

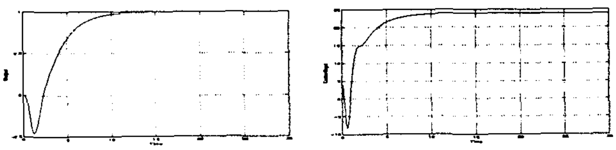
Consider the system with one RHP real zero as follows.

$$\begin{aligned} G(s) &= \frac{-2s+1}{(s+1)(s-2)(s+3)(s-4)} \\ &= \frac{-2s+1}{s^4 - 2s^3 - 13s^2 + 14s + 24}. \end{aligned} \qquad (21)$$

Note that the given system has poles at $s = -3, -1, 2, 4$ and zero at $s = 0.5$ in the complex plane. Thus the system occur large initial undershoot phenomenon because the RHP zero is nearer at the imaginary axis than poles and system with odd RHP zero. The system can be represented by the controllable canonical form as follows :

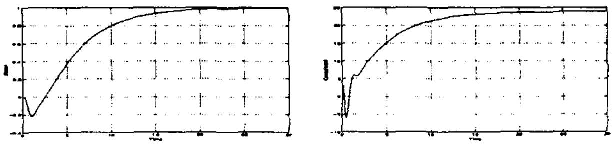
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -24 & -14 & 13 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad (22)$$
$$C = \left[\ 1\ \ -2\ \ 0\ \ 0\ \right], \qquad D = \left[\ 0\ \right].$$

Thus the augmented state space is described as follows :



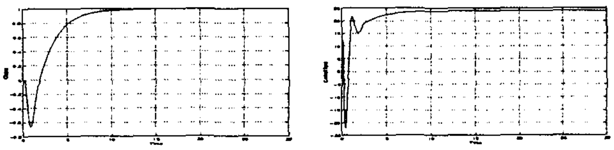
(a) Step Response (b) Control Input

Figure 1: $Q_{x_m} = Q_{e_I} = 1$



(a) Step Response (b) Control Input

Figure 2: $Q_{x_m} = 15$ and $Q_{e_I} = 1$



(a) Step Response (b) Control Input

Figure 3: $Q_{x_m} = 1$ and $Q_{e_I} = 15$

Table 1: The Closed Loop Poles and Zeros

	Closed Loop Poles	Closed Loop Zeros
$Q_{x_m} = 1$, $Q_{e_I} = 1$	-4.6418±1.1990j -1.9205±2.9448j -0.7091 -0.4964	-1.0000 0.5000
$Q_{x_m} = 15$, $Q_{e_I} = 1$	-5.5791±1.7557j -2.3136±4.2489j -0.5006 -0.2495	-1.0000 0.5000
$Q_{x_m} = 1$, $Q_{e_I} = 15$	-5.5635±1.7575j -2.2969±4.2700j -0.9682 -0.4998	-1.0000 0.5000

$$A_a = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -24 & -14 & 13 & 2 & 0 & 0 \\ 1 & -2 & 0 & 0 & -1 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (23)$$
$$B_a = \left[\ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \right]^T,$$
$$B_r = \left[\ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \right]^T,$$
$$C_a = \left[\ 1\ \ -2\ \ 0\ \ 0\ \ 0\ \ 0\ \right], \quad D_a = \left[\ 0\ \right].$$

Let us take $Q_x = 0$ and $R = 0.0001$. First, we perform the simulation in the case of $Q_{x_m} = Q_{e_I}$, and Fig.1 shows the output and control input trajectory. It is shown that the undershoot phenomenon is very large for expectation. Fig.2 shows output and control input in the case of $Q_{x_m} = 15$ and $Q_{e_I} = 1$ in order that undershoot phenomenon decrease at the price of large rising time. Also, Fig.3 shows the output and the control input in case of $Q_{x_m} = 1$ and $Q_{e_I} = 15$, which are taken to make the rising time short even though the undershoot is large. Table.1 shows approximation values of closed loop poles and zeros. From Table.1, we know that the system zero $s = 0.5$ is not change.

4.2 System with One RHP Real Zero and Two RHP Complex Zeros

Consider the system with one RHP real zero and two RHP complex zeros as follows.

$$\begin{aligned} G(s) &= \frac{(-2s+1)\{-(1+j)s+1\}\{-(1-j)s+1\}}{(s+1)(s-2)(s+3)(s-4)} \\ &= \frac{-4s^3 + bs^2 - 4s + 1}{s^4 - 2s^3 - 13s^2 + 14s + 24}. \end{aligned} \qquad (24)$$

Note that given system has poles at $s = -3, -1, 2, 4$ and zero at $s = 0.5,\ 0.5(1 \pm j)$. Thus the system has a large initial undershoot phenomenon because RHP zeros near the imaginary axis more than poles and system with odd RHP zeros. The controllable canonical form of the system is as follows :

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -24 & -14 & 13 & 2 \end{bmatrix},\quad B=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \tag{25}$$
$$C=\begin{bmatrix} 1 & -4 & 6 & -4 \end{bmatrix},\qquad D=\begin{bmatrix} 0 \end{bmatrix}.$$

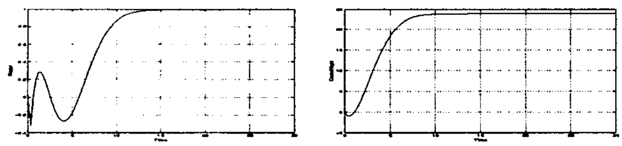
Thus augmented state space model has the coefficient matrix as follows :

$$A_a=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -24 & -14 & 13 & 2 & 0 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & -1 & -3 & -3 & 0 \\ -1 & 4 & -6 & 4 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{26}$$
$$B_a=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$B_r=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T,$$
$$C_a=\begin{bmatrix} 1 & -4 & 6 & -4 & 0 & 0 & 0 & 0 \end{bmatrix},\quad D_a=\begin{bmatrix} 0 \end{bmatrix}.$$

