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Cartesian Space Nonlinear PD Control for the Multi-Link Flexible Manipulators

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Abstract

There have been many control strategies for the exact joint position tracking of flexible manipulators, but direct cartesian space tracking control methods are not developed well. In this paper, we propose a PD control method based on the cartesian error in the end point trajectory tracking. The proposed conroller is composed of PD control combined with nonlinear saturation term but has a very simple form. The effect of this term is continuous suppression of vibration which is induced by the coupling of rigid motion. This control works both on the regulation and on the tracking cases. The performance and validity of this control method is shown by simulation examples.

1 Introduction

In the past few decades, numerous attempts have been made on the control of structurally flexible manipulators due to the requirements on the light weight arm. Unfortunately, such flexible manipulators always incorporate vibration which limits the global performance.

Various control methods have been tested for the control of flexible manipulators since the initial experiments using linear quadratic approach by Cannon[4]. The inverse dynamic control [2, 3, 8, 9] or similarly inversion technique [7, 11, 15] was studied to exactly follow a preplanned output trajectory using feedforward joint torque. Naturally, the inverse dynamics shows unstable nature due to the nonminimum phase property in the nonlinear setting. To cope with this problem, plant was divided into stable and unstable parts and unstable parts are properly handled to give a stable joint solution trajectory in the time domain[3, 9] or in the frequency domain[8]. Iterative feedforward torque calculation using virtual link was done by Asada[2]. Another technique is the feedback of slight modified output[5, 11, 14] which makes the closed loop system stable or passive. Since such feedback information is not the exact output, there is a little performance degradation. The stability of this feedback was verified by investigating the eigenvalues of zero dynamics. In [5], Damaren showed the nonlinear passivity result between the μ tip rate and input torque considering the special dynamic and kinematic conditions. The singular perturbation approach was also studied in [10, 13].

Most of works mentioned above are mainly focused on the joint based control schemes and only a few works that challenge in the direct cartesian space methods can be listed[5, 14]. This may come from the fact that the establishment of cartesian space dynamics is very hard. Also, even if we can obtain the cartesian dynamics, the direct design of stable cartesian control is nearly impossible due to the cartesian force coupling by Jacobian relation. In this paper, we will present a nonlinear PD type cartesian space set point and trajectory tracking controller combining vibration suppression scheme without loosing collocation feedback. Since this control does not require any dynamic inversion or iterative schemes, the implementation is so simple.

This paper is organized as follows; in section 2, some definitions and problems are given and regulation and tracking control is proposed in section 3. Conclusion is in section 4.

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2 Problem Statement

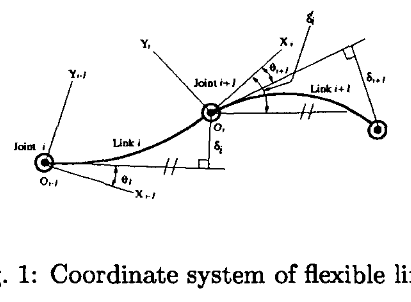

Fig. 1: Coordinate system of flexible link

Fig.1 shows the schematics of flexible link coordinate system for the successive two links. Using separation of variables and truncation of higher order terms, the vibration can be written as

$$\delta_i(r,t) = \sum_{i=1}^{\infty} \phi_{i,k}(x)v_{i,k}(t) \approx \sum_{i=1}^{m_i} \phi_{i,k}(x)v_{i,k}(t), \qquad (1)$$

where $\phi(x)$ and $v(t)$ are mode shape and time function, respectively, and the index i and k denote link number and mode number. Then the generalized coordinate of this system can be constructed as

$$q = [\theta_1 \cdots \theta_n \; v_{1,1} \cdots v_{1,m_1} \cdots v_{n,m_n}]^T = \left[\theta^T \; v^T\right]^T,$$

where $\theta \in \mathbb{R}^n$ and $v \in \mathbb{R}^{(m=\sum_{i=1}^n m_i)}$ are rigid joint angle and flexible variable, respectively. According to the kinematic relation, we can write as

