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Discrete Structural Design of Reinforced Concrete Frame by Genetic Algorithm

안 지 현* 이 차 돈**
Ahn, Jeehyun Lee, Chadon

ABSTRACT

An optimization algorithm based on Genetic Algorithm(GA) is developed for discrete optimization of reinforced concrete plane frame by constructing databases. Under multiple loading conditions, discrete optimum sets of reinforcements for both negative and positive moments in beams, their dimensions, column reinforcement, and their column dimensions are found. Construction practice is also implemented by linking columns and beams by group 'Connectivity' between columns located in the same column line is also considered. It is shown that the developed genetic algorithm was able to reach optimum design for reinforced concrete plane frame construction practice.

1. INTRODUCTION

Recently, discrete optimization of structures has been performed by introduction of a Genetic Algorithm. GAs are search procedures based on the mechanics of natural genetics and natural selection.(1) The main advantages of GA over the conventional optimization techniques can be summarized as: (1) GAs do not require gradient computations; (2) GAs do not require that the constraints should be expressed explicitly in terms of design variables; (3) GAs take advantage of carrying out optimization processes in a stochastic framework; and (4) GAs are not limited by restrictive assumptions about search space, such as continuity or the existence of derivatives.

The present paper describes the genetic algorithm-based approach taken to optimize two dimensional reinforced concrete frame subject to multiple loading conditions. For the process of GAs natural selection, databases for beams and columns are constructed. Each section in each database is assigned with unique identification number and is produced to meet.

2. REVIEW OF GENETIC ALGORITHM

GAs use three basic randomized operators in place of the usual deterministic ones: reproduction, crossover, and mutation.(1)

(1) reproduction

Let n_p be the number of chromosomes. The i-th chromosome, with fitness value of f_i, is made as a candidate chromosome for reproduction according to the following rule:

* 중앙대학교 건축공학과 석사과정
** 중앙대학교 건축공학과 교수

$$\text{Let , } q_i = \sum_{m=1}^{i} \frac{f_m}{\sum_{j=1}^{n_p} f_j} \quad i = 1, 2, \cdots, n_p, \; q_0 = 0 \tag{1}$$

Do until the selected number of chromosome is equal to n_p,

Generate random number r_i in [0,1].

If $q_{i-1} \leq r_i \leq q_i$, select the i-th chromosome as a reproduction candidate for mating pool.

Otherwise, repeat.

(2) crossover

The probability of crossover (p_c) is defined as the ratio of the number of offspring produced in each generation to the population size (n_p).

Let r_i be a random number generated in [0,1] for the i-th chromosome.

Do until the number of mating chromosomes becomes $p_c \times n_p$.

If $r_i \leq p_c$, then the i-th chromosome is chosen for mating and put into the mating pool.

Otherwise, the i-th chromosome succeeds to the child generation.

The remaining chromosomes in the mating pool are randomly matched in pair, swapping strings in each chromosome and the resulting chromosomes succeed to the child generation.

(3) mutation

Probability of mutation (p_m) is defined as the percentage of the total number of genes in the population. The number of mutation (n_m) becomes:

$$n_m = \text{round}\,(p_m \times N_{alle}) \tag{2}$$

where, $N_{alle} = n_p \times S_L$;

S_L = string length of a chromosome ; and

round ($\cdot$) = round to the nearest integer.

Each mutation is performed by swapping the randomly selected position values (or values of alleles) from 0 to 1 or vice versa of randomly selected chromosome in the population. This process repeats until the total number of mutation becomes n_m.

