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The nonlinear stability of density fronts in the ocean

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1. Introduction

Fronts are regions of large horizontal gradients of certain properties, usually including density. Fronts are important natural barriers in the ocean. While they hinder horizontal transfers of heat, momentum and other properties, they play a crucial role in enhancing vertical exchanges. They are also a large reservoir of available potential energy, which is often released in the form of eddies and rings. Density or temperature fronts and isolated eddies are common features in many coastal regions of the world oceans.

There are two important kinematic characteristics that must be accounted for in a complete dynamical description of the currents associated with density fronts. The first focuses on the fact that the isopcynal deflections associated with density currents and solitary cold eddies are not small in comparison with the scale height of the hydrostatic geopotential. The second characteristic is the fact that these flows are strongly baroclinic.

Cushman-Roisin (1986) first gave a one-layer frontal geostrophic model for describing the dynamics of surface density fronts in the ocean. His main idea is, from primitive equation, similar to the procedure of establishment of quasigeostrophic model to obtain the frontal geostrophic model, but in the derivation of the frontal geostrophic equations the Rossby number is made small by requiring that the length scale be larger than the radius of deformation due to the characteristic of the finite interfacial displacements. Due to the baroclinic of fronts in the ocean, Cushman-Roisin et al. (1992) established the two-layer frontal geostrophic model on the basis of the one-layer one. This model overcomes the disadvantages of the form models, and is suitable for both of the baroclinic and the finite interfacial displacements of fronts in the ocean. Thus it plays a important role in studying the occurring, evolving, destroying and bursting of fronts in the ocean.

In this paper, we first obtain the two-layer frontal geostrophic equations for density fronts on the coastal regions by considering the bottom topography is a sloping continental shelf. We study the nonlinear stability of surface fronts in the ocean and fronts on a sloping bottom by using the frontal geostrophic models. Then the nonlinear stability criteria for the two kinds of fronts are obtained by using Arnol'd (1965, 1969) variational principle and a prior estimate method. It is shown that for surface fronts in the ocean the nonlinear stability criterion is first obtained, and for fronts in a sloping bottom our result is better than the form ones.

2. Derivation of the Governing Equations

2.1 The model for fronts on a sloping continental shelf

In order to keep some degree of clarity in the derivations, the analysis starts with a series of assumptions aimed at limiting the number of parameters. These are two layer, f-plane, a sloping bottom, rigid lid, and the upper layer at least as deep as the lower layer (Fig.1). The equations that govern an oceanic system with one interfacial degree of freedom are those of the two-layer shallow water model in f-plane:

$$\vec{V}_{it}+(\vec{V}_i\cdot\nabla)\vec{V}_i+f_0\vec{k}\times\vec{V}_i=-\nabla p_i,\quad i=1,2 \tag{1}$$

$$h+\nabla\cdot(h_i\vec{V}_i)=0,\quad i=1,2 \tag{2}$$

where f_0 is the Coriolis parameter (constant), $\vec{V}_i=(u_i,v_i)$, $i=1,2$ is horizontal velocity vector, $\nabla=(\partial_x,\partial_y)$ with (x,y) the horizontal coordinates and t is time, subscripts with respect to (x,y,t) indicate partial differentiation, $\vec{k}$ is unit vertical vector, and h_i is the ith layer depth. And by the assumptions, the pressure p_i can be written as

$$p_1=\pi,\quad p_2=\pi+g'(\eta-\alpha^*y) \tag{3}$$

where π is the pressure in the upper layer and η the upward interfacial displacement, $g'=g(\rho_2-\rho_1)/\rho_2$ is the reduced gravity (stable stratification), ρ_1 and ρ_2 are the densities in the upper layer and the lower layer, and α^* is the slope parameter.

$$h_1=H-H_2-\eta,\quad h_2=H_2+\eta-\alpha^*y \tag{4}$$

where H is the total depth of fluid, H_2 is the lower layer depth.

By using some assumptions and geostrophic balance $\vec{V}_1=\vec{k}\times\nabla p_1$, we can obtain the flowing equations

$$(\nabla^2\pi+h)_t+J(\pi,\nabla^2\pi+h)=0 \tag{5}$$

$$h_t-J(h-y,\pi+(h-y)\nabla^2(h-y)+\frac{1}{2}\nabla(h-y)\cdot\nabla(h-y))=0 \tag{6}$$

Eqs.(5) and (6) are the two-layer frontal geostrophic model for fronts on a sloping continental shelf. For this model, we can obtain the conservations by using the boundary conditions,

$$\frac{d}{dt}\int\int F(\frac{\varepsilon^2}{\delta}\nabla^2\pi+h)dxdy=0 \tag{7}$$

$$\frac{d}{dt}\int\int G(h-y)dxdy=0 \tag{8}$$

where F and G are two arbitrary integrable functions.

