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A Study on the Structural Controllability of Chemical Processes Based on Relative Order Analysis

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Abstract

The control performance of a chemical process is determined by process structure as well as the performance of controllers. Therefore, the concept of "controllability" should be introduced in the early design stage. Structural information makes controllability assessment possible by giving insights on the pathways of disturbances in the process. In this study, a simple procedure to evaluate controllability is suggested to screen out design alternatives using relative order analysis and structural decomposition. The effectiveness of the proposed method was validated by comparing the results with the case of dynamical simulation.

1. Introduction

Chemical processes are subject to various uncertainties during operations such as changes in operating conditions, feedstock, and product specifications. The objective of control is to keep such processes safe and economical. However, the control performance of a process is determined by the process structure itself as well as controller. Controllability is defined as a capability of process to operate economically, and, at the same time, safely without violating constraints to achieve various design objectives in the presence of uncertainties. Many works have been done to develop the controllability assessment techniques, but the methods for designing process with better controllability or improving it lack in literature [9].

As controllability is an inherent property of process itself, it should be considered at the design stage before control system design is fixed. The most preferable way to consider controllability at the design stage is to include controllability as one of the design objectives as well as traditional economic ones in design optimization problem. However, many controllability assessment methods and indices are based on and defined in different domains from those of design problems. To make matters worse, chemical processes are usually highly nonlinear, complex, and multi-variable systems. In addition, the chemical processes tend to have complex structure due to the reasons such as heat integration and recycle streams.

The methods to evaluate controllability are complicated and the required information is usually missing or unknown in the early design stage. This is one of the prominent reasons why controllability is not considered when designing chemical processes. Therefore, it is highly beneficial to research methods to evaluate and/or enhance controllability easily during the design stage.

Controllability evaluation methods may be classified into three by the model used in the evaluation procedure: steady-state model, linear dynamic model, and nonlinear dynamic model. Another way to evaluate controllability quantitatively and explicitly is performing dynamic simulations of the target process. However, these methods have disadvantages. Nonlinear process characteristics may be lost in the linearization, while nonlinear dynamic model is not utilized in evaluating controllability directly. Detailed dynamic simulation requires detailed information on the system, which is not known until the late part of design stage. There are other quantitative controllability measures such as RGA (relative gain array), Nierderlinski index, singular value analysis, condition number, etc. RGA is widely used as a controllability index of steady-state systems, but it cannot be effectively applied to dynamic systems for evaluating controllability.

Structural information makes controllability assessment possible by giving insights into the pathways of disturbances in the process, and this concept is used to define "structural controllability" [6]. When analyzing structural controllability, information such as operating conditions, process variables, and detailed simulation models is not required, which is used in dynamic simulation. Therefore, it is possible to evaluate controllability with structural analysis in the early design stage. By doing so, it is also possible to rank design alternatives and eliminate uncontrollable design alternatives. Thus, the number of design alternatives to consider can be reduced.

Structural analysis has been used to synthesize control structure of a linear system and to evaluate its feasibility. The methods used for the structural analysis are based on the structural controllability suggested by Lin et al. [7], and use graph theory. The advantages of these methods are that the results are not system-specific, and that the required information is not extensive.

In this study, a simple procedure to evaluate controllability is suggested to screen out design alternatives using relative order analysis and structural decomposition.

2. Structural Controllability

If each flow in a process is not interconnected and independent, then the disturbance in a flow does not propagate through other parts of the process, and thus this process is controllable. However, such a process is very unprofitable, and therefore is not designed. It is common practice to make process topology complex to maximize the heat recovery of process flows, etc. Therefore, it is difficult to evaluate controllability in the design stage intuitively, and the finally designed processes may be found to be impossible or very difficult to control.

The existence of complex interconnection between process flows implies that there are many propagation paths for disturbances. Such diverse propagation paths may affect controllability of a process adversely. This relationship between controllability and process flows enables the evaluation of process controllability by studying process streams and/or structures. Structural controllability is the controllability depending on the process structure. Structural controllability was defined by Lin et al. as follows [7].

(i) A process is structurally controllable if a disturbance does not propagate into other parts of the process.