Let us take $Q_x = 0$ and $R = 0.0001$ like Section.4.1. First, we investigate in the case of $Q_{x_m} = Q_{e_I}$. Fig.4 gives output and control input in case of $Q_{x_m} = Q_{e_I} = 1$. It is shown that the undershoot phenomenon is very large for expectation. Fig.5 shows the output and the control input in case of $Q_{x_m} = 15$ and $Q_{e_I} = 1$, which says that the undershoot phenomenon decreases at the price of large rising time. Also, Fig.6 shows the output and the control input in case of $Q_{x_m} = 1$ and $Q_{e_I} = 15$, which says that the rising time becomes small even though the undershoot is large. Table.2 shows approximation value of closed loop system poles and zeros, and it says that the system zero $s = 0.5$, $0.5(1 \pm j)$ is not changed.

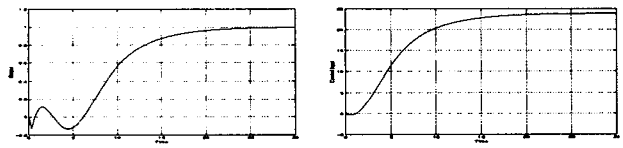
5. Conclusion

In this paper, we formulate a linear quadratic type problem to deal with the effects of the right half plane zeros in the



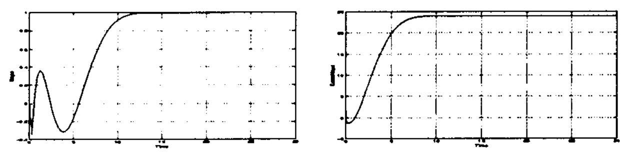
(a) Step Response (b) Control Input

Figure 4: $Q_{x_m} = Q_{e_I} = 1$



(a) Step Response (b) Control Input

Figure 5: $Q_{x_m} = 15$ and $Q_{e_I} = 1$



(a) Step Response (b) Control Input

Figure 6: $Q_{x_m} = 1$ and $Q_{e_I} = 15$

Table 2: The Closed Loop Poles and Zeros

	Close Loop Poles	Closed Loop Zeros
$Q_{x_m} = 1$ $Q_{e_I} = 1$	-17.0487±16.5917j -1.0038± 0.4289j -0.5020± 0.4996j -0.5956 -0.4963	0.5000±0.5000j -1.0000 -1.0000±0.0000j 0.5000
$Q_{x_m} = 15$ $Q_{e_I} = 1$	-28.4246±28.1451j -0.8842± 0.4988j -0.5004± 0.5001j -0.5003 -0.2421	0.5000±0.5000j -1.0000±0.0000j 0.5000
$Q_{x_m} = 1$ $Q_{e_I} = 15$	-28.4167±28.1527j -1.0680± 0.1991j -0.5002± 0.4999j -0.8203 -0.4999	0.5000±0.5000j -0.9827±0.0262j 0.5000

nonminimum phase systems. It is shown that there exists the trade-off relation between undershoot and rising time performances in nonminimum phase systems by using the new performance index with tracking error and new state. And, the usefulness of the proposed method is shown by two examples of the system with one RHP real zero and, with one RHP real zero and two RHP complex zeros. We can know that the RHP zero is not changed for feedback, and it is possible to design a controller which adjusts the trade-off relation between undershoot and rising time performance.

The global characterization of the effects for the RHP zeros upon the step response of the system, both real and complex, still remains as an open problem. Thus, it is difficult to control systems with RHP zeros. The global characterization of undershoot phenomenon will require the further research.

References

[1] J. B. Burl, *Linear Optimal Control, H_2 and H_∞ Methods*, Addison-Wesley Longman, Inc., 1999.

[2] G. F. Franklin, J. D. Powell and A. Emami-Naeini, *Feedback Control of Dynamic Systems, 2nd edition*, Addison-Wesley Pub., 1991.

[3] S. Hara, N. Kobayashi and T. Nakamizo, "Design of non-undershooting multivariable servo systems using dynamic compensators", *Int. J. of Control*, vol. 44, no. 2, pp. 331-342, 1986.

[4] I. Horowitz and Y. Liao, "Limitation of non-minimum-phase feedback systems", *Int. J. of Control*, vol.40, no. 5, pp. 1003-1013, 1984.

[5] I. Horowitz and M. Sidi, "Optimum synthesis of non-minimum phase feedback systems with plant uncertainty", *Automatica*, vol. 27, no. 2, pp. 281-292, Mar., 1991.

[6] B. A. León de la Barra, "On undershoot in SISO systems", *IEEE Trans. on Automat. Contr.*, vol. 39, no. 3, pp. 578-581, 3. 1994.

[7] R. H. Middleton, "Trade-offs in Linear Control System Design", *Automatica*, vol. 27, no. 2, pp. 281-292, Mar., 1991.

[8] T. Mita and H. Yoshida, "Undershooting phenomenon and its control in linear multivariable servomechanisms", *IEEE Trans. on Automat. Contr.*, vol. 26, no. 2, pp. 402-407, 1981.

[9] L. Qiu and E. J. Davison, "Performance limitations of non-minimum phase systems in the servomechanism problem", *Automatica*, vol. 29, no. 2, pp. 337-349, Mar., 1993.

[10] M. Vidyasagar, "On undershoot and nonminimum phase zeros", *IEEE Trans. on Automat. Cont.*, vol.31, no.5, p.440, May., 1986.