# 해석적 주파수종속 무한요소를 사용한 시간영역해석의 지반-구조물의 상호작용을 고려한 지진해석

## Time Domain Soil-Structure Interaction Analysis for Earthquake Loadings Based on Analytical Frequency-Dependent Infinite Elements

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### **ABSTRACT**

This paper presents a time domain method for soil-structure interaction analysis for seismic loadings. It is based on the finite element formulation incorporating analytical frequency-dependent infinite elements for the far field soil. The dynamic stiffness matrices of the far field region formulated using the present method in frequency domain can be easily transformed into the corresponding matrices in time domain. At first, the equivalent earthquake forces are evaluated along the interface between the near and the far fields from the free-field response analysis carried out in frequency domain, and the results are transformed into the time domain. An efficient procedure is developed for the convolution integrals to evaluate the interaction force along the interface, which depends on the response on the interface at the past time instances as well as the concurrent instance. Then, the dynamic responses are obtained for the equivalent earthquake force and the interaction force using Newmark direct integration technique. Since the response analysis is carried out in time domain, it can be easily extended to the nonlinear analysis. Example analysis has been carried out to verify the present method in a multi-layered half-space.

### 1. INTRODUCTION

The dynamic responses of massive structures, such as nuclear power plants, dams, liquid-storage tanks, and large underground structures, may be influenced by the soil-structure interaction as well as the dynamic characteristics of the exciting loads and the structures. The effect of the soil-structure interaction is noticeable especially for stiff and massive structures resting on relatively soft ground. It may cause the dynamic characteristics of the structural response altered significantly. Therefore, the interaction effects have to be considered in the dynamic analysis of the structure in a semi-infinite soil medium.<sup>[1,2]</sup>

Two important characteristics that may distinguish the soil-structure interaction system from the general structural dynamic system are the unbounded nature and the nonlinear behavior of the soil medium. The radiational damping in an unbounded soil medium can be described more easily in frequency domain than in time domain, since it is dependent on the excitation frequency. On the other hand, the nonlinear behavior of the soil medium can be considered more easily in time domain. Thus

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supplementary treatments of the soil-structure interaction in frequency and time domains may be needed to consider both characteristics of the soil medium.

At present, most of the well-known computer software packages for soil-structure interaction analyses are based on the frequency domain analysis, and utilize the equivalent linearization technique to consider the material nonlinearity of the soil medium.<sup>[6]</sup> However, in recent years, several time domain methods have been proposed to study nonlinear behaviors of the soil medium, effects of pore water, and nonlinear conditions along the interface between soil and structure. One method is the coupling of the boundary and the finite element methods.<sup>[3,4]</sup> In this method the structure and the near field soil region are modeled using finite elements, while the far field soil region is represented using boundary elements. However, it is generally difficult to derive fundamental solutions in layered soils and to couple the boundary elements with the finite elements. Another method is the one using the transformation of the dynamic stiffness matrix into the terms in time domain.<sup>[1,5]</sup> However, the dynamic stiffness matrix for the far field region is usually obtained numerically at each frequency by the conventional methods. Therefore, the transformation has to be carried out numerically using discrete Fourier transform or discrete z-transform, which requires tremendous computational time and huge computer-memory for realistic problems.

In this paper, a soil-structure interaction analysis method for earthquake loadings in time domain is presented. It is based on the finite element formulation incorporating analytical frequency-dependent infinite elements for the far field soil.<sup>[7,8]</sup> The dynamic stiffness matrices of the far field region formulated in frequency domain by the present method can be easily transformed into the corresponding matrices in time domain. At first, the equivalent earthquake forces are evaluated along the interface between the near and the far fields from the free-field response analysis carried out in frequency domain,<sup>[9,10]</sup> and the results are transformed into time domain through the inverse Fourier transform. An efficient procedure is developed for the convolution integrals to evaluate the interaction force along the interface, which depends on the response on the interface at the past time instances as well as the concurrent instance. Then the dynamic responses are obtained for the equivalent earthquake force and the interaction force using Newmark direct integration technique. Example analysis has been carried out to verify the present method in a multi-layered half-space. The present method in time domain can be easily extended to the problems with the nonlinear behaviors in the soil medium and the structure.

