Effects of Cell Residence Time Distributions in Cellular Mobile Communication Systems

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ABSTRACT

We present a simulation result to the analysis of the effects of cell residence time distributions upon the expected channel occupancy time based on an analytic mobility model. Numerical examples show that exponential distribution provides upper and lower bound to the expected channel occupancy times of new calls and handoff calls. This fact reveals that the assumption of exponential distribution as the cell residence time distribution may over- or under-estimate cellular mobile systems.

1. INTRODUCTION

In mobile cellular communication systems, during communications with other people, mobile subscribers seize radio channels for certain amount of time while moving arbitrarily in a cell. So the time interval that mobiles spend in a cell (i.e., the cell residence times) affects the availability of channels in a cell. Because the channel occupancy time is closely related to the cell residence time distribution, it may over- or under-estimate real systems to assume a particular distribution for the cell residence time distribution without analytic bases. Previously, for the sake of simplicity, in the absence of any proven probability distribution, most of researches dealing with the mobility problem have assumed, either explicitly or implicitly, the cell residence time to be an exponentially distributed random variable [1] - [5]. [6] has shown that the cell residence time can be described by the generalized gamma distribution. But the cell residence time may have different distributions according to a wide range of mobility characteristics of cell regions. So "what is the proper distribution of the cell residence time" is a somewhat open question. The present work is motivated by the recognition of this characteristic. [7] used *phase-type* distributions in order to describe the call times and the cell residence times in the environment of mixed platforms and mixed call types. Because it modeled system states as a multidimensional birth-death process, the size of the state space grows quickly while the number channels increases.

Our mobility model reduces the short-coming of the *phase-type* model, assuming that each platform-type has a different cell residence time distribution which has a general form. For the simulation, we test three distributions: gamma, normal, and exponential.

2. EXPECTED CHANNEL OCCUPANCY TIMES

There are K platforms on which mobile units can be mounted. The distribution of the cell residence time Y_k for platform k is given as $G_k, \cdots, K$, and the corresponding *pdf* is denoted by $g_k(\cdot)$. The call duration X of any call is assumed to be an exponential distribution with mean $\frac{1}{\gamma}$. If we let the residual cell residence time of platform k Z_k, the distribution of Z_k is the equilibrium distribution of G_k which is denoted by G_k^e.. We use the notation $g_k^*(s)$ as *Laplace* transform of $g_k(\cdot)$, which is $\int_0^\infty e^{-st} d\,G_k(t)$.

There may exist four kinds of ongoing calls in a cell. The first two are *handoff* calls from the neighboring cell (cell A): type 1 is to terminate its call session in the current cell (cell B) and type 2 is to continue its call session over the other neighboring cell (cell C). The last two are *new* calls generated in the current cell: type 3 is to complete its call duration in the current cell and type 4 is a handoff call into the other neighboring cell (cell C). These four call types are illustrated in Fig. 1. Here $E[Si]$ $(i=1,2,3,4)$ denotes the expected channel occupancy times of type i call.

Given that a requested call is a handoff call from platform k, it will be type 1 if its cell residence time is greater than its call duration. So we can derive the type 1 probability using the *Laplace* transform of the cell residence time distribution. The probability that a handoff call is type 1 is

$$\begin{aligned} p_1 &= P\{Y_k > X\} \\ &= \int_0^\infty P\{Y_k > x\}\gamma e^{-\gamma x}dx \\ &= \int_0^\infty (1 - G_k(x))\gamma e^{-\gamma x}dx \\ &= 1 - \int_0^\infty \gamma e^{-\gamma x}\int_0^x g_k(y)dydx \\ &= 1 - \int_0^\infty g_k(y)\int_y^\infty \gamma e^{-\gamma x}dxdy \\ &= 1 - g_k^*(\gamma). \end{aligned} \tag{1}$$

