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# 동특성 민감도 해석을 이용한 전단형 철골구조물의 다목적 다단계 최적설계

# Multi-Objective and Multi-Level Optimization for Steel Frames Using Sensitivity Analysis of Dynamic Properties

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## ABSTRACT

An improved optimization algorithm for multi-objective and multi-level (MO/ML) optimum design of steel frames is proposed in this paper. In order to optimize the steel frames under seismic load, two main objective functions need to be considered for minimizing the structural weight and maximizing the strain energy. For the efficiency of the proposed method, well known multi-level optimization techniques using decomposition method that separately utilizes both system-level and element-level optimizations and an artificial constraint deletion technique are incorporated in the algorithm. And also dynamic analysis is executed to evaluate the implicit function of structural strain energy at each iteration step. To save the numerical efforts, an efficient reanalysis technique through sensitivity analysis of dynamic properties is proposed in the paper. The efficiency and robustness of the improved MO/ML algorithm, compared with a plain MO/ML algorithm, is successfully demonstrated in the numerical examples.

## 1. Introduction

Most of the design problems in practice have more than one important objective function. Recent trends when designing steel structures have shown more than two objectives. For example, cost saving by decreasing structural weight may be the one objective; and increasing structural strain energy to endure earthquakes or minimizing deflection may be the other. An optimum design model for more than two objective functions requires a multi-objective optimization method, which has recently been developed to adjust many conflicting designs to structural optimizations. Pareto (1896) presented the first study on multi-objective optimization. However, wider interests in optimization theory and operation research was not studied until the late 1960s, and from then on, several studies on these subjects have been presented [1, 2, 3, 4]. Most of the studies were concerned mainly with the theory, but applications to engineering design can hardly be found. Not until recently did a few studies on multi-objective optimization appear in structural optimization [5, 6].

Only recently, studies on multi-objective optimizations using multi-level optimization [7, 8, 9] started to improve their efficiency because too much of computing times are usually required with multi-objective optimization. As one of the most recent study, Gang Li et al. (1999) has proposed a plain algorithm for multi-objective and multi-level (MO/ML) optimization for eight-story, one-bay shear steel frame, which has the objective functions of weight and structural strain energy [11]. The plain MO/ML algorithm is the latest development for multi-level and multi-objective optimization with dynamic property. However, the algorithm shows robust results only when an initial value is set near an optimum value. The algorithm also did not perform the sensitivity analysis of dynamic properties, and thus repeated dynamic analysis had to be followed. In the paper, an improved MO/ML algorithm for multi-objective and multi-level optimization is proposed to overcome these drawbacks in the plain MO/ML algorithm [11]. In the improved MO/ML algorithm, an artificial constraint deletion is introduced to increase its robustness and efficiency.

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In addition, the sensitivity analysis of dynamic properties is applied to increase the computing efficiency. The new optimization algorithm is applied to a ten-story, one-bay steel frame numerical example, and it is demonstrated that the improved MO/ML algorithm has better robustness and efficiency than the plain MO/ML algorithm [11].

## 2. General Formulations

For steel frames, the multi-objective and multi-level optimization problem can be stated in the following form as in the reference [11].

For the system-level optimization:

Find $X$ such that minimize $\{-U(X), W(X)\}$ (1)

subject to $G_j(X) \le 0 \quad j = 1,2,\dots,n_s \; X^L \le X \le X^U$

where $X$ is the design variables at system-level, which are the moment of inertia; $U(X)$ is the structural strain energy part of multi-objective function; $W(X)$ is the structural steel weight part of multi-objective function; $G_j(X)$ is j-th constraint; $n_s$ is the number of constraint; and $X^L$ and $X^U$ are the lower and upper bounds of the system variables, respectively.