(ii) Structural controllability is good if a undesirable disturbance does not propagates.

Structural analyses have been widely used to synthesize control structures of linear processes and their feasibility. To this end, graph theory is usually used.

3. Digraph of a Chemical Process

Process dynamic model of a chemical process can be described with the following model.

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

Alternatively, state space model of a chemical process can be represented with a digraph. The digraph in Fig. 1 describes a dynamic system with the following structural model.

$$
\begin{aligned}
\dot{x}_1 &= f_1(x_1, x_2, x_3) + g_1(x)u \\
\dot{x}_2 &= f_2(x_1, x_2) \\
\dot{x}_3 &= f_3(x_1, x_2, x_3) + g_2(x)d \\
y &= h(x_2)
\end{aligned}
\tag{2}
$$

The definition of the relative order is as follows: the relative order r_{ij} of the output y_i with respect to a manipulated input u_j is defined as the smaller integer for which

$$L_{g_j} L_f^{r_{ij}-1} h_i(x) \neq 0 \tag{3}$$

or $r_{ij} = \infty$ if such an integer does not exist.

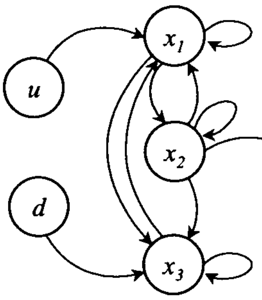


Fig. 1. A typical digraph.

There exists the following relationship between r_i and r_{ij}.

$$r_i = \min(r_{i1}, r_{i2}, \cdots, r_{im}) \tag{4}$$

where r_I is the relative order of the output y_i with respect to the manipulated input vector u.

The relative order r_{ij} between two variables is found to follow the following relationship with the shortest length of path l_{ij} by Daoutidis and Kravaris [2].

$$r_{ij} = l_{ij} - 1 \tag{5}$$

The relative order quantifies how "direct" the effect of an input variable is on an output variable, and "physical closeness" of these two variables [10].

Relative order matrix M_r is defined as

$$M_r = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mm} \end{bmatrix} \tag{6}$$

As chemical processes are modeled with differential algebraic equations (DAEs), it should be converted into state space model to evaluate structural controllability. The number of differentiation required by a DAE to transform into an ordinary differential equation is called 'index'. As modeling techniques advance, many chemical processes are modeled with high-index DAEs. Systems with index 1 are easily transformed into equivalent ODE systems, and therefore are easy to analyze and design control structure. In case of high-index systems, it is impossible to solve them directly with respect to algebraic variables and additional constraints existing among the variables. By using Hirschorn's inversion algorithm [3], transforming high-index system into state space model is possible [4].

4. Characteristic Matrix and Relative Order Matrix

The characteristic matrix *C(x)* is defined by

$$C(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \vdots & & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix} \tag{7}$$

As $L_{g_j} L_f^k h_i(x)$ is independent of coordinate transformation, characteristic matrix depends only on the input/output characteristics of the system.

The Lie derivative L is required to obtain characteristic matrix, and, in turn, obtaining characteristic matrix takes much calculation and is difficult to automate. In this study, a simple procedure is suggested using relative order matrix.

When evaluating controllability, only the structural matrix of characteristic matrix is required. In addition, functional relations may not be known clearly in the early design stage. However, from the observation of the definitions of characteristic matrix and the relative order, it is possible to obtain structural matrix of the characteristic matrix.

For example, the characteristic matrix of a system with the given relative order matrix can be represented as follows.

$$M_r = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & \infty \\ \infty & 3 & 2 \end{bmatrix} \Rightarrow C(x) = \begin{bmatrix} x & x & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix} \tag{8}$$

As relative order matrix indicates how much a system is decoupled, the following procedure is used to compare the design alternatives [5].

(i) Obtain the relative order matrices and the characteristic matrices of design alternatives.

(ii) Rearrange relative order matrices so that the smallest elements in a row are diagonally located, and calculate the sum of the diagonal elements.

