A Simplified Estimation of Stress Intensity Factor on the Hertzian Contact

헤르츠접촉하에서 응력확대계수의 간단한 계산법

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초록 - 헤르츠 접촉하에 있는 반무한체에서의 표면균열을 살펴보았다. 시편의 응력확대계수 K를 구하기 위해 사용되는 간단화된 방법을 이 논문에 쓰여진 모델에 적용시켰다. 기존에 알려진 결과에 비해 상당히 만족스런 결과를 얻었으며 다른 방법보다 이 방법이 훨씬 더 편리함이 입증되었다.

I. Introduction

It was reported by Hornbogen that the wear rate is significantly affected by the fracture toughness of the material[1]. The experiments for revealing fatigue process of crack in rolling contact have carried out by Way[2]. After the works of Way, various theoretical models dealing with fatigue failure behavior of the surface crack were proposed. The propagation behavior of the surface crack of semi-infinite elastic body under Hertzian contact was investigated and the method of predicting the fatigue life of rolling contact component using the Paris'criteria of propagation life was proposed by Keer et al.[3]. In recent years, this problem was discussed by many investigators.

In the framework of the theory of elasticity, numerous analytical methods have been developed to solve the crack problems. Most investigator have been simulated the behavior of cracks for the problem of asperity contact using finite element method. The other prevailing methods are the method of continuous distribution of edge dislocations and weight function method. However, the previous methods have a common disadvantage, which is that the complicated finite element calculation must be repeated and a large amount of numerical calculations are involved.

The main objective of this paper is to suggest a simplified method which provide a more efficient methodology in the analysis of crack problems based on the linear fracture mechanics to determine stress intensity factor. The simplified method will be demonstrated at first and some known results will be studied to validate the accuracy and efficiency of this method. The current analysis may facilitate further exploration of the behavior of surface crack in semi-infinite elastic body under Hertzian contact.

II. Analytical model and method

2.1 Analytical model

Consider a Hertzian contact moving across the surface of a semi-infinite elastic body containing a surface crack from right to left, as shown in Fig.1. The surface traction at the asperity contact may expressed as:

$$P(X_1) = -P_0\sqrt{1-(X_1/c)^2}, \quad P_0 > 0$$

$$Q(X_1) = \mu P(X_1)$$

where P_0 is the maximum normal loading acting at the center of the Hertzian contact origin, μ is the coefficient of friction.

Furthermore, it is also assumed that the analytical model is under the plane-strain state

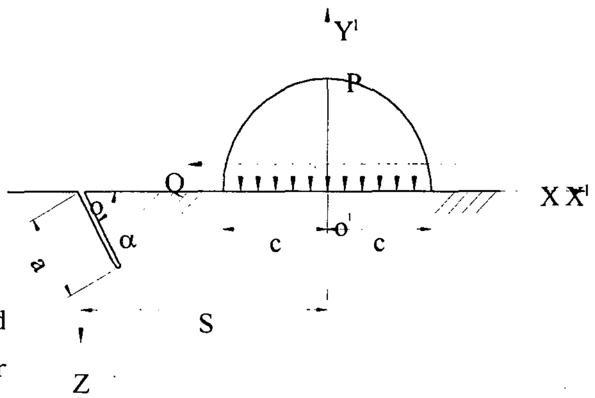

Fig.1 Analytical model and coordinate systems

2.2 Analytical method

The current analysis is based on the contact mechanics and the linear elastic fracture mechanics(LEFM).

2.2.1 Linear elastic fracture mechanics background

The stress intensity factor depends on loading, crack size, crack shape and geometric boundaries, with the general form given by

$$K = f(g)\sigma\sqrt{\pi a} \tag{1}$$

where σ = remote or nominal stress applied to component, a = crack length, $f(g)$ =correction factor depending on specimen and crack geometry

Stress intensity factors for a single loading mode can be added algebraically. Consequently, stress intensity factors for complex loading conditions of the same mode can be determined from the superposition of simpler results, such as those readily obtainable from handbooks.

Correction factor $f(g)$ is a universal function for a given crack geometry and composition. If the $f(g)$ is obtained from a single simple loading case, it can be used to calculate stress intensity factor for any other complicated loading system of the same cracked geometry. In this case, $f(g)\sqrt{\pi a}$= constant.