## 2. EOUVALENT EARTHQUAKE FORCES FROM FREE FIELD ANALYSIS

A soil-structure interaction system subjected to seismic loadings shown in Figure 1 can be represented by decomposing into two sub-systems; the scattering sub-system and the radiation sub-system as shown in Figure 2. The two sub-systems are coupled by the interaction force  $\mathbf{f}_e$  along the interface  $\Gamma_e$  between the near and the far field regions. The boundary conditions at the interface of the radiation subsystem are defined so that displacements in the interior regions (S, I, and L) of the sub-system may be identical to those of the complete system. It is noted that the displacement in the interior region of the scattering sub-system is zero. Accordingly, if one is only interested in the responses of the interior region, the complete system may be replaced by the simpler radiation sub-system, of which the equation of motion can be written in frequency domain as

$$\begin{bmatrix} \mathbf{S}_{ss}^{S} & \mathbf{S}_{sa}^{S} & 0 & 0 & 0 & 0 \\ \mathbf{S}_{ss}^{S} & \mathbf{S}_{sa}^{SI} & \mathbf{S}_{ai}^{I} & \mathbf{S}_{ab}^{I} & 0 & 0 \\ 0 & \mathbf{S}_{ia}^{I} & \mathbf{S}_{ii}^{I} & \mathbf{S}_{ib}^{I} & 0 & 0 \\ 0 & \mathbf{S}_{ba}^{I} & \mathbf{S}_{bi}^{I} & \mathbf{S}_{bb}^{IL} & \mathbf{S}_{bc}^{L} \\ 0 & 0 & 0 & \mathbf{S}_{bb}^{L} & \mathbf{S}_{lc}^{L} & \mathbf{S}_{cc}^{L} + \mathbf{S}_{cc}^{F} \\ 0 & 0 & 0 & \mathbf{S}_{cb}^{L} & \mathbf{S}_{cl}^{L} & \mathbf{S}_{cc}^{L} + \mathbf{S}_{cc}^{F} \\ 0 & \mathbf{I}_{c} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{I}_{c} \end{bmatrix}$$

where S's are the dynamic stiffness matrices;  $\mathbf{u}$  denotes the total displacement field;  $\mathbf{f}_c$  is the reaction force on the fixed interface boundary in the scattering sub-system subjected to the earthquake excitation, which can be considered as the equivalent earthquake force on the interface for the radiation sub-system; and superscripts (S, I, L, and F) and subscripts (s, a, i, b, l, e, and f) are defined as shown in Figure 1.

In this study, the soil-structure interaction system in two dimensional space is modeled using the finite elements and the infinite elements as in Figure 3.<sup>17,8,10]</sup> The structure and the near field soil region are modeled using 9-node plane strain finite elements, and the remaining far field soil region is represented using horizontal, vertical, left corner, and right corner infinite elements; i.e., HIE, VIE, LCIE, and RCIE. The present horizontal and vertical infinite elements have three nodes each on the interface between the near and the far field soil regions, while the corner infinite elements have one node as shown Figure 3(b).

The equivalent earthquake force  $\mathbf{f}_c$  can be evaluated along  $\Gamma_e$  from the free-field responses using the rigid exterior boundary method as [9,10]

$$\mathbf{f}_{e}(\omega) = \mathbf{S}_{ee}^{F}(\omega)\mathbf{u}_{e}^{f}(\omega) - \mathbf{A}\mathbf{t}_{e}^{f}(\omega)$$
(2)

where  $\mathbf{S}_{ce}^F(\omega)$  is the dynamic stiffness matrix of the far field region which can be easily calculated using the dynamic infinite elements;  $\mathbf{u}_e^f$  and  $\mathbf{t}_e^f$  are the displacement and the traction along  $\Gamma_e$  for the free-field soil medium subjected to the earthquake excitation; and  $\mathbf{A}$  is the matrix to transform the traction into the force. Since the total displacement of the nonlinear soil region 'I' can be obtained directly, the present method appears attractive for the nonlinear analysis which requires the evaluation of the stress associated with the total displacement in the soil medium.

## 3. EARTHQUAKE RESPONSE ANALYSIS IN TIME DOMAIN

Equation (1) can be rewritten into a simpler form introducing the interaction force  $\mathbf{f}_b(\omega)$  along  $\Gamma_e$  as<sup>[1]</sup>

$$\begin{bmatrix}
\mathbf{S}_{nn}(\omega) & \mathbf{S}_{nc}(\omega) \\
\mathbf{S}_{en}(\omega) & \mathbf{S}_{ce}(\omega)
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_{n}(\omega) \\
\mathbf{u}_{e}(\omega)
\end{bmatrix} = 
\begin{bmatrix}
\mathbf{0} \\
\mathbf{f}_{c}(\omega) + \mathbf{f}_{b}(\omega)
\end{bmatrix}$$
(3)

$$\mathbf{f}_b(\omega) = -\mathbf{S}_{cc}^F(\omega)\mathbf{u}_c(\omega) \tag{4}$$

where  $\mathbf{f}_b(\omega)$  is the interaction force on  $\Gamma_e$  from the far field soil region; and subscript n stands for the nodes of the structure and the near field soil region, while e denotes those on the interface  $\Gamma_e$ .