$$x = h(\theta, v) = h(q), \quad \dot{x} = J_\theta \dot{\theta} + J_v v = J\dot{q},$$
$$\ddot{x} = J_\theta \ddot{\theta} + \dot{J}_\theta \dot{\theta} + J_v \ddot{v} + \dot{J}_v \dot{v} = J\ddot{q} + \dot{J}\dot{q}$$

where $x \in \mathbb{R}^r$ and $J \in \mathbb{R}^{r \times (n+m)}$ are the cartesian position and kinematic Jacobian, respectively. According to Lagrangian dynamic formalism, the equations of motion can be compactly written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D\dot{q} + Kq + G(q) = B\tau, \qquad (2)$$

where M, C, D, K, G and B are the inertia, Coriolis and centripetal, viscous damping, stiffness, gravity and input matrices of appropriate dimension, respectively. The Coriolis and centripetal matrix C is chosen to satisfy the skew symmetric property such as $\dot{q}^T(\dot{M} - 2C)\dot{q} = 0$. Various dynamic and mathematical properties related above equation are given in [1]. If we rewrite Eq.(2) in detail, then,

$$\begin{bmatrix} M_{rr} & M_{rf} \\ M_{fr} & M_{ff} \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{v} \end{bmatrix} + \begin{bmatrix} C_{rr} & C_{rf} \\ C_{fr} & C_{ff} \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ \dot{v} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & D_{ff} \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ \dot{v} \end{bmatrix} +$$
$$\begin{bmatrix} 0 & 0 \\ 0 & K_{ff} \end{bmatrix}\begin{bmatrix} \theta \\ v \end{bmatrix} + \begin{bmatrix} g_r \\ g_f \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix}, \qquad (3)$$

where r and f denote rigid and flexible parts. We assume that the viscous damping term exists only in the flexible part. If we consider the imaginary rigid link robot which produces no vibration, the kinematic relation can be written as

$$x_0 = h(\theta, 0), \quad \dot{x}_0 = J_\theta \dot{\theta}, \quad \ddot{x}_0 = J_\theta \ddot{\theta} + \dot{J}_\theta \dot{\theta}.$$

Similarly, by fixing all the flexible coordinates to zero ($v = \dot{v} = \ddot{v} = 0$), we can obtain the cartesian coordinate dynamics[6] such that

$$\Lambda \ddot{x} + \Gamma \dot{x} + \Pi = f, \qquad (4)$$

where

$$\Lambda \triangleq J_\theta^{-T} M_{rr} J_\theta^{-1}, \; \Gamma \triangleq J_\theta^{-T}\left(C_{rr} - M_{rr} J_\theta^{-1} \dot{J}_\theta\right) J_\theta^{-1}$$
$$\Pi \triangleq J_\theta^{-T} g_r, \quad f \triangleq J_\theta^{-T} \tau.$$

Whenever one wants to perform the output tracking, especially in the flexible manipulator, it will be better to control the errors defined in the cartesian space besides the availability of the error state. The cartesian error can be obtained using the strain gauge signals. This direct cartesian control will also save the efforts on the calculation of inverse dynamics that usually requires very complex procedures and time-consuming iteration. In the following section, we will present the cartesian space PD control with vibration damping. When the link is also flexible in the gravity force direction, the equilibrium position for given joint angle θ has always static deflection that equilibrates the following algebraic equation[12].

$$K_{ff} v + g_f(\theta, v) = 0.$$

To avoid this configuration-dependent static deflection problem, we will assume that the manipulator motions and vibrations are allowed only in the perpendicular direction to gravity ($G(q) = 0$). This simplification does not lose the validity of our results and one can easily generalize them.

3 Control Strategy

As was mentioned in the previous section, the direct cartesian design of controller has some advantages over the joint space control schemes. In this section, we will propose a nonlinear PD control which involves vibration damping. This method works both on the end effector regulation and on trajectory tracking problems.