(iii) If there are more than two alternatives with the same sum of diagonal elements, then calculate decoupling index $DI = \sum_i \sum_{j \neq i} r_i / r_{ij}$ and select the one with smallest DI.

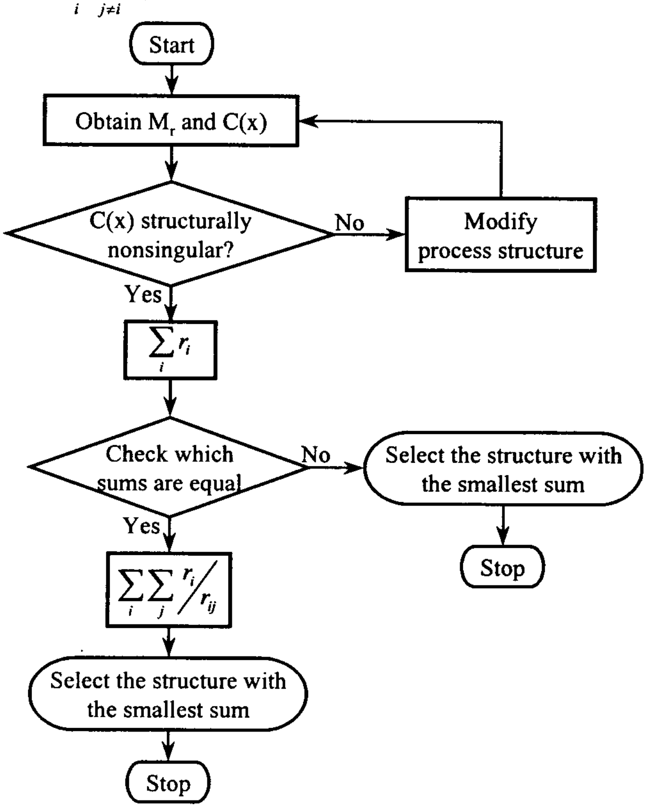


Fig. 2. Structural controllability evaluation procedure.

5. Case Study

Let us consider the water-ethanol separation system. This process was studied by Chiang and Luyben to evaluate controllability of heat-integrated distillation columns [1].

By using ideal binary distillation column, the column can be described with the following structural relationships. Some basic assumptions are used in the ideal distillation column. Relative volatility is assumed to be constant; heat loss and changes in temperature are negligible; vapor flow is constant throughout the column [8].

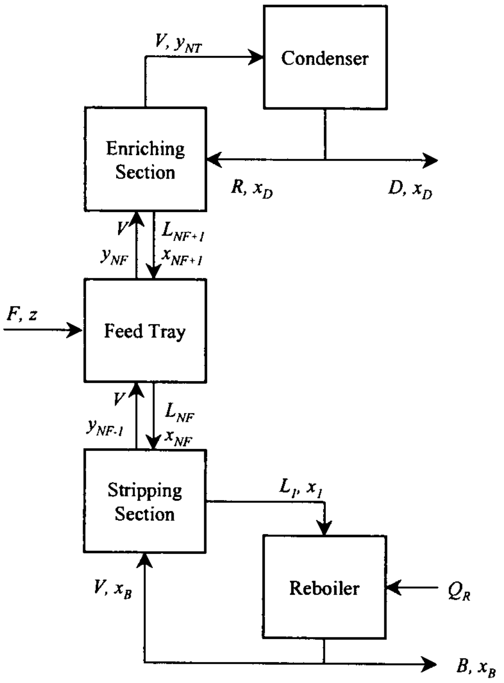


Fig. 3. Ideal binary distillation column.