3. CONSTRUCTION OF DATABASE

3.1. Beams

Predefined discrete beam sections are generated. Minimum beam width (b_{min}), maximum beam width (b_{max}), minimum ratio of beam depth to beam width (β_{min}), and maximum ratio of beam depth to beam width (β_{max}) are defined before the generation of the database. Discrete increment for beam width (Δb) and beam depth (Δh) are also given. Cross sectional dimensions for beams in the database are then automatically generated with these values. The following procedure is adopted to generate sections in the beam database.

$$\text{Generate } b_i = b_{min} + (i-1) \cdot \Delta b, \quad i = 1, 2, \cdots, n_b \tag{3}$$

Calculate $h_{min,i} = \beta_{min} \times b_i$ and $h_{max,i} = \beta_{max} \times b_i$ and

Let $h_{1,i}$ = larger of ($h_{\min}$, $h_{\max}$)
Generate $h_{j,i}=h_{1,j}+(j-1)\cdot\Delta h$ until $h_{j,i}$ is equal to or smaller of ($h_{\max}$, $h_{\max,i}$), $j=1, 2, \cdots, n_p$
where, n_b = number of beam width; and n_h = number of beam depth.

3.2 Columns

Minimum column width($w_{\min}$), maximum column width($w_{\max}$), and discrete increment for column width(Δw) are given in advance in order to generate candidates for column sections.

4. REINFORCEMENTS

As GA selects member dimensions from databases, appropriate reinforcements for beams and columns are assigned according to code provisions. For a specific frame configuration, selecting process through a code provisions would lead to a unique mappings between frame configurations and fitness value.

4.1. Beams

The required reinforcing bar areas for given moment M_u can be found from;

$$A_{s,\min} \le A_s = \frac{\phi f_y \cdot d - \sqrt{(\phi f_y \cdot d)^2 - \frac{4\phi}{1.7}\cdot\frac{f_y^2}{f_c'\cdot b}\cdot|M_u|}}{\frac{2\phi}{1.7}\cdot\frac{f_y^2}{f_c'\cdot b}} \le A_{s,\max} \quad (4)$$

The obtained number of reinforcing bars n_s must be less than or equal to:

$$n_s = 2\times \text{round}\left\{\frac{b-(2\times(t_c+d_s)+s_b)}{d_b+s_b} - 0.5\right\} \quad (5)$$

where, d_s = stirrup diameter; d_b = flexural reinforcing bar diameter; s_b = larger of (d_b, 3/4 max. aggregate size, 2.5cm); and t_c = cover thickness.

4.1. Columns

The minimum area of reinforcing bars in column section is determined as:

larger of (4 × one reinforcing bar area, $0.008\times A_g$, and $\dfrac{0.85f_{c'}\cdot A_g - \frac{P_u}{0.8\phi}}{A_g(0.85f_{c'}+f_y)}$).
where, A_g = column sectional area.

The maximum area of reinforcing bars in a column section is limited either by maximum number of reinforcing bars ($n_{c,\max}$) in a layer or by maximum reinforcement ratio:

$$n_{c,\max} = 2\times \text{round}\left\{\frac{w-(2\times(t_c+d_t)+s_c)}{d_b+s_c} - 0.5\right\} \quad (6)$$

where, d_t = diameter of tie bar; s_c = spacing between logituding bars in column= larger of ($1.5d_b$, 3/4 max. aggregate size, 2.5cm); and w = column size.

As sectional dimensions and possible number of reinforcing bars are determined, characteristic points on the P-M interaction curve for each candidate column are evaluated and stored in the database. The P-M interaction curve can be approximately constructed by linearly connecting characteristic points(see Fig.1).

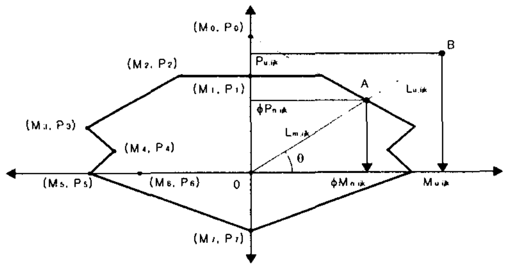

Fig. 1. Linearized P-M interaction curve by connecting characteristic points.

5. FORMULATION OF OPTIMIZATION

In the following, the subscripts i, j, and k stand for group number, member number in group, and load case, respectively.