For the element-level optimization:

Find $x$ such that minimize $W_i(x)$ (2)

subject to $g_j(X_i^*, x) = 0$; $g_k(x) \le 0 \quad k = 1,2,\dots,n_i$; $x^L \le x \le x^U$

where $x$ is the design variables at element-level, which are the member cross-sectional dimensions; $W_i(x)$ is the objective function, which is the weight of i-th member; $g_k(x)$ is k-th constraint; $n_i$ is the number of the constraints; $x^L$ and $x^U$ are the lower and upper bounds of the element variables, respectively; and $g_j(X_i^*, x)$ is the additional relative equality constraint that connects design variables of the two levels and thus simplifies the coupling between them.

### 2.1 Design variables

In general, the design variables of a steel frame at system-level $\{X\}$ and element-level $\{x\}$ are taken as the moments of inertia of each member and element cross-sectional dimensions respectively, which may be given in vector forms as

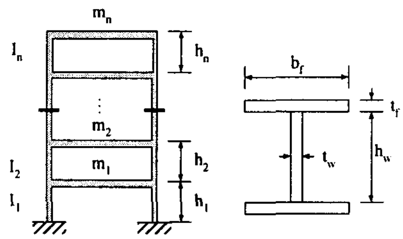


**Fig. 1. A shear steel frame with I Section**

$$X = (X_1, X_2, \dots, X_n) = (I_1, I_2, \dots, I_n) \tag{3}$$

$$x = (x_1, x_2, x_3, x_4) = (h_w, b_f, t_f, t_w) \tag{4}$$

Noting that the design variables are moments of inertia of frame members, the relationships between cross sectional area A, section modulus S and moment of inertia I must be given in a multi-level optimization. The following relations can be obtained [11].

$$A(I) = \left(\frac{A_0}{\sqrt{I_0}}\right)\sqrt{I}\,, \quad S(I) = \left(\frac{I}{I_0}\right)^{0.75} S_0 \qquad \text{(5a), (5b)}$$

where $A_0$ and $I_0$ are the initial values of the cross-sectional area and moment of inertia of an element at the beginning of the system-level optimization in a single iteration cycle.

### 2.2 Objective function

Both maximum structural strain energy and minimum structural weight are simultaneously considered as the objective function in the system-level optimization as in the reference [11]. The multi-objective function often has a linearly weighting form by employing the weighting method for system-level optimization as

$$F(I) = w_1\left(\frac{W(I)}{W_0}\right) + w_2\left(-\frac{U(I)}{U_0}\right) \; ; \; (w_1, w_2 > 0, \text{ and } w_1 + w_2 = 1) \tag{6}$$

where $U(I)$ and $W(I)$ are structural strain energy and weight, respectively; $U_0$ and $W_0$ are initial values at the beginning of each iteration; and $w_1$ and $w_2$ are weight factors to be determined by considering engineering problems in practice.

For a shear steel frame under earthquake load, the structural strain energy can be derived as

$$U(I) = \sum_{j=1}^{m} \sum_{i=1}^{n} \frac{1}{2} Q_{ij}\, \delta_{ij} \tag{7}$$

where and are shear force and story drift of the i-th element with respect to the j-th mode of vibration, respectively; m is the total number of modes of vibration; and n is number of elements.

Based on Eq. (5a), the structural weight can be written as

(8)

where is material mass density; and and are cross-sectional area and height of the i-th element, respectively.

Besides, note that the objective function in the element-level becomes the weight of the element:

(9)

### 2.3 Constraints

The basic and design constraints required for optimal (seismic) design of steel frame according to the AISC-LRFD and AISC seismic provisions may be summarized as in Table 1.