Condenser and reflux drum:

$$M_D' = f(V, R, D) \qquad (9)$$

$$(M_D x_D)' = f(V, R, D, y_{NT}, x_D)$$

Top tray:

$$M_D' = f(R, L_{NT}) \qquad (10)$$

$$(M_{NT} x_{NT})' = f(R, L_{NT}, V, y_{NT-1}, y_{NT}, x_{NT})$$

n-th tray:

$$M_n' = f(L_{n+1}, L_n) \qquad (11)$$

$$(M_n x_n)' = f(L_{n+1}, L_n, V, y_{n-1}, y_n, x_{n+1}, x_n)$$

Feed tray (n=N_F)

$$M_n' = f(L_{NF}, L_{NF}, F) \qquad (12)$$

$$(M_{NF} x_{NF})' = f(L_{NF+1}, L_{NF}, V, F, z, y_{NF}, y_{NF-1}, x_{NF+1}, x_{NF})$$

Bottom tray (n=1):

$$M_1' = f(L_2, L_1) \qquad (13)$$

$$(M_1 x_1)' = f(L_2, L_1, V, y_1, y_B, x_2, x_1)$$

Reboiler:

$$M_B' = f(L_1, V, B) \qquad (14)$$

$$(M_B x_B)' = f(L_1, V, B, y_B, y_1, x_1, x_B)$$

Vapor-liquid equilibrium:

$$y_n = f(x_n) \qquad (15)$$

Tray hydraulic equation:

$$L_n = f(M_n) \qquad (16)$$

Level controllers:

$$B = f(M_B) \qquad (17)$$

$$D = f(M_D)$$

Feedback controllers

$$R = f(x_D) \qquad (18)$$

$$V = f(Q_R)$$

These structural relationships between variables can be represented with a digraph (Fig. 4). As this structural relationship is the same for a binary system, it can be duplicated for other binary system.

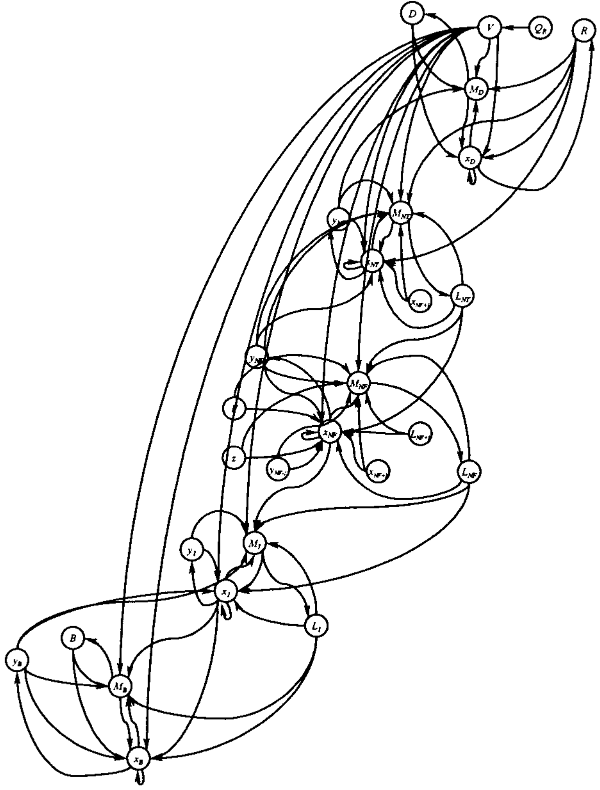
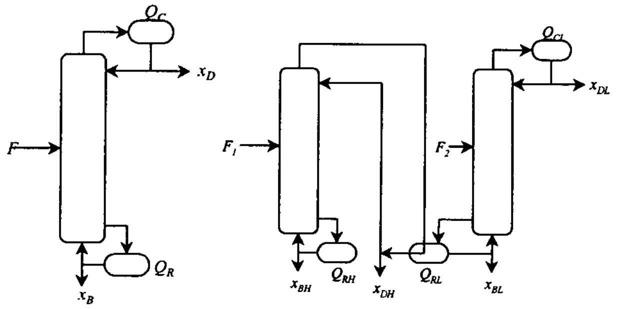


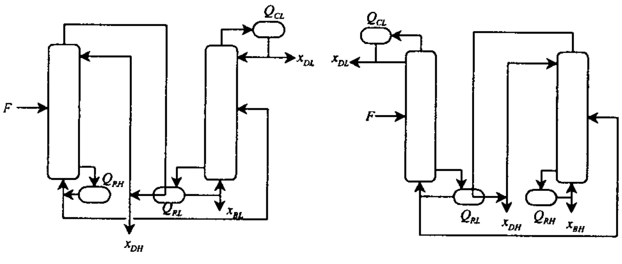
Fig. 4. Digraph of ideal binary distillation column.