The constraints are normalized and used for constituting unconstrained objective function:

$$\langle g^{+}_{M,ijk}\rangle = \frac{|M^{+}_{u,ijk}|}{\phi M^{+}_{n,ijk}} - 1 \ge 0;$$

$$\langle g^{-}_{Ml,ijk}\rangle = \frac{|M^{-}_{ul,ijk}|}{\phi M^{-}_{nl,ijk}} - 1 \ge 0; \text{ and } \langle g^{-}_{Mr,ijk}\rangle = \frac{|M^{-}_{ur,ijk}|}{\phi M^{-}_{nr,ijk}} - 1 \ge 0. \tag{7}$$

Safety of a column is evaluated by taking the ratio of distances from the origin to the loading point in P-M interaction plane (Fig. 1).

Let $L_{m,ijk}$ and $L_{u,ijk}$ be the distance between the origin O and the point A and the distance between the origin and the point B on P-M interaction envelope, respectively:

$$L_{m,ijk} = \sqrt{(\phi P_{n,ijk})^2 + (\phi M_{n,ijk})^2}\,; \text{ and } L_{u,ijk} = \sqrt{(\phi P_{u,ijk})^2 + (\phi M_{u,ijk})^2}\,. \tag{8}$$

The penalty function for column strength is then expressed in normalized form as:

$$\langle g_{PM,ijk}\rangle = \frac{L_{u,ijk}}{L_{m,ijk}} - 1 \ge 0 \tag{9}$$

Let t and b represent two different column group numbers in the l-th connectivity condition. If all the columns in group b with size w_b are located lower than those columns in group t with w_t, connectivity for these column sizes is evaluated by:

$$\langle g_{c,l}\rangle = \frac{w_{t,l}}{w_{b,l}} - 1 \ge 0 \tag{10}$$

Similarly, if n_t and n_b represent the number of reinforcing bars of columns in group t and group b, respectively, then connectivity for reinforcing bars in columns is evaluated by:

$$\langle g_{s,l}\rangle = \frac{n_{t,l}}{n_{b,l}} - 1 \ge 0 \tag{11}$$

The final form of unconstrained objective function can be expressed as follows:

Minimize:

$$F = W_g \cdot \frac{G}{M \cdot (W_M + W_{PM} + W_{CON})} + W_c \cdot \frac{C}{C_{initial}} \quad (12)$$

where,

$$G = \sum_{k=1}^{NDLC} \sum_{i=1}^{NGB} \sum_{j=1}^{NMBG[i]} (W_M \cdot \langle g^{+}_{M,ijk} \rangle + W_M \cdot \langle g^{+}_{M,ijk} \rangle) + \sum_{k=1}^{NDLC} \sum_{i=1}^{NGC} \sum_{j=1}^{NMCG[i]} (W_{PM} \cdot \langle g_{PM,ijk} \rangle) + \sum_{l=1}^{NCON} (W_{CON} \cdot NC_l(\langle g_{c,l} \rangle + \langle g_{s,l} \rangle))$$

$C_{initial}$ =initial cost; W_g, W_c=weight for G and C, respectively; M=total number of members; W_M, W_{PM}, W_{CON} = weight of penalty function for beam, column, and connectivity of column, respectively; $NDLC$ =number of different loading conditions; and NC_l =number of columns related to l-th condition.

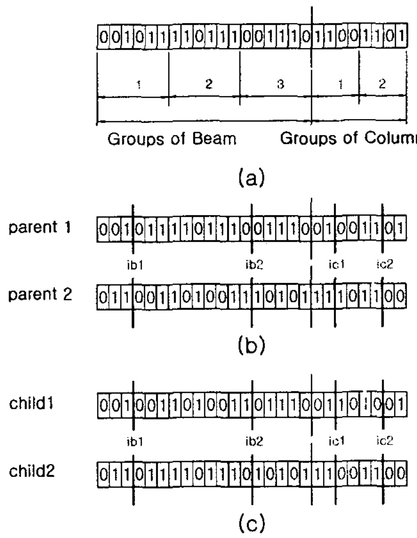

Fig.2. Representative chromosome and generation of children by crossover:
(a) groups of genes in a chromosome;
(b) cross sites in parents' chromosome; and
(c) generated children after crossover

6. APPLICATION OF GENETIC ALGORITHM

The GA adopting niche concept, for optimizing reinforced concrete frame, is briefly summarized below:

(1) Determine string length S_L for chromosome. Generate n_p chromosomes having string length S_L and randomly assign 0 or 1 to each allele in each chromosome.

