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Robust Parallel Compensator Design for Static Output Feedback Stabilization of Plants with Multiple Uncertainty

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Abstract

This paper presents a design scheme of robust parallel compensator for plants with multiple uncertainty, which realizes strict positive realness of the closed-loop system by using static output feedback. Further, an approximate relation between the static output feedback control system with the proposed compensator and the $PID \cdots D^{\gamma-1}$ control system is shown.

1. Introduction

For ensuring asymptotic stability of the control system, strict positive realness (SPRness) condition is often required. In this paper we discuss the design problem of a robust parallel compensator (RPC) such that the closed-loop system realizes SPRness by using static output feedback (SOF). That is, we discuss the existence of an RPC which ensures the desired SOF making the closed-loop system SPR.

In general, it is difficult to guarantee the existence of a proper (or strictly proper) stable controller to make the closed-loop system SPR for most real plants. Addressing the limitation, three approaches have been developed. The first approach is the adaptive parallel compensation scheme described by Mareels [1] in which the compensator is constructed by adaptive high gain. The second one is the backstepping method described by Krstic *et al.* [2] in which a new stabilizing function and a new tuning function are defined. The third one presents a relatively simple design scheme based on a fixed parallel compensator. That is, parameter estimation and adaptive observer are not required. This paper focuses on the third approach. Bar-Kana [3] suggested that the non-SOF plant could be made virtually SOF-able by implementing a parallel compensator on the plant and gave a design scheme of the compensator. However, the method requires *a priori* knowledge of the dynamic feedback compensator which stabilizes the plant, and concrete approaches concerning the design of such compensators have not yet been shown. [4] discussed a parallel compensator design from a different viewpoint such that the extended plant with parallel compensator satisfies the sufficient condition of SOF stabilization (also called almost strictly positive real condition) [5]. In their work, it is not practical to select the design parameters of the compensator. Recently a systematic design scheme of the parallel compensator was developed by [6, 7, 8]. Further, the realization of parallel compensator for plants with uncertainties was discussed by [9, -, 12].

This paper deals with a systematic and quantitative RPC design scheme for plants with multiple uncertainty. A systematic design scheme which includes determining procedure of the parameters of the proposed compensator is developed. Further, analysis of static output feedback control system with the proposed PRC is given, and an approximate relation between the design parameters of the control system and the parameters of the $PID \cdots D^{\gamma-1}$ control system is also shown.

2. Preliminaries

Consider a single-input single-output time-invariant linear plant with multiple uncertainty described by

$$y(t) = G(s)[u(t)] \quad (1)$$

$$G(s) = G_0(s)(1 + \Delta(s)) \quad (2)$$

where $G_0(s)$ and $\Delta(s)$ are the real rational functions, $G_0(s)$ denotes the modelled part of the plant and $\Delta(s)$ denotes the multiple uncertainty of the plant. The notation $G(s)[u(t)]$ denotes the output of a plant at time t with transfer function $G(s)$ and input $u(t)$. Further, $G_0(s)$ is written by

$$G_0(s) = \frac{N(s)}{D(s)} \quad (3)$$

where $N(s)$ and $D(s)$ (monic) are mth and nth order polynomials, respectively.

Now, we start by stating our assumptions.

Assumption 1:
(1) $N(s)$ is a Hurwitz polynomial.
(2) The relative degree ($\gamma = n - m$) is greater than or equal to 2.
(3) $\Delta(s) \in RH_\infty$ and it is not exactly known but there exists a known rational function $r(s) \in RH_\infty$ such that $|\Delta(j\omega)| \leq |r(j\omega)|$, $\forall\omega$.

Now, the problem is to construct an RPC such that the augmented plant:

$$G_a(s) = G(s) + F(s) \tag{4}$$

satisfies the sufficient condition of SOF stabilization[4, 6].