The following heat-integrated structures shown in Fig. 5 are considered for the controllability evaluation.



(a) Single Distillation Column

(b) Feed Split



(c) Light Split/Forward

(d) Light Split/Reverse

Fig. 5. Structures of Heat-Integrated Columns.

The pairings of controlled and manipulated variables are as given in Table 1.

Configuration	Controlled Variable	Manipulated Variable
Single Column	x_D, x_B	R, Q_R
FS	x_{DH}, x_{BH}, x_{DL}, x_{BL}	R_H, Q_{RH}, R_L, F_1/F_2
LSF	x_{DH}, x_{DL}, x_{BL}	R_H, R_L, Q_{RH}
LSR	x_{DL}, x_{DH}, x_{BH}	R_L, R_H, Q_{RH}

Table 1. Pairings of controlled and manipulated variables.

The digraphs of FS, LSR, and LSF structures can be drawn easily by decomposing and modifying the digraph of single distillation column. Relative order matrix of each configuration is obtained by analyzing its digraph.

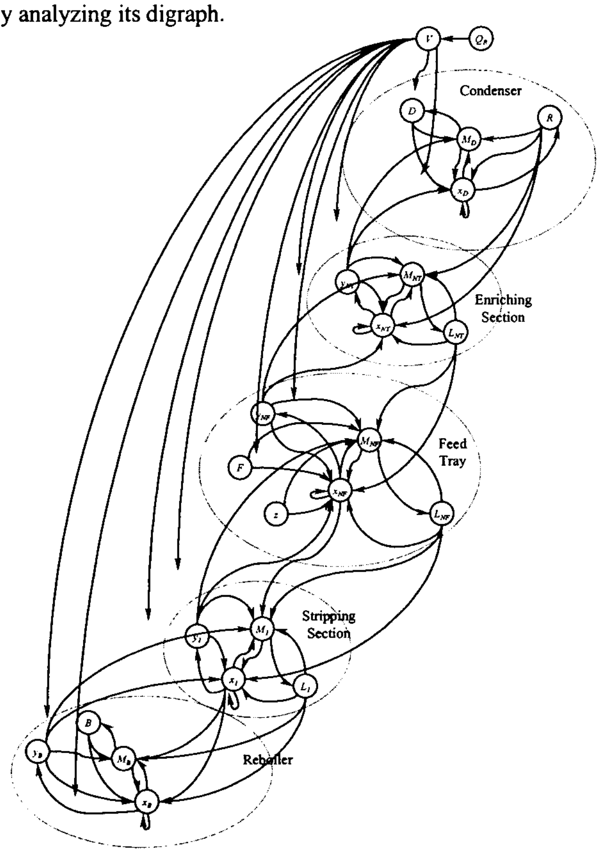


Fig. 6. Decomposed digraph of single distillation column.

Sum of the relative order and DI of each design alternative are as in Table 2.

Configuration	Σr_i	DI
Single Column	3	-0.833
FS	8	9.650
LSF	4	-1.458
LSR	4	-1.307

Table 2. Calculation results of Σr_i and DI

The single column has the smallest Σr_i and the smallest DI, while FS configuration has the largest values of Σr_i and DI. Therefore, the single distillation column is expected to have the best control characteristics, and FS configuration is the most difficult to control. LSF and LSR configurations are expected to show similar dynamic behavior. Chiang and Luyben used dynamic simulation to compare the four configurations, and their study showed the identical results. As dynamic simulation requires much more information in detail than structural information, the proposed method can be useful to screen out process structures with undesirable control performance.

6. Conclusions

A new procedure to evaluate controllability is suggested to select design alternatives. The proposed method uses only structural information, and can be used at the early step in design stage. It is useful in screening out the design alternatives using only structural information, and therefore can save effort before using detailed controllability evaluation methods such as dynamic simulation, which requires very detailed information that may not be available in the early design stage. The result shows good agreement with the previous work by Chiang and Luyben [1].

Acknowledgements

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