(2) Convert binary numbers encoded in each chromosome into decimal numbers and identify appropriate beam group numbers and column group numbers by mapping these numbers to a member identification number in databases for beams and columns. Generate n_p frames by the use of information in n_p chromosomes.

(3) Evaluate the objective function value F for each frame by Eq. (12). Let F_i be the value of F for frame i. Evaluate the fitness value for i-th frame (f_i) by the following rule:

$$fit_i = \frac{[F_{\max} + F_{\min}] - F_i}{f_{avg}}, \quad i = 1, 2, \cdots, n_p \quad (13)$$

where, $F_{\max}$ =maximum of F_i ; $F_{\min}$ =minimum of F_i ; and $f_{avg} = \sum_{i=1}^{n_p} F_i \Big/ n_p$.

(4) Using f_i values obtained in step (3), operate the reproduction by the method presented in section 2.

(5) Perform the crossover as described in section 2. For a randomly chosen pair of chromosomes in the mating pool, four positions-two in strings for beam group and two for column group-are selected uniformly at random (Fig.2). A pair of children are generated by exchanging mapping strings between parent chromosomes. Mapping strings are those substrings in the middle, separated by two cut-off positions i.e., substrings between ib_1 and ib_2 for beams and between ic_1 and ic_2 for

columns (Fig.2).

(6) Apply niche concept.

(7) Check the convergence criterion and stop if it is met. Otherwise mutate randomly and proceed to the next generation. Repeat from step (2) until convergence is obtained.

7. ILLUSTRATIVE EXAMPLES

The performance of the developed algorithm is investigated for reinforced concrete plane frames having a different number of stories. Lateral equivalent static earthquake loads (E) are applied as joint loads. Uniform gravity loads are assumed for dead load (D) and live load (L).

Different loading cases are considered as suggested in structural design code for strength design[8]:

$$U = 1.4D + 1.7L; \text{ and}$$
$$U = 0.75(1.4D + 1.7L \pm 1.87E). \quad (17)$$

Assumed concrete strength and yield strength of reinforcing bars in these examples are f_c' =240 kg/cm^2 and f_y =4000 kg/cm^2, respectively, for all frames. The cost of concrete, forming, and reinforcing steels are given as 700 won/kg, 121,660 won/m^3, and 19,200 won/m^2, respectively.

7.1. 3-Bay, 9-Story Reinforced Concrete Frame

The frame shown in Fig. 3 is composed of 3 beam groups and 4 column groups. Initially 500 frames were randomly generated. With crossover probability of p_c=0.5, different values of mutation probability(p_m=0.0% and 0.1%) are tried for comparison. It was observed that GAs with p_m

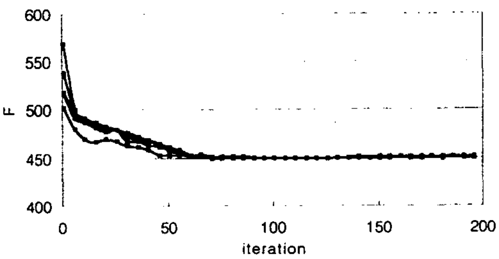

Fig. 4. Convergence trend for 3-bay, 9-story reinforced concrete frame.

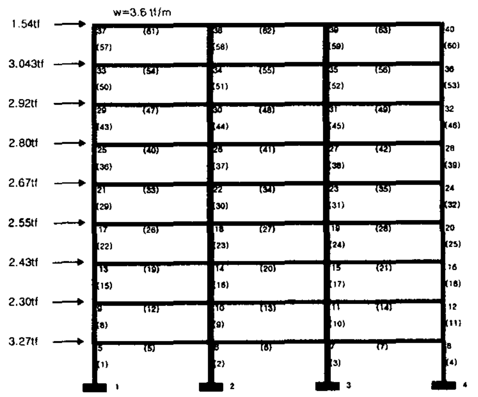

Fig. 3. 3-bay, 9-story reinforced concrete frame subject to gravity and lateral load. (span=9m, story height=3.6m)