For time domain analysis, the interaction force can be transformed as

$$\mathbf{f}_{b}(t) = -\int_{0}^{t} \mathbf{S}_{cc}^{F}(t-\tau)\mathbf{u}_{c}(\tau)d\tau$$

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(5)

in which  $S_{ee}^F(t)$  is the matrix of the impulse response functions which is defined as the inverse Fourier transform of  $S_{ee}^F(\omega)$  as

$$\mathbf{S}_{ee}^{F}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathbf{S}_{ee}^{F}(\omega) e^{i\omega t} d\omega \tag{6}$$

In general, the dynamic stiffness matrix for the far field  $S_{ee}^F(\omega)$  is obtained numerically, hence it is very time consuming to compute  $S_{ee}^F(t)$  numerically using the discrete Fourier transform. Therefore the analytical frequency-dependent infinite elements recently proposed by the present authors<sup>[7,8]</sup> are utilized, of which the dynamic stiffness matrix can be derived in an analytical form of the exciting frequency and the constant matrices as

$$S_{ee}^{F}(\omega) = S_{0}^{F} + i\omega S_{1}^{F} + \frac{1}{a + i\omega} S_{2}^{F} + \frac{1}{(a + i\omega)^{2}} S_{3}^{F}$$
 (7)

where  $S_0^F$ ,  $S_1^F$ ,  $S_2^F$ , and  $S_3^F$  are constant matrices.

Then the impulse response function matrix can be obtained in an analytical form as

$$\mathbf{S}_{ee}^{F}(t) = \mathbf{S}_{0}^{F}\delta(t) + \mathbf{S}_{1}^{F}\dot{\delta}(t) + \mathbf{S}_{2}^{F}e^{-at}H(t) + \mathbf{S}_{3}^{F}te^{-at}H(t)$$
(8)

where H(t) is Heviside step function; and  $\delta(t)$  and  $\dot{\delta}(t)$  are Dirac-delta function and its derivative respectively. From equations (5) and (8), the interaction force on  $\Gamma_e$  can be also obtained in an analytical form as

$$\mathbf{f}_{b}(t) = -\mathbf{S}_{0}^{F} \mathbf{u}_{e}(t) - \mathbf{S}_{1}^{F} \dot{\mathbf{u}}_{e}(t) - \int_{0}^{t} \{\mathbf{S}_{2}^{F} + (t - \tau)\mathbf{S}_{3}^{F}\} e^{-a(t - \tau)} \mathbf{u}_{e}(\tau) d\tau$$
(9)

Finally, the time domain equation of motion for the soil-structure interaction system can be derived from equations (3) and (9) as

$$\begin{bmatrix} \mathbf{M}_{nn} & \mathbf{M}_{nc} \\ \mathbf{M}_{en} & \mathbf{M}_{ec} \end{bmatrix} \begin{pmatrix} \ddot{\mathbf{u}}_{n}(t) \\ \ddot{\mathbf{u}}_{e}(t) \end{pmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_{1}^{F} \end{bmatrix} \begin{pmatrix} \dot{\mathbf{u}}_{n}(t) \\ \dot{\mathbf{u}}_{e}(t) \end{pmatrix} + \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{nc} \\ \mathbf{K}_{en} & \mathbf{K}_{ee} + \mathbf{S}_{0}^{F} \end{bmatrix} \begin{pmatrix} \mathbf{u}_{n}(t) \\ \mathbf{u}_{e}(t) \end{pmatrix} = \begin{cases} \mathbf{0} \\ \mathbf{f}_{e}(t) + \bar{\mathbf{f}}_{b}(t) \end{cases}$$
(10)

where  $\mathbf{M}_{nn}$ ,  $\mathbf{M}_{ne}$ ,  $\mathbf{M}_{en}$ , and  $\mathbf{M}_{ee}$  are mass matrices of the inner fields;  $\mathbf{K}_{nn}$ ,  $\mathbf{K}_{ne}$ ,  $\mathbf{K}_{en}$ , and  $\mathbf{K}_{ce}$  are stiffness matrices of the inner fields; and  $\bar{\mathbf{f}}_b(t)$  is the third term of the interaction force  $\mathbf{f}_b(t)$  in equation (10) as

$$\overline{\mathbf{f}}_{b}(t) = -\int_{a}^{t} \{\mathbf{S}_{2}^{F} + (t - \tau)\mathbf{S}_{3}^{F}\} e^{-a(t - \tau)} \mathbf{u}_{a}(\tau) d\tau \tag{11}$$