Since $p_1 + p_2 = 1$, the probability that a handoff call is type 2 is

$$p_2 = g_k^*(\gamma). \tag{2}$$

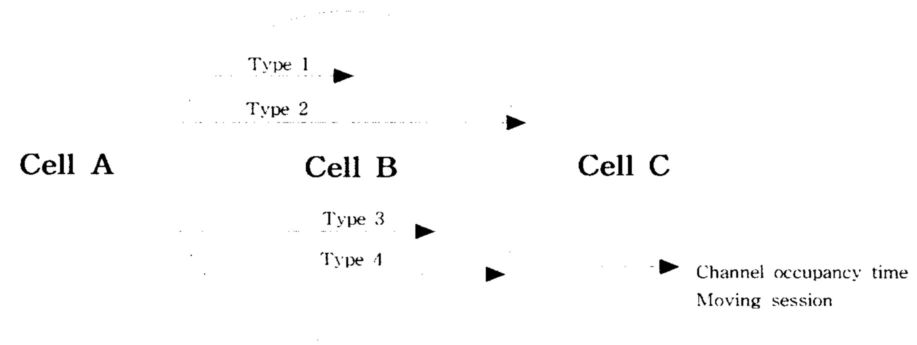

Fig. 1. Four call types in a cell

Similarly, when an entered call request is a new call, it will end its call session if the residual cell residence time is greater than its call duration. Hence, the probability that a new call is type 3 is

$$\begin{aligned} p_3 &= P\{Z_k > X\} \\ &= \int_0^\infty P\{Z_k > x\}\gamma e^{-\gamma x}dx \\ &= \int_0^\infty (1 - G_k^e(x))\gamma e^{-\gamma x}dx \\ &= 1 - a_k^*(\gamma) \\ &= 1 - \frac{1 - g_k^*(\gamma)}{\gamma E[Y_k]}. \end{aligned} \tag{3}$$

where $a_k^*(s)$ denotes the *Laplace* transform of the equilibrium *pdf* of the cell residence time $(= \int_0^\infty e^{-st}dG_k^e(t))$. Since $p_3 + p_4 = 1$, the probability that a new call is type 4 is given as

$$p_4 = a_k^*(\gamma) = \frac{1 - g_k^*(\gamma)}{\gamma E[Y_k]}. \tag{4}$$

If we S_{ki} be the channel occupancy time of type i call from platform k, then the expected call duration of each type in a cell is obtained by

$$E[S_{k1}] = E[X \mid X < Y_k], \tag{5a}$$

$$E[S_{k2}] = E[Y_k \mid X > Y_k], \tag{5b}$$

$$E[S_{k3}] = E[X \mid X < Z_k], \tag{5c}$$

$$E[S_{k4}] = E[Z_k \mid X > Z_k]. \tag{5d}$$

The conditional expectations in (5a) ~ (5d) can be derived as follows.

$$\begin{aligned} E[X \mid X < Y_k] &= \frac{\int_0^\infty x\gamma e^{-\gamma x}\int_x^\infty g_k(y)dydx}{1 - g_k^*(\gamma)} \\ &= \frac{1}{\gamma} - \frac{\int_0^\infty ye^{-\gamma y}g_k(y)dy}{1 - g_k^*(\gamma)}, \end{aligned} \tag{6a}$$

$$E[Y_k \mid X > Y_k] = \frac{\int_0^\infty ye^{-\gamma y}g_k(y)dy}{g_k^*(\gamma)} \tag{6b}$$

$$\begin{aligned} E[X \mid X < Z_k] &= \frac{\int_0^\infty x\gamma e^{-\gamma x}\int_x^\infty G_k^e(y)dydx}{1 - a_k^*(\gamma)} \\ &= \frac{1}{E[Y_k](1 - a_k^*(\gamma))}\left(\frac{1}{\gamma}\int_0^\infty ye^{-\gamma y}g_k(y)dy \right. \\ &\quad \left. - \frac{2(1 - g_k^*(\gamma))}{\gamma^2} + \frac{E[Y_k]}{\gamma}\right), \end{aligned} \tag{6c}$$

$$\begin{aligned} E[Z_k \mid X > Z_k] &= \frac{1}{E[Y_k]a_k^*(\gamma)}\left(\frac{1 - g_k^*(\gamma)}{\gamma^2} \right. \\ &\quad \left. - \frac{1}{\gamma}\int_0^\infty ye^{-\gamma y}g_k(y)dy\right). \end{aligned} \tag{6d}$$

If we denote S_{kh} to be the channel occupancy time of handoff calls of k-platform, its expectation is

$$\begin{aligned} E[S_{kh}] &= p_1E[S_{k1}] + p_2E[S_{k2}] \\ &= \frac{1 - g_k^*(\gamma)}{\gamma}. \end{aligned} \tag{7}$$

Similarly, the expectation of the channel occupancy time of a new call from k-platform in a cell, denoted by S_{kn}, is obtained by

$$\begin{aligned} E[S_{kn}] &= p_3E[S_{k3}] + p_4E[S_{k4}] \\ &= \frac{1}{\gamma} - \frac{1 - g_k^*(\gamma)}{\gamma^2 E[Y_k]}. \end{aligned} \tag{8}$$

3. NUMERICAL EXAMPLES

To investigate the effects of cell residence time distributions upon the expected channel occupancy time, we test three distributions: 1) gamma, 2) truncated normal and 3) exponential distribution. The parameters are chosen to have the same mean and variance at the same mean level (in exponential case, only the mean is considered). To give a consistency, coefficient of variation (c_s^2) is adjusted to the same value at each mean level. Fig. 2 ~ Fig. 4 represent simulation results when c_s^2 = 0.25, 0.49 and

0.81, respectively.