**Table 1. Constraints for Strength and Local Buckling**

| Level | Design Constraints | | Remarks |
|---|---|---|---|
| System | Strength | For ,<br>For , | and are required and nominal axial compressive or tensile strength, respectively; and are required and nominal flexural strength, respectively; and and are resistance factors for the axial and flexure. |
| Element | Local Buckling | For the flange,<br>For the web,<br>If ,<br>If , | is yield strength of member and flange (ksi); and is nominal yield strength. |

__Frequency constraints__

The frequency constraints are used to make the natural frequencies avoid a range called restricted zone of frequency, which can be given in the following form [11]:

(10)

where is the j-th natural frequency; and and are the lower and upper bound of the restricted zone, respectively. Three alternative cases can be considered assuming that

1) In case all the structural natural frequencies are less than the restricted zone, the frequency constraint Eq. (10) can be changed into the following form:

(11a)

2) In case all the structural natural frequencies are greater than the restricted zone, the frequency constraint Eq. (10) can be changed into the following form:

(11b)

3) In case when a couple of adjacent frequencies called concerned frequencies, and . which can cover the entire restricted zone, the frequency constraint can be written as

(11c)

**Additional relative constraint**

In order to assure the consistency between system and element design variables, the coupling between the system-level and element-level is treated by an additional relative constraint in the element-level optimization. The appropriate additional relative constraint is very important in a multi-level optimization problem because it affects both the convergence of the optimization procedure and the final optimal solutions. The equality constraint of moment of inertia of an element is introduced during the element-level optimization process to treat the coupling effectively. The additional relative constraint can be written as:

(12)

where is the optimum moment of inertia for the i-th element at system-level.

**Side Constraints**

The side constraints prescribe design variables in a certain range, which can be stated as:

(13a), (13b)

The variable limits should be carefully chosen based on engineering requirements in practice.

## 3. Multi-objective and Multi-level Optimization Algorithm

### 3.1 Plain MO/ML Algorithm for Multi-objective and Multi-level Optimization

In the reference [11], a plain optimum design algorithm shown in Fig. 2 proposed for a multi-objective and multi-level optimization for steel framed structures. However, it is found that the following problems are incurred when the algorithm is applied to numerical examples:

- Only in case an initial value is not far from a near-optimum value in the algorithm, robust results could be obtained. If an initial value is feasible but far from the optimum, unreliable results may be produced.
- The dynamic analysis is performed to evaluate objective function and constraint in each iterative step without sensitivity analysis. Dynamic reanalysis in each iterative step will be more time-consuming in a large structure, and therefore, the computational efficiency can not be expected when it is applied to large structures.

### 3.2 Improved MO/ML Algorithm for Multi-Objective and Multi-Level Optimization

An Improved MO/ML algorithm shown in Fig. 3 is proposed for an efficient multi-objective and multi-level optimization, which shows better performance compared to the plain MO/ML algorithm [11]. In the improved MO/ML algorithm, a sensitivity analysis of dynamic properties and an artificial constraint deletion technique are introduced to carry out optimum design efficiently for fast convergence.

**1) Artificial Constraint Deletion**

In the improved MO/ML algorithm, the artificial constraint deletion for frequency constraint is introduced to improve the efficiency and the robustness of the optimization algorithm. After optimization is carried out for all constraints except a frequency constraint, the feasibility for frequency constraint is checked. If feasibility is satisfied with frequency constraint, it is assumed that the optimization process arrived at a near optimum design. Otherwise, sensitivity analysis for eigenvalue and eigenvector has to be carried out and an optimization with frequency constraints is executed.

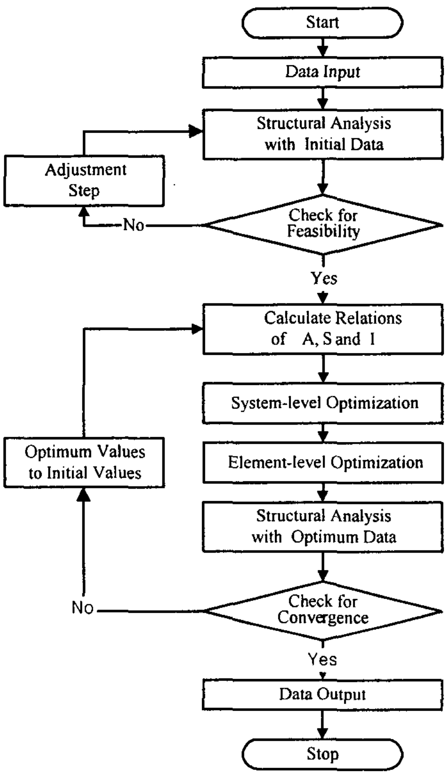


Fig. 2. Plain MO/ML Algorithm (Li, et al.)