The interaction force  $\bar{\mathbf{f}}_h(t)$  in equation (11) can be decomposed for the computational efficiency as

$$\bar{\mathbf{f}}_b(t) = \bar{\mathbf{f}}_{b1}(t) + \bar{\mathbf{f}}_{b2}(t) + \Delta \bar{\mathbf{f}}_b(t)$$
(12)

where

$$\bar{\mathbf{f}}_{b1}(t) = -\int_0^{t-\Delta t} \mathbf{S}_2^F e^{-a(t-\tau)} \mathbf{u}_e(\tau) d\tau$$
 (13a)

$$\bar{\mathbf{f}}_{b2}(t) = -\int_0^{t-\Delta t} (t-\tau) \mathbf{S}_3^F e^{-a(t-\tau)} \mathbf{u}_c(\tau) d\tau$$
 (13b)

and  $\Delta \overline{\mathbf{f}}_b(t) = -\int_{t-\Delta t}^t \{\mathbf{S}_2^F + (t-\tau)\mathbf{S}_3^F\} e^{-a(t-\tau)} \mathbf{u}_c(\tau) d\tau$  (13c)

Then, the above equations (13a), (13b), and (13c) can be approximately rewritten into discrete time

forms at  $t = n\Delta t$  as [8]

$$\mathbf{f}_{b1}(n\Delta t) = e^{-a\Delta t} \{ \mathbf{f}_{b1}((n-1)\Delta t) - \mathbf{S}_{2}^{F} \mathbf{u}_{e}((n-1)\Delta t) \Delta t \}$$
(14a)

$$\mathbf{f}_{b2}(n\Delta t) = e^{-a\Delta t} \{ 2\mathbf{f}_{b2}((n-1)\Delta t) - e^{-a\Delta t} \mathbf{f}_{b2}((n-2)\Delta t) - \mathbf{S}_{3}^{F} \mathbf{u}_{c}((n-1)\Delta t)(\Delta t)^{2} \}$$
 (14b)

$$\Delta \mathbf{f}_b(n\Delta t) = -\mathbf{S}_2^F \mathbf{u}_c(n\Delta t) \Delta t \tag{14c}$$

From equations (13) and (14), it can be observed that  $\bar{\mathbf{f}}_{b1}(t)$  and  $\bar{\mathbf{f}}_{b2}(t)$  can be evaluated from the known past displacements, while  $\Delta \mathbf{f}_b(t)$  is related to the unknown concurrent displacement at t.

Incorporating equations (14a), (14b), and (14c), equation (10) can be rewritten as

$$\begin{bmatrix} \mathbf{M}_{nn} & \mathbf{M}_{nc} \\ \mathbf{M}_{en} & \mathbf{M}_{ec} \end{bmatrix} \begin{pmatrix} \ddot{\mathbf{u}}_{n}(t) \\ \ddot{\mathbf{u}}_{e}(t) \end{pmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_{1}^{F} \end{bmatrix} \begin{pmatrix} \dot{\mathbf{u}}_{n}(t) \\ \dot{\mathbf{u}}_{e}(t) \end{pmatrix} + \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{nc} \\ \mathbf{K}_{en} & \mathbf{K}_{cc} + \mathbf{S}_{0}^{F} + \mathbf{S}_{2}^{F} \Delta t \end{bmatrix} \begin{pmatrix} \mathbf{u}_{n}(t) \\ \mathbf{u}_{e}(t) \end{pmatrix} = \begin{cases} \mathbf{0} \\ \mathbf{f}_{e}(t) + \overline{\mathbf{f}}_{b1}(t) + \overline{\mathbf{f}}_{b2}(t) \end{cases}$$
(15)

In this paper, Newmark direct integration procedure is applied to solve equation (15). It is noteworthy that the convolution integrals for  $\bar{\mathbf{f}}_{b1}(t)$  and  $\bar{\mathbf{f}}_{b2}(t)$  can be evaluated recursively as finite sums of a few past terms of  $\bar{\mathbf{f}}_{b1}(t)$ ,  $\bar{\mathbf{f}}_{b2}(t)$ , and  $\mathbf{u}_e(t)$  as in equations (14a) and (14b). Hence the present time domain formulation based on the analytical frequency-dependent infinite elements is very straight forward and computationally very efficient in comparison with the methods using numerical transforms such as discrete Fourier transform or discrete z-transform, which usually require huge computational efforts.