The exponential case provides upper bound for the expected channel occupancy time of new calls, and lower bound for that of handoff calls. The simulation result shows that as the expected cell residence time is larger, the gamma and normal cases converge to the exponential case.

Our result shows that it may over- or under-estimate cellular mobile systems to assume exponential distribution for the cell residence time distribution. Therefore, Markovian approaches assuming exponential distribution for the cell residence time distribution can not provide an effective model.

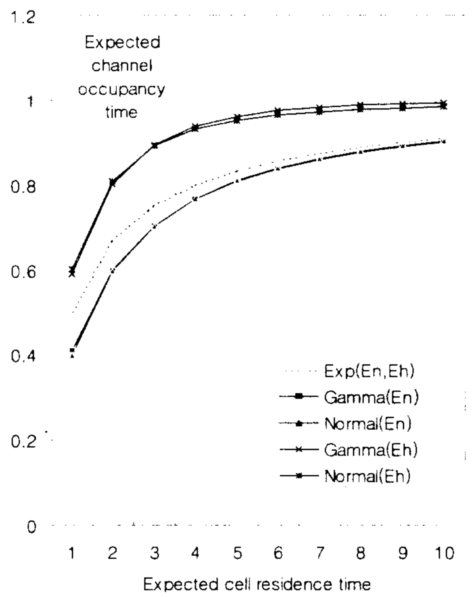

Fig. 2. Effects of cell residence time distributions on expected channel occupancy time (c_s^2=0.25)

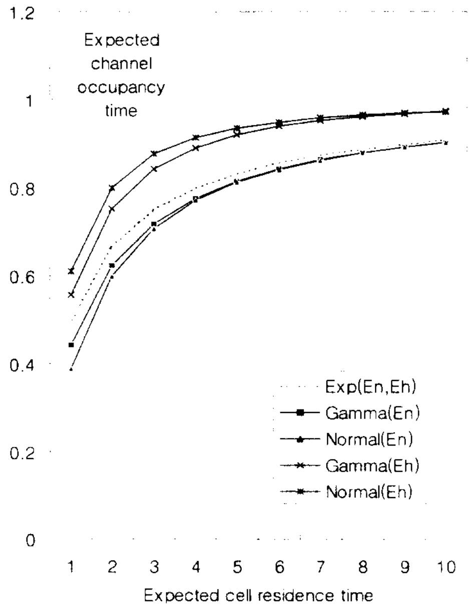

Fig. 3. Effects of cell residence time distributions on expected channel occupancy time (c_s^2=0.49)

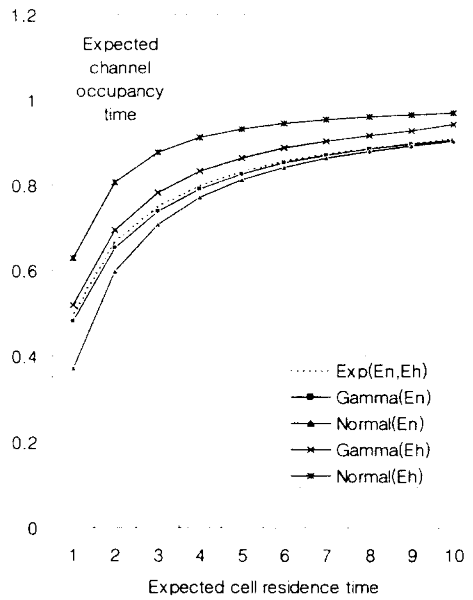

Fig. 4. Effects of cell residence time distributions on expected channel occupancy time (c_s^2=0.81)

4. CONCLUSIONS

We present a simulation result to the analysis of the effects of cell residence time distributions upon the expected channel occupancy time based on an analytic mobility model. Our result shows that exponential distribution is not appropriate for the cell residence time distribution. Additionally, exponential case provides upper and lower bound to the expected channel occupancy times of new calls and handoff calls.

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