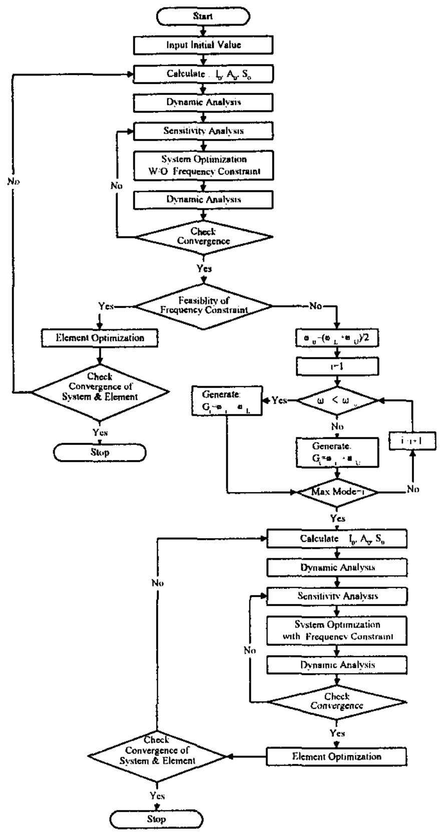


Fig. 3. Improved MO/ML Algorithm

## 2) Sensitivity Analysis of Dynamic Properties

Sensitivity analyses of dynamic properties are required for an efficient reanalysis of eigenvalue and eigenvector. Detailed descriptions of sensitivity analysis are shown in the reference [10], and thus only key equations are briefly described herein. The basic problem is to compute derivatives of eigenvalues and eigenvectors with respect to design variables or system parameters. For convenience, consider the following general eigenvalue problem:

(14)

where is an eigenvalue; eigenvector, the stiffness matrix; and the mass matrix corresponding to the design vector

The equation of sensitivity analysis for eigenvalue and eigenvector can be obtained by differentiation of Eq. (14) as

(15)

(16)

where

## 4. Numerical Example

In this section, a ten-story, one-bay shear steel frame with I-sections (Fig. 1), which is almost the same example used in the reference [11], is optimized by an optimum design program developed for the multi-objective and multi-level optimization based on the theory and formulations discussed above. The basic data for design are summarized in Table 2.

**Table 2. Basic Data for Design**

| Item | Data |
|---|---|
| Building story and Story height | N = 10, = 5.5 m |
| Concentrated mass and Mass density of steel | = 4000 kg, = 7850 kg/ |
| Elastic modulus and Yield strength | E = 200,000 Mpa, $F_y$ = 248 Mpa (36 ksi) |
| Restricted zone of Frequency | [ , ] =[15, 20] |
| Bounds of system Variables ($10^{-4}m^4$) | = (7.0, 7.0, 5.0, 5.0, 4.0, 4.0, 3.0, 3.0, 3.0, 3.0) and<br>= (1.5, 1.5, 1.0, 1.0, 0.7, 0.7, 0.5, 0.3, 0.5, 0.3) |
| Bounds of element variables ($10^{-2}m$) | = (80, 50, 3.0, 1.6) and = (25, 15, 1.0, 0.5) |