Using the similar method discussed in [11], we first present the following preliminary. We have from (2) that

$$\begin{aligned} G_a(s) &= G(s) + F(s) \\ &= (G_0(s) + F(s))\left(1 + \frac{\Delta(s)}{1 + H(s)}\right) \qquad (5) \\ H(s) &= G_0(s)^{-1}F(s) \qquad (6) \end{aligned}$$

Let

$$G_{a0}(s) = G_0(s) + F(s) \tag{7}$$

be regarded as a new nominal plant, and

$$\Delta_a(s) = \frac{\Delta(s)}{1 + H(s)} \tag{8}$$

be regarded as a new multiple uncertainty of the plant (7). Then, under Assumption 1, if $G_0(s) + F(s)$ satisfies the sufficient condition of SOF stabilization and $\|\Delta_a(s)\|_\infty < 1$, then $G_a(s)$ also satisfies the sufficient condition of SOF stabilization [9]. Further, under Assumption 1(3), it follows that

$$\|\Delta_a(s)\|_\infty \leq \left|\frac{1}{1 + H(j\omega)}\right| |r(j\omega)|, \ \forall\omega \tag{9}$$

3. Robust Parallel Compensator Design

In this section we propose the following RPC such that $G(s) + F(s)$ satisfies the sufficient condition of SOF stabilization.

$$F(s) = \frac{s}{R(s)} + \frac{1}{k'} \tag{10}$$

$$k' = l^\gamma k_0', k_0' > 0 \tag{11}$$

$$\begin{aligned} R(s) &= \sum_{i=2}^{\gamma} l^{i-2} r_{i-3} s^{\gamma-i+3} \\ &+ l^{\gamma-1}(r_0' s^2 + r_1' s + r_2') \qquad (12) \end{aligned}$$

$$\begin{cases} r_{i-3} = 0, i < 3 \\ r_{i-3} > 0, i \geq 3 \end{cases} \tag{13}$$

where, $R(s)$ is stable and satisfying the following design conditions:

$$\begin{cases} (1)\ r_0' > 0, r_1' > 0, r_2' > 0. \\ (2)\ p_1(s) = \sum_{i=2}^{\gamma} r_{i-3} s^{\gamma-i+1} + r_0' \textit{ is stable.} \\ (3)\ p_2(s) = s^{\gamma-2}(\sum_{i=2}^{\gamma} r_{i-3} s^{\gamma-i+3} + r_0' s^2 + k_0' s) \\ \quad +k_0' a_0(\sum_{i=2}^{\gamma} r_{i-3} s^{\gamma-i+1} + r_0') \textit{ is stable.} \\ (4)\ \left|\frac{1}{1+H(j\omega)}\right| |r(j\omega)| < 1 \ , \ \forall\omega. \end{cases}$$

where, a_0 is the leading coefficient of $N(s)$.

To derive the main results, we first give the following lemma.

Lemma 1: Consider the RPC defined by (10) $\sim$ (13) under Assumption 1(1),(2). Suppose that the design conditions (1) $\sim$ (3) are satisfied. Then, there exists a positive constant l^* such that $G_{a0}(s)$ satisfies the sufficient condition of SOF stabilization for any $l > l^*$.

Proof: Consider Eqs. (3), (10) and define $R(s) = lR'(s)$. Then $G_{a0}(s)$ can be represented by

$$\begin{aligned} G_{a0}(s) &= \frac{N(s)}{D(s)} + \frac{s}{R(s)} + \frac{1}{k'} \\ &= \frac{k'N(s)R'(s) + D(s)p_3(s)}{k'D(s)R'(s)} \qquad (14) \end{aligned}$$

$$\begin{aligned} p_3(s) &= \sum_{i=2}^{\gamma} l^{i-3} r_{i-3} s^{\gamma-i+3} + l^{\gamma-2} r_0' s^2 \\ &+ (l^{\gamma-1} k_0' + l^{\gamma-2} r_1') s + l^{\gamma-2} r_2' \qquad (15) \end{aligned}$$

Define the zero polynomial of $G_{a0}(s)$

$$\begin{aligned} N_{a0}(s) = k'N(s)R'(s) + D(s)\{\sum_{i=2}^{\gamma} l^{i-3} r_{i-3} s^{\gamma-i+3} \\ + l^{\gamma-2} r_0' s^2 + (l^{\gamma-1} k_0' + l^{\gamma-2} r_1') s + l^{\gamma-2} r_2'\} \end{aligned} \tag{16}$$

Since $\deg D(s) = n$ and $\deg R'(s) = \gamma$, it follows that

$$\deg\{N_a(s)\} = n + \gamma \tag{17}$$

$$\deg\{k'D(s)R'(s)\} = n + \gamma \tag{18}$$

It is obvious that the relative degree of $G_{a0}(s)$ is zero, and the leading coefficient of $N_{a0}(s)$ is positive.