2.2 The model for surface density fronts in the ocean

In this case, we consider β -plane i.e., $f=f_0+\beta y$ and the lower boundary is flat (Fig.2). Similar to Section 2.1, we can obtain the two-layer frontal geostrophic model for surface density fronts in the ocean as follows

$$h_t+J(\pi+h\nabla^2h+\frac{1}{2}\nabla h\cdot\nabla h,h)=0 \tag{9}$$

$$(\nabla^2\pi+h+y)_t+J(\pi,\nabla^2\pi+h+y)=0 \tag{10}$$

and we have the conservations as follows

$$\frac{d}{dt}\int\int F^*(\nabla^2\pi+h+y)dxdy=0 \tag{11}$$

$$\frac{d}{dt}\int\int G^*(h)dxdy=0 \tag{12}$$

where F^* and G^* are two arbitrary integrable functions.

3. Conclusion and discussion

In this paper, the frontal geostrophic model, which was first developed by Cushman-Roisin (1986) and Cushman-Roisin et al. (1992), was extended to describe the dynamic system of density fronts on a sloping continent shelf. The new frontal geostrophic model is a two-layer with a sloping bottom. The nonlinear stability of two kinds fronts in the ocean, i.e., surface density fronts and fronts in coastal regions, was discussed. And nonlinear stability criteria of two kinds of two-layer frontal geostrophic models were obtained. It is shown that for fronts in the sloping bottom, our result is better than that of Swaters(1993); and for surface fronts, we first give a nonlinear stability criterion. It should be pointed, we only focused on the theoretical analyses on this problem. How to apply the theoretical results to the actual oceanic problem is our future aim.

Acknowledgements

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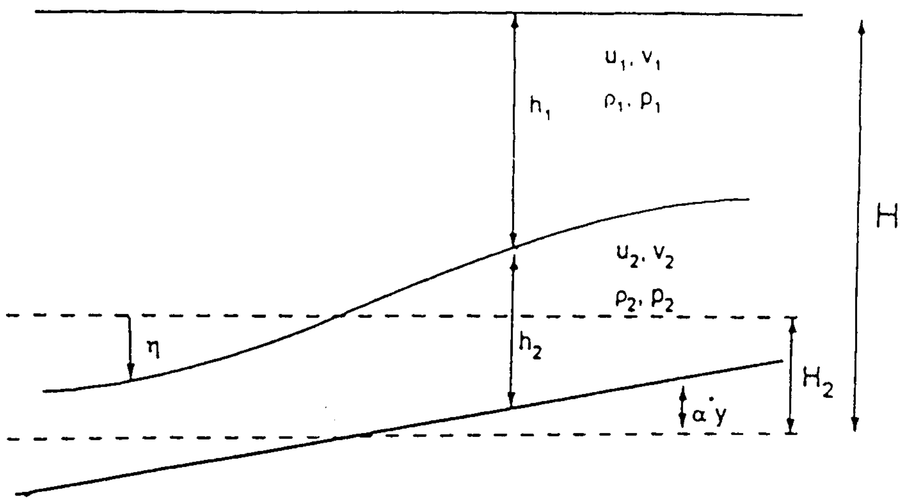

Fig. 1. Geometry of the two-layer model for fronts on a sloping bottom.

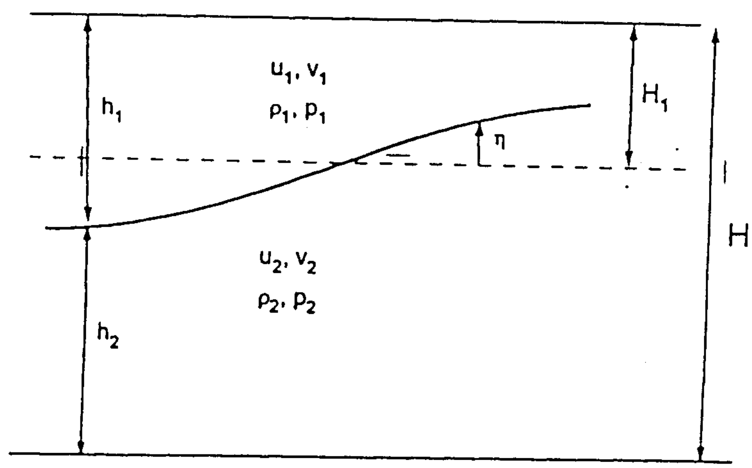

Fig. 2. Geometry of the two-layer model for surface fronts in the ocean.