In determining stress intensity factor, numerical methods (including finite element methods) have been widely used. Determination methods for stress intensity factor tend to be approximate. In general, values for $f(g)$ in Eq.(1) tend to be between 1 and 1.4, with the value for many engineering situations being between 1 and 1.2. Errors in K-factor may be small compared to uncertainties in a fatigue analysis, such as material properties, load levels, load history and service environment.

The compliance function $f_{\mathrm{I}}(g)$ for some case is presented as follow[4]:

Edge crack loaded in tension

$$f_{\mathrm{I}}(g)=1.12-0.231\frac{a}{b}+10.55\left(\frac{a}{b}\right)^2-21.72\left(\frac{a}{b}\right)^3+30.39\left(\frac{a}{b}\right)^4 \tag{2}$$

Edge crack loaded in bending

$$f_{\mathrm{I}}(g)=1.122-1.40\frac{a}{b}+7.33\left(\frac{a}{b}\right)^2-13.08\left(\frac{a}{b}\right)^3+14.0\left(\frac{a}{b}\right)^4 \tag{3}$$

For small cracks($a/b \ll 1$), the higher order terms in the above equations can be ignored and they are converted into

$$f_{\mathrm{I}}(g) = 1.12$$

to express the stress in tensity factor as

$$K_{\mathrm{I}} = 1.12\sigma\sqrt{\pi a} \tag{4}$$

Similar expressions for small cracks($a/b \ll 1$)can be used for Modes II and Modes III.

For Mord II the stress intensity factor is expressed by

$$K_{II} = 1.0\sigma\sqrt{\pi a}\ , \qquad f_{II}(g) = 1.0 \tag{5}$$

2.2.2 An useful formulation of contact mechanics

By theory of contact mechanics the stress field in the semi-infinite solid due to the Hertzian distributed normal and tangential contact load are obtained as:

$$\left.\begin{aligned}
\sigma_{xx} &= \frac{q_0}{\pi}\left[(2x^2 - 2c^2 - 3z^2)\phi + 2\pi\frac{x}{c} + 2(c^2 - x^2 - z^2)\frac{x}{c}\bar{\phi}\right] \\
&\quad - \frac{p_0}{\pi} z\left(\frac{c^2 + 2x^2 + 2z^2}{c}\bar{\phi} - \frac{2}{c} - 3x\phi\right) \\
\sigma_{zz} &= \frac{q_0}{\pi} z^2\phi - \frac{p_0}{\pi} z(c\bar{\phi} - x\phi) \\
\sigma_{xz} &= \frac{q_0}{\pi}\left[(c^2 + 2x^2 + 2z^2)\frac{z}{c}\bar{\phi} - 2\pi\frac{z}{c} - 3xz\phi\right] - \frac{p_0}{\pi} z^2\phi
\end{aligned}\right\} \tag{6}$$

in which $\phi = \dfrac{\pi}{k_1} \dfrac{1 - (k_2/k_1)^{1/2}}{(k_2/k_1)^{1/2}\{2(k_2/k_1)^{1/2} + (k_1 + k_2 - 4c^2)/k_1\}^{1/2}}$

$$\bar{\phi} = \frac{\pi}{k_1} \frac{1 + (k_2/k_1)^{1/2}}{(k_2/k_1)^{1/2}\{2(k_2/k_1)^{1/2} + (k_1 + k_2 - 4c^2)/k_1\}^{1/2}}$$

where $\quad k_1 = (c+x)^2 + z^2 \qquad\qquad k_2 = (c-x)^2 + z^2$

2.3 Analytical procedure

Based on the previous results, the behavior of the surface crack in semi-infinite elastic body due to Hertzian contact can be studied.

We extend the concept as shown in Eq.(1) for dynamic stress fields in the model employed due to moving contact. Consider the non-crack semi-infinite elastic body. It is assumed that the applied stresses distribution is impartial in the small range which contained crack. The applied stress represented as that at point of crack tip.

be obtained from (6) as follows:

$$\sigma = \sigma_{xx}(-s, a) \qquad \tau = \sigma_{xz}(-s, a) \tag{7}$$

where s and a represent as the distance and crack length

Substituting (7) into (1), and for correction factor using the approximate value $f_{\mathrm{I}}(g)=1.12$ and $f_{\mathrm{II}}(g)=1.0$, the dimensionless stress intensity factor K_{I} and K_{II} can be written respectively as

$$K_{\mathrm{I}} / P_0\sqrt{\pi a} = \frac{1.12}{P_0}\sigma$$

$$K_{\mathrm{II}} / P_0\sqrt{\pi a} = \frac{1.0}{P_0}\tau \tag{8}$$

The variation of dimensionless K-factor is significantly attributed to the variation of applied stress σ or τ due to Hertzian contact.