Consider the same degree in l, there exists a positive constant l^* such that all the roots of the polynomial $N_a(s)$ are located in the left half-plane for any $l > l^*$[7], [8]. □

Theorem 1: Suppose that the plant (1) satisfies Assumption 1. Consider the following augmented plant:

$$G_a(s) = G(s) + F(s) \tag{19}$$

here, $F(s)$ is given by Eqs. (10) $\sim$ (13). If Lemma 1 is held, and $F(s)$ satisfies the design condition (4), Then, $G_a(s)$ satisfies the sufficient condition of SOF stabilization for any $l > l^*$.

Proof: According to Lemma 1, we have $\Delta_a(s) \in RH_\infty$. From (9) and design condition (4), $\|\Delta_a(s)\|_\infty < 1$ is satisfied. Therefore, the proposition holds. □

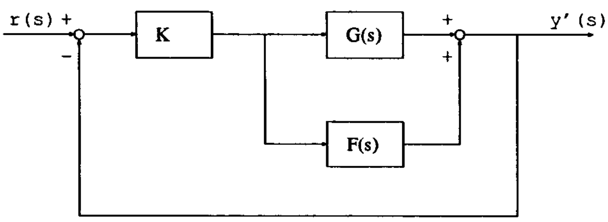

Fig.1 Overall block-diagram of the static output feedback control system

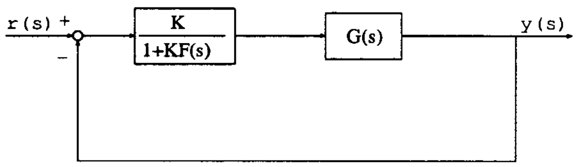

Fig.2 Equivalent block-diagram of the control system

4. Analysis of static output feedback control system with the proposed RPC

When the sufficient condition of SOF stabilization is satisfied for the controlled plants, we can easily construct simple output feedback-based control system. This sufficient condition can simplify the task of designing a control system. For example, we can design the simple adaptive control which requires neither full state feedback nor adaptive observers [5, 13]. However, in most industrial control systems, classical PID controllers are still used. The popularity and wide acceptability of the method are due to its structural simplicity and robustness. Here, a connection between the static output feedback control system with the proposed RPC and $PID \cdots D^{\gamma-1}$ control system is analyzed as follows.

Consider the static output feedback control system with the proposed RPC is illustrated in Fig. 1. Fig. 1 can be further re-expressed in the form of Fig. 2.

From Fig. 2, the equivalent controller is presented by

$$
\begin{aligned}
C_e(s) &= \frac{K}{1+KF(s)} \\
&= \frac{1}{\frac{s}{R(s)} + \frac{1}{k'} + \frac{1}{K}}
\end{aligned} \tag{20}
$$

If k' and K are large (relatively speaking), Eq. (20) gives a $PID \cdots D^{\gamma-1}$ controller as follows.

$$
C_e(s) \approx \frac{R(s)}{s} \tag{21}
$$

Define $PID \cdots D^{\gamma-1}$ control system

$$
\begin{aligned}
&C_{PID\cdots D^{\gamma-1}}(s) \\
&= k_P + \frac{k_I}{s} + k_D s + \cdots + k_{D^{\gamma-1}} s^{\gamma-1}
\end{aligned} \tag{22}
$$

Then, we have ($\gamma \geq 3$)

$$
\begin{aligned}
k_P &= l^{\gamma-1} r'_1 \\
k_I &= l^{\gamma-1} r'_2 \\
k_D &= l^{\gamma-1} r'_0, \quad \cdots \\
k_{D^i} &= l^{i-2} r_{i-3} + r_{\gamma-i-2}, \quad \cdots \\
k_{D^{\gamma-1}} &= l r_0 s^{\gamma-1}
\end{aligned} \tag{23}
$$

and ($\gamma = 2$)

$$
\begin{aligned}
k_P &= l r'_1 \\
k_I &= l r'_2 \\
k_D &= l r'_0
\end{aligned} \tag{24}
$$

Remark: Equivalent block-diagram representation of Fig. 2 only needs to consider the relation between the static output feedback control system and $PID \cdots D^{\gamma-1}$ control system and not the usual feedback controller.

5. Conclusions

In this paper, RPC design scheme for plants with multiple uncertainty was considered such that the closed-loop system realizes strict positive realness by using static output feedback. It was also shown that an approximate relation between the static output feedback control system with the proposed compensator and the $PID \cdots D^{\gamma-1}$ control system exists.

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