**Table 3. Initial Values**

| | Story | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Value 1 | $h_w$ (cm) | 100.0 | 90.0 | 80.0 | 75.0 | 70.0 | 65.0 | 55.0 | 50.0 | 20.0 | 5.0 |
| | $b_f$ (cm) | 30.0 | 30.0 | 30.0 | 25.0 | 20.0 | 15.0 | 11.0 | 10.0 | 5.0 | 5.0 |
| | $t_f$ (cm) | 1.5 | 1.5 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | $t_w$ (cm) | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 0.5 | 1.0 | 1.0 |
| | Frequency (rad/sec) | 233.45, 165.38, 131.26, 103.52, 78.54, 58.74, 37.82, 18.80, 10.03, 2.60 | | | | | | | | | |
| Initial Value 2 | $h_w$ (cm) | 80.0 | 80.0 | 80.0 | 70.0 | 65.0 | 60.0 | 45.0 | 30.0 | 12.0 | 8.0 |
| | $b_f$ (cm) | 15.0 | 20.0 | 15.0 | 15.0 | 14.0 | 8.0 | 8.0 | 9.0 | 5.0 | 5.0 |
| | $t_f$ (cm) | 2.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | $t_w$ (cm) | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 0.5 | 0.5 | 0.5 |
| | Frequency (rad/sec) | 163.89, 126.75, 98.17, 74.65, 54.65, 38.55, 24.95, 14.11, 8.20, 3.40 | | | | | | | | | |

The new improved and the plain MO/ML algorithms are applied to the above design example. The results of the application using the two algorithms are comparatively shown in Table 4. From the results summarized in Table 4 and convergence of the optimum solution in Fig. 4 and Fig. 5, essential features of the proposed algorithm are comparatively discussed with emphasis on robustness and efficiency of the algorithms. However, the parametric study of and , that shows about same as the results given in the reference [11], is not described in the paper.

A robust algorithm should produce a consistent solution regardless of its initial value. To investigate the robustness of the two algorithms, each optimization is individually performed with two different sets of initial values of Table 3. For each optimization, the convergence history of optimum process is presented in Fig. 5. As shown in Table 4, the plain MO/ML algorithm produces diverse results depending on whether the initial value is properly assumed or not. When the optimization is carried out using the plain algorithm, an optimum design is obtained only with the initial values that satisfy feasible conditions. By using the frequency obtained from the eigenvalue analysis, the type of frequency constraint is determined. If the initial values are far away from the optimum then the type of frequency constraint in the algorithm is not reasonably selected and thus a real optimum solution cannot be obtained. However, the improved MO/ML algorithm by using artificial constraint deletion can produce reasonable frequency type, regardless of the initial values. Therefore, since reasonable optimum results are obtained regardless of initial values, it may be argued that the new improved MO/ML algorithm is more robust than the plain one.

An efficient algorithm could be defined as the one that reaches the convergence within a reasonable number of iterations, which is also important factor to prove superiority of the algorithm. In order to demonstrate the efficiency of the improved MO/ML algorithm, the number of iterations of the algorithm are compared with those of the plain MO/ML algorithm. As shown in Table 4 and Fig. 4, the number of reanalysis of improved MO/ML algorithm is much less than those of the plain MO/ML algorithm. In the improved MO/ML algorithm, efficient reanalysis technique is introduced to decrease the number of analyses, whereas it is not in the plain algorithm. It may be noted in this example that applying the improved MO/ML algorithm could reduce a large number of analyses. Moreover, noting that the more a large scale structure is to be optimized, the more efficient optimization algorithm is required, it may be stated that the improved MO/ML algorithm is expected to be a lot more efficient in the optimization of these large-scale structures, compared with the available algorithm.