First of all, we must define a vector saturation function $\overline{S}(a, \epsilon b)$. The arguments a and b are r dimensional vectors and $\epsilon > 0$ is a scalar constant smaller than unity. $\overline{S}(a_i, \epsilon b_i)$ denotes the i-th element of $\overline{S}(a, \epsilon b)$. Then,

$$\overline{S}(a_i, \epsilon b_i) = \begin{cases} \mathrm{sgn}(a_i)\epsilon|b_i| & \text{if } |a_i| > \epsilon|b_i| \\ a_i & \text{if } |a_i| \le \epsilon|b_i| \end{cases} \quad i = 1 \cdots r.$$

<u>**Theorem 1**</u> **(Regulation)** Consider the following control law

$$\tau = J_\theta^T\left[K_p e_0 + K_p \overline{S}(\tilde{e}, \epsilon e_0) - K_v J_\theta \dot{\theta}\right], \qquad (5)$$

where e_0, e and $\tilde{e}$ represent rigid end point error, real end point error and their difference, respectively, in the task space defined by

$$e_0 = x_0 - x_d, \quad e = x - x_d,$$
$$\tilde{e} = e_0 - e = x - x_0.$$

If the gain matrices K_p and K_v are diagonal positive definite, then, $x = x_d$, $\dot{x} = 0$, and $v = \dot{v} = 0$ are the globally asymptotically stable equilibrium point of closed loop system (2) and (5).

<u>***Proof***</u> Consider the following Lyapunov candidate function.

$$V(e_0, v, \dot{q}) = \frac{1}{2}\dot{q}^T M \dot{q} + \frac{1}{2} v^T K_{ff} v + \frac{1}{2} e_0^T K_p e_0 + \int_0^t \dot{e}_0^T K_p \overline{S}(\tilde{e}, \epsilon e_0)\, dt.$$

Firstly, let's show the positive definiteness of V. If we assume that initial cartesian error is zero ($e_0(0) = 0$), the last two terms can be written as

$$\begin{aligned} V_{e_0} &= \frac{1}{2} e_0^T K_p e_0 + \int_0^t \dot{e}_0^T K_p S(\widetilde{e},\ \epsilon e_0)\ dt \\ &= \sum_{i=1}^{r} \int_{e_{0,i}(0)}^{e_{0,i}(t)} K_{p,i} \left[e_{0,i} + \overline{S}(\widetilde{e}_i,\ \epsilon e_{0,i}) \right] de_{0,i} \end{aligned}$$

where the index i represents the ith element. From the definition of saturation function $\overline{S}(\widetilde{e}_i,\ \epsilon e_{0,i})$, V_{e_0} is bounded such that

$$0 \le \frac{1-\epsilon}{2} e_0^T K_p e_0 \le V_{e_0} \le \frac{1+\epsilon}{2} e_0^T K_p e_0.$$

Therefore, the chosen Lyapunov candidate function is positive definite. If we take time derivative along the closed loop system (2) and (5), then,

$$\begin{aligned} \dot{V}(\dot{x}, \dot{v}, e_0, v) &= \dot{q}^T M \ddot{q} + \frac{1}{2} \dot{q}^T \dot{M} \dot{q} + \dot{v}^T K_{ff} v \\ &\quad + \dot{e}_0^T K_p \left[e_0 + \overline{S}(\widetilde{e},\ \epsilon e_0) \right] \\ &= -\dot{\theta}^T J_\theta^T K_v J_\theta \dot{\theta} - \dot{v}^T D_{ff} \dot{v}\ \le\ 0 \end{aligned}$$

In the above derivation, we implicitly used the skew symmetric property of $\dot{M} - 2C$. Invoking LaSalle's invariant set theorem, the single point $(x_0, v, \dot{q}) = (x_d, 0, 0)$ is globally asymptotically stable equilibirum. Since, whenever $v \equiv 0$, x_0 is same as x, this concludes our proof. ■

In general, the trajectory tracking control requires both feedforward and feedback torques. Using Eq.(4), cartesian feedforward torque can be generated without iterative dynamic inversion.