Since only positive K_{I} values are physically significant, all negative K_{I} were nullified.

Ⅲ. Results and discussion

The simulation were performed on computer using tool program SIGMPLOT. The half-width of contact patch was set equal to 4. The friction coefficients was chosen as 0.02, 0.1 and 0.7 which simulated the cases of fluid lubrication, boundary lubrication and unlubricated respectively. To obtain generalized solutions, all the results obtained were normalized by dividing K by $P_0\sqrt{\pi a}$, and the distance by half-width of contact patch c.

Figs.3 and 4 show the variation of K_{II} with Hertzian contact position and contact friction for a/c=0.02 and a/c=0.2. For the case of μ=0.1, 0.7, a/c=0.02 and μ=0.7, a/c=0.2 , the maxima occurs when the midpoint of the contact is located at the crack. However, for other cases, K_{II} exhibits a cyclic variation, with maxima and minima occurring when the beginning edge and trailing edge of the contact reaches a position

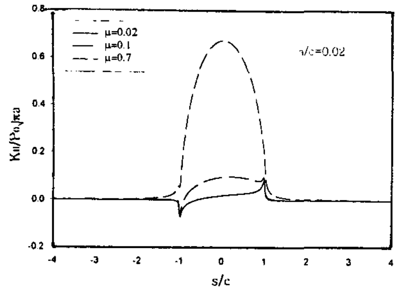

Fig.3 $K_{II}/P_0\sqrt{\pi a}$ versus s/c for a/c=0.02 and μ=0.02, 0.1, 0.7

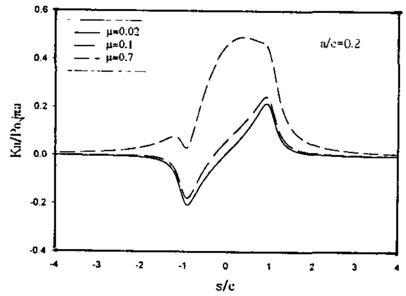

Fig.4 $K_{II}/P_0\sqrt{\pi a}$ versus s/c for a/c=0.2 and μ=0.02, 0.1, 0.7

over the crack. This fluctuation of K_{II} is considered to be the primary reason for fatigue crack growth. The greater K_{II} values is obtained for the case of unlubricated (μ=0.7). The K_{II} curves shown in Figs.3 and 4 reveal that K_{II} values increase with contact friction and crack length. Fig.5 shows the variation of K_I with Hertzian contact position and contact friction for μ=0.7 and a/c=0.02, 0.2, 1. The maximum of K_I occurs when the trailing edge of the contact reaches nearly a position over the crack. K_I increases rapidly to the maximum value and subsequently decreases gradually to zero as the contact moves further to left. The figure also implies that the K_I magnitudes were rapidly reduced with increasing crack length. The K_I values corresponding to the case of fluid lubrication and boundary lubrication are either zero or very small at all contact position. The variation of K_I is sensitive with respect to friction and crack length.

The results presented here compared with some known Kim's results[5,6](see appendix) and it is found that are in good agreement.

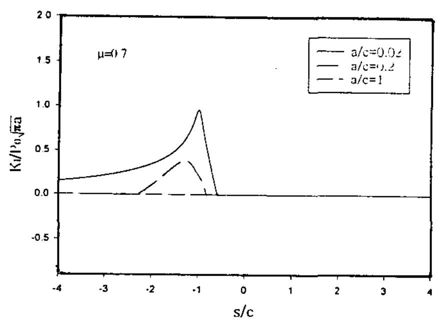

Fig.5 $K_I/P_0\sqrt{\pi a}$ versus s/c for μ=0.7 and a/c=0.02, 0.2, 1

IV. Conclusion

Behavior of the surface crack in the

semi-infinite elastic body due to Hertzian contact was studied using a simplified method. We make a comparison with Kim[5,6]and the results are in excellent agreement. It is proved that the simplified method is more convenient and efficient than other methods . The more significant results of this study are as follows:

1. For unlubricated condition, cracks may initiate at the surface because of the presence of stress intensity factor K_{I} caused by friction forces.