Table. 1. Results of optimum design for 3-bay, 9-story reinforced concrete frame

	Group no.	Story level	Member no.	Optimization results: sectional dimensions (cm) width	depth	reinforcements (f_y=4000kgf/cm^2) positive moment	negative moment
Beam	1	1-3 (Beam)	5,6,7,12,13, 14,19,20,21	30	65	2-D22	4-D22
	2	4-6 (Beam)	26,27,28,33, 34,35,40,41,42	30	48	3-D22	6-D22
	3	7-9 (Beam)	47,48,49,54, 55,56,61,62,63	30	48	3-D22	6-D22
Column	1	1-4 (exterior column)	1,4,8,11,15, 18,22,25	45		6-D25	
	2	1-4 (interior column)	2,3,9,10,16, 17,23,24	65		10-D25	
	3	5-9 (exterior column)	29,32,36,39,43, 46,50,53,57,60	40		6-D25	
	4	5-9 (interior column)	30,31,37,38,44, 45,51,52,58,59	50		4-D25	
initial deisng	Max. F		F	670815			
			Cost	20,567,700 (won)			
	Min. F		F	501			
			Cost	25,685,500 (won)			
optimum design			F	451			
			Cost	21,335,800 (won)			

equal to 0.1% better than GA without mutation.

The size of domain space and sampling space are found to be: $(2^{N_{B,allw}})^{NGB} \times (2^{N_{C,allw}})^{NGC} = (2\times10^6)^3 \times (2\times10^4)^4 = 1.7\times10^{10}$ for domain space and $n_p \times$ iteration number at convergence= $500\times201=1.0\times10^5$ for sampling space, respectively. It is worth mentioning that relative size of domain being used for GAs selection process is 1.0×10^5 / 1.7×10^{10} = 5.9×10^{-6}, which is in the order of $O(10^{-6})$.

Table 1 shows the optimized results. The value of minimum objective function (F) and its cost at initial stage are reduced from 501 and 25,685,500 to 451 and 21,335,800 at final design, respectively.

7.2. 3-Bay, 20-Story Building

The 20 story frame (Fig. 5) subject to both uniformly distributed gravity loads and equivalent static lateral earthquake loads is composed of 5 beam groups and 8 column groups. Compared to the previous 9 story frame, sectional dimensions associated with b_{max} and h_{max} for beam and w_{max} for column are increased to 50cm, 100cm and 120cm, respectively. The size of domain space turns out to be $(2^7)^5\times(2^5)^8=3.8\times10^{22}$. Starting from a randomly distributed 500 initial designs in the domain space, GA could successfully yield optimized design at 500 iterations. The convergence trend are shown in Fig. 6. Probabilities of crossover (p_c) and mutation (p_m) are given as 0.5 and 0.001, respectively. The order of sampling space relative to domain space is $(500\times500)/\ 3.8\times10^{22}=6.6\times10^{-18}=O(10^{-18})$. Table 2 shows the result of optimum design. It seems that depending on the locations of beams and columns, their dimensions and amount of reinforcements are properly designed. The value of minimum object function (F) and its cost at initial stage are reduced from 612 and 99,065,600 to 475 and 71,753,100 at final design stage, respectively.