Theorem 2 **(Tracking)** Consider the following control law

$$\begin{aligned} \tau = J_\theta^T \left[\Lambda \ddot{x}_d + \Gamma \dot{x}_d + K_p e_0 + \right. \\ \left. K_p \overline{S}(\widetilde{e},\ \epsilon e_0) + K_v \dot{e}_0 \right]. \end{aligned} \tag{6}$$

If the gain matrix K_p and K_v are diagonal positive, all the system states $e, \dot{e}, v$ and $\dot{v}$ are globally uniformly bounded in the closed loop system (2) and (6).

Proof Consider the Lyapunov candidate function as

$$\begin{aligned} V(e_0, v, \dot{s}) = &\ \frac{1}{2} \dot{s}^T M \dot{s} + \frac{1}{2} v^T K_{ff} v + \frac{1}{2} e_0^T K_p e_0 + \\ &\int_0^t \dot{e}_0^T K_p \overline{S}(\widetilde{e},\ \epsilon e_0)\ dt, \end{aligned}$$

where s denotes the joint space tracking error which is defined by

$$s = \begin{bmatrix} s_\theta \\ s_v \end{bmatrix} = \begin{bmatrix} \theta_d - \theta \\ -v \end{bmatrix}.$$

Since the desired flexible variables v_d and $\dot{v}_d$ are set to zero, the desired joint solution then can be given by

$$\begin{aligned} &\theta_d = h^{-1}(x_d),\ \ \dot{\theta}_d = J_\theta^{-1} \dot{x}_d, \\ &\ddot{\theta}_d = J_\theta^{-1} \left(\ddot{x}_d - \dot{J}_\theta J_\theta^{-1} \dot{x}_d \right). \end{aligned}$$

Although the desired joint space solutions were defined above, we do not have to calculate them in the real implementation provided that we have the knowledge of the explicit form of Λ and Γ. As was in the regulation case, this Lyapunov candidate function is surely a positive one. Plugging the control law (6) into the dynamic equation (2), the closed loop system becomes

$$M\ddot{s} + C\dot{s} + D\dot{s} + Ks = \begin{bmatrix} -\tau_{fb} \\ M_{fr}\ddot{\theta}_d + C_{fr}\dot{\theta}_d \end{bmatrix}, \tag{7}$$

where τ_{fb} is the feedback torque term defined by

$$\tau_{fb} = J_\theta^T \left[K_p e_0 + K_p \overline{S}(\widetilde{e},\ \epsilon e_0) + K_v \dot{e}_0 \right].$$

Taking the time derivative of V along Eq.(7), then,

$$\begin{aligned} \dot{V} &= \dot{s}^T M \ddot{s} + \frac{1}{2} \dot{s}^T \dot{M} \dot{s} + \dot{v}^T K_{ff} v + \\ &\quad \dot{e}_0^T K_p e_0 + \dot{e}_0^T K_p \overline{S}(\widetilde{e},\ \epsilon e_0) \\ &= -\dot{s}_\theta^T J_\theta^T K_v J_\theta \dot{s}_\theta - \dot{s}_v^T D_{ff} \dot{s}_v + \\ &\quad \dot{s}_v^T (M_{fr}\ddot{\theta}_d + C_{fr}\dot{\theta}_d). \end{aligned}$$

Whenever the motion trajectory is sufficiently smooth and the rigid joints follow the desired trajectories well enough, we can assume that $\dot{\theta}_d \approx \dot{\theta},\ \ \ddot{\theta}_d \approx \ddot{\theta}$. Then,

$$\begin{aligned} \eta &= \int_0^t \left[\dot{s}_v^T (M_{fr}\ddot{\theta}_d + C_{fr}\dot{\theta}_d) \right] dt \\ &\approx \int_0^t \left[\dot{s}_v^T (M_{fr}\ddot{\theta} + C_{fr}\dot{\theta}) \right] dt \\ &= \int_0^t \left[\dot{v}^T (M_{ff}\ddot{v} + C_{ff}\dot{v} + D_{ff}\dot{v} + K_{ff} v \right] dt \\ &= \mathcal{T}_f(t) + \mathcal{P}_f(t) - \mathcal{T}_f(0) - \mathcal{P}_f(0) + \int_0^t \left[\dot{v}^T D_{ff} \dot{v} \right] dt \\ &\le \mathcal{T}_f(t) + \mathcal{P}_f(t) + \int_0^t \left[\dot{v}^T D_{ff} \dot{v} \right] dt, \end{aligned}$$