### 4. NUMERICAL EXAMPLE AND DISCUSSION

For the verification of the earthquake response analysis procedure, a site-response analysis was carried out for vertically incident SV-waves. The near field soil region is discretized with plane strain finite elements and the remaining far field soil region is modeled by analytical frequency-dependent infinite elements as shown in Figure 4. Two body wave components and 4 Rayleigh wave components are used in the infinite element formulation. The properties of the free-field system are taken to the same as those of the far field soil layers shown in Table 1. The acceleration shown in Figure 5(a), which is the NS-component of January 20, 1994 earthquake measured at A15 sensor location of Hualien LSST site, Taiwan<sup>[11]</sup>, was used as the control motion on the ground surface. The peak ground acceleration (PGA) of the control motion is 0.0318g.

The acceleration time histories at several selected points obtained from the present earthquake analysis are compared with those of the free-field analysis which are considered as the target solutions and also with the those from the frequency domain analysis. Comparisons of the acceleration time histories shown in Figures 5 – 7 and the response spectra in Figures 8 – 9 indicate that the calculated earthquake responses in frequency domain are almost identical to those of the free-field analysis, but those in time domain are slightly different with the others. The differences seem to come from the approximation in the procedure of the direct integration in time domain. In this analysis, mass, damping, and stiffness matrices of the total soil-structure interaction system are real constant, since only geometric damping of the unbounded medium is introduced without considering the equivalent material damping.

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#### 5. CONCLUSIONS

This paper presents a time domain analysis method for soil-structure interaction subjected to seismic loadings. It is based on the finite element formulation incorporating analytical frequency-dependent infinite elements for the far field soil. The dynamic stiffness matrices of the far field region formulated in frequency domain by the present infinite elements can be easily transformed into the corresponding matrices in time domain. At first, the equivalent earthquake forces are evaluated along the interface between the near and the far fields from the free-field response analysis carried out in frequency domain, and the results are transformed into the time domain. An efficient procedure is developed for the convolution integrals to evaluate the interaction force along the interface, which depends on the response on the interface at the past time instances as well as the concurrent instance. Then, the dynamic responses are obtained for the equivalent earthquake force and the interaction force using Newmark direct integration technique. Since the response analysis is carried out in time domain, it can be easily extended to nonlinear analysis. The results of the example analysis indicate that the present time domain method yields very good earthquake responses.

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Table 1 Material properties of the soil region shown in Figure 4

Soil Region	Layer Depth (m)	Shear Wave Velocity (m/sec)	Mass Density (kg/m³)	Poisson's Ratio	Hysteretic Damping Ratio
Layer I	2.0	133	1.69	0.38	0.0
Layer 2	3.0	231	1.93	0.48	0.0
Half Space	∞	333	2.42	0.47	0.0

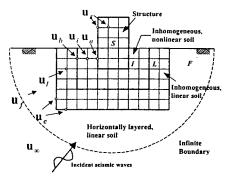
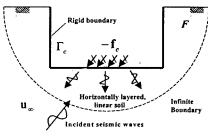
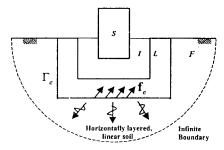


Figure 1 A complete soil-structure system and the definition of displacement fields

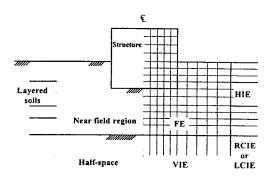


(a) scattering sub-system

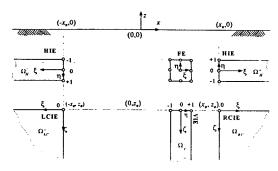


(b) radiation sub-system

Figure 2 Soil-structure sub-systems



(a) analysis model



(b) global and local coordinates

Figure 3 Soil-structure interaction system in two dimensional soil medium

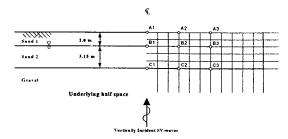


Figure 4 Finite and infinite element mesh for free-field earthquake response analysis