**Table 4. Results of Optimum Design of Improved and Plain MO/ML Algorithm** ( =0.8, =0.2)

| Algorithm | Building Story | Moment of Inertia($cm^4$) | $h_w$ | $b_f$ | $t_f$ | $t_w$ | Weight (kg) | Energy (N-m) | Number of Analysis |
|---|---|---|---|---|---|---|---|---|---|
| plain MO/ML Algorithm<br>Initial value 1 | 1 | 143270.00 | 79.99 | 17.71 | 1.68 | 1.04 | 6688.67 | 838.24 | 155 |
| | 2 | 109779.10 | 80.00 | 15.45 | 1.28 | 1.04 | | | |
| | 3 | 87351.93 | 78.72 | 13.10 | 1.10 | 1.02 | | | |
| | 4 | 65912.84 | 71.65 | 14.15 | 1.00 | 0.93 | | | |
| | 5 | 47404.20 | 66.71 | 11.31 | 1.00 | 0.87 | | | |
| | 6 | 31885.36 | 57.90 | 11.37 | 1.00 | 0.75 | | | |
| | 7 | 18336.83 | 49.50 | 9.25 | 1.00 | 0.65 | | | |
| | 8 | 9579.79 | 38.36 | 9.33 | 1.00 | 0.50 | | | |
| | 9 | 4396.77 | 33.03 | 5.00 | 1.00 | 0.50 | | | |
| | 10 | 231.84 | 8.13 | 5.00 | 1.00 | 0.50 | | | |
| plain MO/ML Algorithm<br>Initial value 2 | 1 | 148655.40 | 80.00 | 11.53 | 2.65 | 1.04 | 6438.72 | 721.42 | 56 |
| | 2 | 127306.60 | 80.00 | 13.58 | 1.83 | 1.03 | | | |
| | 3 | 103534.20 | 80.00 | 12.92 | 1.39 | 1.03 | | | |
| | 4 | 73353.72 | 75.19 | 11.94 | 1.12 | 0.97 | | | |
| | 5 | 47525.73 | 67.35 | 10.21 | 1.06 | 0.87 | | | |
| | 6 | 30908.62 | 60.01 | 7.25 | 1.24 | 0.78 | | | |
| | 7 | 14795.02 | 49.65 | 5.00 | 1.25 | 0.66 | | | |
| | 8 | 5100.75 | 35.22 | 5.00 | 1.00 | 0.50 | | | |
| | 9 | 1169.20 | 18.17 | 5.00 | 1.00 | 0.50 | | | |
| | 10 | 268.51 | 8.79 | 5.00 | 1.00 | 0.50 | | | |
| Improved MO/ML Algorithm<br>Initial value 1 | 1 | 155954.30 | 80.00 | 23.06 | 1.46 | 1.03 | 6425.72 | 716.18 | 30 |
| | 2 | 117217.70 | 80.00 | 17.48 | 1.26 | 1.04 | | | |
| | 3 | 90894.39 | 79.02 | 8.49 | 1.76 | 1.03 | | | |
| | 4 | 66127.80 | 72.17 | 13.72 | 1.00 | 0.94 | | | |
| | 5 | 45134.24 | 63.15 | 13.55 | 1.00 | 0.82 | | | |
| | 6 | 28100.78 | 57.59 | 9.39 | 1.00 | 0.75 | | | |
| | 7 | 14349.50 | 47.27 | 7.58 | 1.00 | 0.63 | | | |
| | 8 | 6208.67 | 37.13 | 5.61 | 1.00 | 0.50 | | | |
| | 9 | 1163.40 | 18.12 | 5.00 | 1.00 | 0.50 | | | |
| | 10 | 237.18 | 8.23 | 5.00 | 1.00 | 0.50 | | | |
| Improved MO/ML Algorithm<br>Initial value 2 | 1 | 148655.40 | 80.00 | 11.53 | 2.65 | 1.04 | 6438.72 | 692.14 | 12 |
| | 2 | 127306.60 | 80.00 | 13.58 | 1.83 | 1.03 | | | |
| | 3 | 103534.20 | 80.00 | 12.92 | 1.39 | 1.03 | | | |
| | 4 | 73353.72 | 75.19 | 11.94 | 1.12 | 0.97 | | | |
| | 5 | 47525.73 | 67.35 | 10.21 | 1.06 | 0.87 | | | |
| | 6 | 30908.62 | 60.01 | 7.25 | 1.24 | 0.78 | | | |
| | 7 | 14795.02 | 49.65 | 5.00 | 1.25 | 0.66 | | | |
| | 8 | 5100.75 | 35.22 | 5.00 | 1.00 | 0.50 | | | |
| | 9 | 1169.20 | 18.17 | 5.00 | 1.00 | 0.50 | | | |
| | 10 | 268.51 | 8.79 | 5.00 | 1.00 | 0.50 | | | |