2. The magnitudes of the stress intensity factors increase with the increasing friction coefficients.

3. When the crack propagate to a certain depth from the surface, further crack growth is attributed to K_{II}.

4. The driving force for crack growth is K_{I} for unlubricated condition and K_{II} for both fluid and boundary lubricated condition.

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appendix

Some Kim's results got from reference[6]

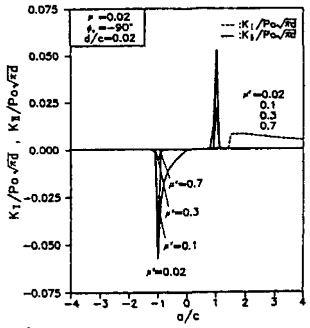

Fig. 2(a) $K_I/P_o\sqrt{\pi d}$ and $K_{II}/P_o\sqrt{\pi d}$ versus a/c for $\mu=0.02$, $\phi_o=-90°$, $d/c=0.02$, $\mu^c=0.02$, 0.1, 0.3, 0.7

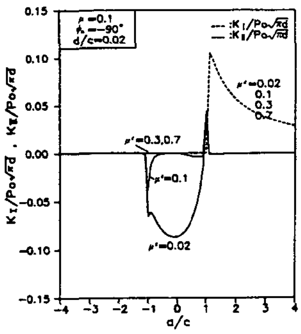

Fig. 3(a) $K_I/P_o\sqrt{\pi d}$ and $K_{II}/P_o\sqrt{\pi d}$ versus a/c for $\mu=0.1$, $\phi_o=-90°$, $d/c=0.02$, $\mu^c=0.02$, 0.1, 0.3, 0.7

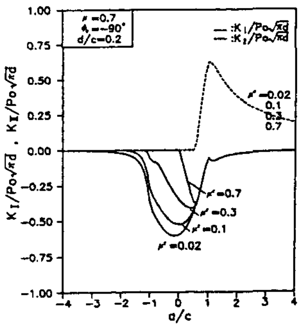

Fig. 7(a) $K_I/P_o\sqrt{\pi d}$ and $K_{II}/P_o\sqrt{\pi d}$ versus a/c for $\mu=0.7$, $\phi_o=-90°$, $d/c=0.2$, $\mu^c=0.02$, 0.1 0.3, 0.7

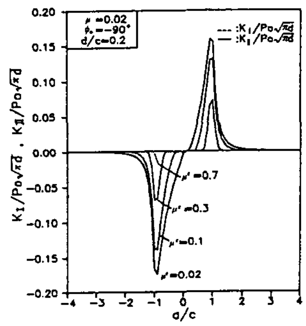

Fig. 5(a) $K_I/P_o\sqrt{\pi d}$ and $K_{II}/P_o\sqrt{\pi d}$ versus a/c for $\mu=0.02$, $\phi_o=-90°$, $d/c=0.2$, $\mu^c=0.02$, 0.1, 0.3, 0.7

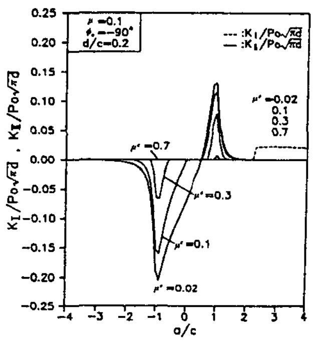

Fig. 6(a) $K_I/P_o\sqrt{\pi d}$ and $K_{II}/P_o\sqrt{\pi d}$ versus a/c for $\mu=0.1$, $\phi_o=-90°$, $d/c=0.2$, $\mu^c=0.02$, 0.1, 0.3, 0.7

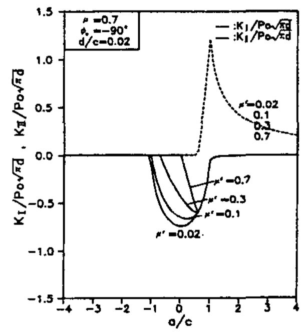

Fig. 4(a) $K_I/P_o\sqrt{\pi d}$ and $K_{II}/P_o\sqrt{\pi d}$ versus a/c for $=0.7$, $\phi_o=-90°$, $d/c=0.02$, $\mu^c=0.02$, 0. 0.3, 0.7