where the $\mathcal{T}_f$ and $\mathcal{P}_f$ are kinetic and potential energy due to vibration such that

$$\mathcal{T}_f = \frac{1}{2} \dot{v}^T M_{ff} \dot{v},\ \ \mathcal{P}_f = \frac{1}{2} v^T K_{ff} v$$

using the skew symmetric property of $\dot{M}_{ff} - 2C_{ff}$. Since $\dot{\theta}_d$ and $\ddot{\theta}_d$ are bounded and energy is continuously dissipated by viscosity, the flexible variable $v,\ \dot{v}$ and $\ddot{v}$ are bounded and consquently so are its kinetic and potential energy. Therefore, we get the conclusion that

$$\eta = \int_0^t \left[\dot{s}_v^T (M_{fr}\ddot{\theta}_d + C_{fr}\dot{\theta}_d) \right] dt < \infty\ \ \forall t \ge 0\ .$$

Since the kinetic energy $V_{\dot{s}} = \frac{1}{2} \dot{s}^T M \dot{s}$ satisfies

$$\lambda_{min}(M) \parallel \dot{s} \parallel^2 \le V_{\dot{s}} \le \lambda_{max}(M) \parallel \dot{s} \parallel^2,$$

we can write

$$\dot{V} \le -\gamma V_{\dot{s}} + \dot{s}_v^T (M_{fr}\ddot{\theta}_d + C_{fr}\dot{\theta}_d)$$

for some $\gamma > 0$. Integrating the above equation,

$$V(t) - V(0) \leq -\gamma \int_0^t V_{\dot{s}} dt + \eta, \qquad (8)$$

which implies the global uniform boundedness of V and thus $V_{\dot{s}}$. Then,

$$\gamma \int_0^t V_{\dot{s}} dt \leq V(0) + \eta$$

due to the positive definiteness of V. Since $V_{\dot{s}}$ is uniformly bounded and also uniformly continuous, it follows from the Barbalat's lemma that $\lim_{t\to\infty} V_{\dot{s}}(t) = 0$, which means that $\lim_{t\to\infty} \| \dot{s} \| = 0$. From (8), all the states are uniformly bounded as much as to satisfy the following inequality.

$$V(t) \leq V(0) + \eta \;\; \forall t > 0$$

Therefore, we come to a conclusion that the proposed control law (6) guarantees the global uniform bounded stability of Eq.(2). ■

REMARKS 3.1 *Although the saturation term $\overline{S}(\tilde{e}, \epsilon e_0)$ does not play any crucial role in the stability proof of Theorem 1 or 2, it improves the capability for the suppression of vibration induced by the coupling of rigid motion. Since the $\tilde{e}$ is the difference from real tip position to imaginary rigid tip position, by the addition of $\tilde{e}$ in the feedback loop, the overall motion is commanded to simultaneously follow the desired trajectory and regulate the concurrent vibration. Allowing the saturation of $\tilde{e}$, we can guarantee the closed loop stability same as that of pure PD control.*

4 Concluding Remarks

In this paper, we proposed a cartesian based nonlinear PD control. Actually, in the flexible manipulators, we had a very limited usage of PD control due to the lack of rigorous stability proof for the tracking problems. Our results, in view of this fact, can enlarge the usage of PD control without any hindrance. The stability and performance were tested and verified by simulation examples. The structure of this control law is so simple that one can easily implement in the real system. Another advantage in real implementation is that it does not require the derivative of strain gauge signals for the vibration suppression. Due to the high capability of vibration reduction, this scheme can also be used in parallel with any other control schemes which are short of this damping chracteristic.

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