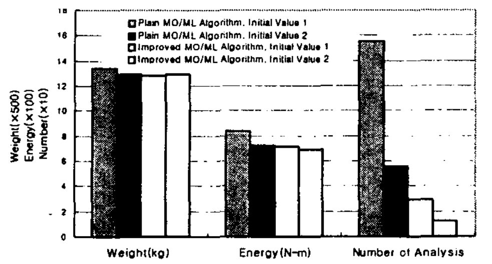


**Fig. 4. Comparison of Optimum Results using Two Algorithm**

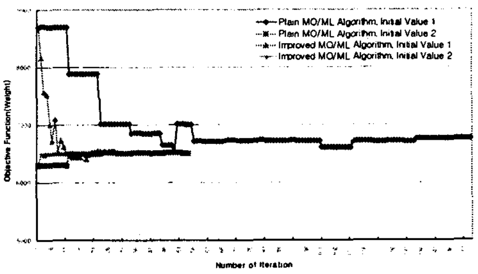


**Fig. 5. Convergence History Diagram**

## 5. Conclusion

In the paper, an improved MO/ML algorithm is proposed for the multi-objective and multi-level optimization using sensitivity analysis of dynamic properties. To optimize steel frames under seismic load, dynamic analysis is required to evaluate frequency constraint and the structural strain energy of the multi-objective function. Since dynamic analyses are time-consuming, an approximation technique using sensitivity analysis is proposed to increase the convergence efficiency. In the improved MO/ML algorithm, an artificial constraint deletion technique is also introduced to increase robustness and efficiency of the algorithm by overcoming the drawback of the frequency constraints in the plain MO/ML algorithm.

In order to demonstrate robustness and efficiency of the improved MO/ML algorithm, a ten-story and one-bay steel frame is used as the numerical example, and the optimum results of the proposed algorithm are compared with those of the plain MO/ML algorithm. Based on the results of the application example, it may be concluded that the improved MO/ML algorithm is considerably more efficient and robust than the plain MO/ML method. Therefore, it may be argued that the improved MO/ML algorithm proposed in this study may be successfully applied to large-scale structural optimization problems with robustness and efficiency.

## Reference


[1] Ignizio J. (1976), Goal Programming and Extensions, Heath, Lexington, Messachusetts

[2] Osyczka, A. (1984), Multicriterion Optimization in Engineering, JOHN WILEY & SONS, New York

[3] Rao S. S. (1987), Game Theory Approach for Multiobjective Structural Optimization, Computer & Structure 25 No.1, pp119-127

[4] Ringuest, J. L. (1992), Multiobjective Optimization : Behavioral and Computational Considerations, KLUWER ACADEMIC PUBLISHERS

[5] Saravanos, D. A. and Chamis, C. C. (1992), "Multiobjective Shape and Material Optimization of Composite Structure Including Damping", AIAA Journal,Vol.30, No.3, March, pp805-813

[6] Ramana V. Grandhi and Geetha Bharatram (1993), "Multiobjective Optimization of Large-Scale Structures", AIAA Journal,Vol.31, No.7, July , pp1329-1337

[7] Kirsch, U. (1975), Multilevel Approach to Optimum structural Design, ASCE *Journal of the Structural Division*, Vol. 101, No. ST4, pp957-974

[8] Hafkta RT. (1984), An Improved Computational Approach for Multilevel Optimization Design, *Journal of structural Mechanics*, Vol 12, pp245-261

[9] Sobieszczanski-Sobieski J. James B. B., Dovi A. R. (1985), Structural Optimization by Multilevel Decomposition, AIAA *Journal*, Vol 23 pp1775-1782

[10] Rogers, L. C. (1970), Derivatives of Eigenvalues and Eigenvectors, AIAA *Journal*, Vol 8, No.5, pp943-944

[11] Gang Li et al (1999), Multiobjective and Multilevel Optimization for Steel Frames, Engineering Structures 21 pp519-529