Measurement of the phase sensitivity of a Mach-Zehnder Interferometer by practical detectors

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In a recent study[1] the phase sensitivity of MZI(Mach-Zehnder Interferometer) was analysed in an ideal situation, where the ideal photodetectors were assumed for the two kinds of measurement schemes i.e., measurement of the photon number of one output and of the coincidence detection at two outputs.

In this paper, we are going to show by the straightforward calculation of the expectation values and the variance of the involved operators that the sensitivity also depends on the quantum efficiencies of two detectors with the quantum efficiencies, μ_1 and μ_2, respectively. The detection schemes are the measurement of half the difference of two output photon numbers ($\hat{J}_z$) and its the square ($\hat{J}_z^2$).

The model of non-ideal photodetection have been studied in detail by Yuen and Shapiro[2]. They showed that for photodetection with quantum efficiency $0 < \mu \leq 1$, the detected field mode is described by a photon annihilation operator, $\hat{a}' = \sqrt{\mu}\hat{a} + \sqrt{(1-\mu)}\hat{v}$, where $\hat{a}$ and $\hat{v}$ are the annihilation operators for the input mode and vacuum-state mode.

Let us assume that we have two photodetectors, D_1 and D_2, with quantum efficiencies of μ_1 and μ_2, respectively, as shown in Fig.1.

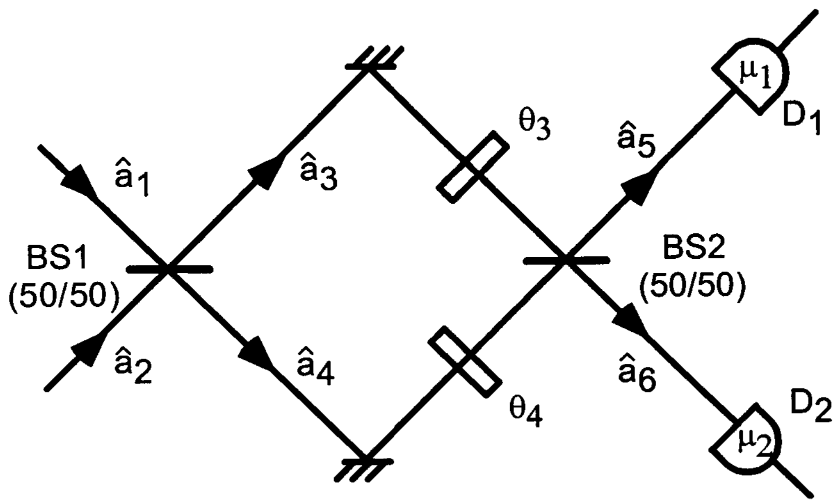

Fig.1 The schematic diagram of a Mach-Zehnder Interferometer

The operators of the absorbed photon numbers by two practical detectors attached at the two output ports are given as follows.

$$\hat{n}_5' = \hat{a}_5'^{+}\hat{a}_5',$$
$$\hat{n}_6' = \hat{a}_6'^{+}\hat{a}_6'.$$

Then the operator for half the difference between two photon numbers will be

$$\hat{J}_z = \frac{1}{2}(\hat{n}_6' - \hat{n}_5'),$$
$$\hat{J}_z^2 = \frac{1}{4}(\hat{n}_6' - \hat{n}_5')^2.$$

We can calculate phase-sensitivity for the two kinds of measurement schemes $\hat{J}_z$, and $\hat{J}_z^2$ by their variances, given by $(\Delta J_z)^2 = \langle \hat{J}_z^2 \rangle - \langle \hat{J}_z \rangle^2$ and $(\Delta J_z^2)^2 = \langle \hat{J}_z^4 \rangle - \langle \hat{J}_z^2 \rangle^2$ for the general Fock states $| n_1, n_2 \rangle$ as the inputs of the interferometer.

In the $\hat{J}_z$ schemes, the phase-sensitivity consists of ideal and non-ideal terms. When the quantum efficiencies of two detectors is unity, phase-sensitivity is exactly the same as the general Fock state input given by ref.[1]. In case of one Fock state input $(n_1 = n \gg 1, n_2 = 0)$, it depends on the quantum efficiency of detectors, while for the twin Fock state input, phase-sensitivity diverges to infinity $\Delta\theta \rightarrow \infty$.

The phase-sensitivity in the $\hat{J}_z^2$ scheme approaches to classical limit, for the one Fock state input $(n_1 = n \gg 1, n_2 = 0)$. In case of a twin Fock state input $(n_1 = n_2 = n)$, phase-shift uncertainty can reach the quantum limit.

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Reference

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[2]. H. P. Yuen and J. H. Shapiro, in *coherence and quantum optics IV*, edited by L. Mandel and E. Wolf (plenum, New York, 1978), p. 719.