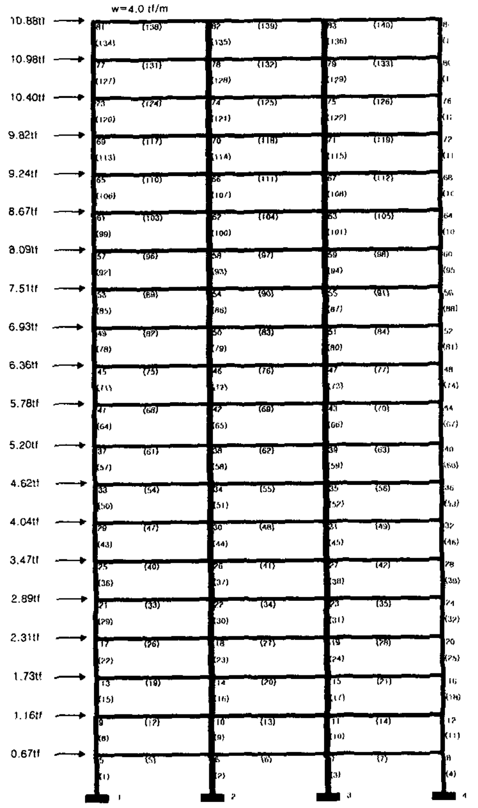

Fig. 5. 3-bay, 20-story reinforced concrete frame subject to gravity and lateral load (span=9m, story height=3.6m).

8. CONCLUSIONS

The following conclusions are made from this study:

(1) The developed GA reinforced by niche operator in addition to its three basic operators - reproduction, crossover, and mutation - successfully led the randomly distributed initial design points in the design space to the optimum design point.

(2) The developed GA reached optimum design for the frames considered in this study, sampling

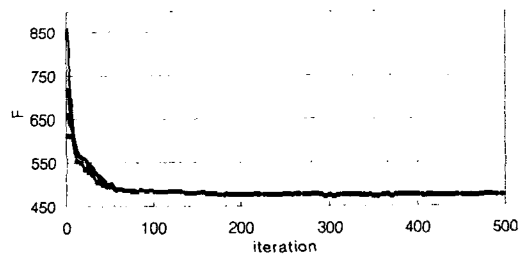

Fig. 6. Convergence trend for 3-bay, 20-story reinforced concrete frame.

small fractions of the domain in the order of $\mathcal{O}(10^{6})$ for 9-story frame, and $\mathcal{O}(10^{18})$ for 20-story frame, respectively.

(3) GAs with p_m equal to 0.1% perform better than GA without mutation.

(4) Although it is applied only to the optimization of reinforced concrete plane frames, the main algorithms developed in this study can also be applied to discrete optimization of three dimensional reinforced concrete frames.

9. ACKNOWLEDGMENT

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Table 2. Results of optimum design for 3-bay, 20-story reinforced concrete frame.

	Group no.	Story level	Member no.	Optimization results			
				sectional dimensions (cm)		Reinforcements (f_y=4000kgf/cm^2)	
				width	depth	negative moment	positive moment
Beam	1	1-4 (Beam)	5,6,7,12,13, 14,19,20,21, 26,27,28	35	60	2-D25	8-D25
	2	5-8 (Beam)	33,34,35,40, 41,42,47,48, 49,54,55,56	35	73	2-D25	9-D25
	3	9-12 (Beam)	61,62,63,68, 69,70,75,76, 77,82,83,84	40	63	2-D25	8-D25
	4	13-16 (Be,a)	89,90,91,96,97 ,98,103,104,105, 110,111,112	33	63	2-D25	8-D25
	5	17-20 (Beam)	117,118,119, 124,125,126, 131,132,133, 138,139,140	30	58	2-D25	5-D25
Column	1	1-5 (exterior column)	1,4,8,11,15, 18,22,25,29,32	85		12-D25	
	2	1-5 (interior column)	2,3,9,10,16, 17,23,24,30,31	105		18-D25	
	3	6-10 (exterior column)	36,39,43,46,50, 53,57,60,64,67	68		8-D25	
	4	6-10 (interior column)	37,38,44,45,51, 52,58,59,65,66	90		14-D25	
	5	11-15 (exterior column)	71,74,78,81,85, 88,92,95,99,102	55		6-D25	
	6	11-15 (interior column)	72,73,79,80, 86,87,93,94, 100,101	78		10-D25	
	7	16-20 (exterior column)	106,109,113,116, 120,123,127, 130,134,137	48		6-D25	
	8	16-20 (interior column)	107,108,114,115, 121,122,128, 129,135,136	60		6-D25	
initial design	Max. F		F	20593			
			Cost	92,133,700 (won)			
	Min. F		F	612			
			Cost	99,065,600 (won)			
optimum design			F	475			
			Cost	71,753,100 (won)			