A Numerical Analysis of The Single Trapped Atom Laser

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Some time ago, Mossberg group at the University of Oregon has experimentally demonstrated a laser action taking place in a nearly homogeneous ensemble of strongly driven two-state atoms[1]. When the atom is *dressed* by the driving field, the atom–field system assumes the *dressed*-level structure [Cf. FIG. 1 of Ref.[1]]. When the driving field is red-detuned from the atom, for instance, the lower dressed-states will approach the lower-state of the *undressed* atom as the detuning becomes redder. Accordingly, there occurs an inversion on dressed-state transition between the lower dressed-state in an n-th manifold and the upper dressed-state in the $(n-1)$-th manifold, thus possibly providing gain at the corresponding transition frequency. With the cavity tuned to this inverted sideband, the transition from fluorescence to lasing was observed as the number density of the atomic beam traversing the cavity increased. In this talk, however, we wish to point out this is not the whole story. A much stronger lasing action can occur at the cavity resonance tuned to a certain frequency, i.e., at $\omega_C \approx \omega_L + g^2/(\omega_A - \omega_L)$ where ω_A and ω_L denote the atomic resonance and the driving field frequency, respectively, which can be quite easily predicted by a semiclassical analysis. Surprise or not, this particular frequency, however, does not seem to be explicable by the foregoing simple (singly) dressed-atom picture. Consider a Jaynes-Cummings system where the two-state atom coupled to the single cavity mode is driven by an external field. The Hamiltonian of the system is

$$H = \frac{1}{2}\hbar\omega_A\sigma_z + \hbar\omega_C a^\dagger a + i\hbar g\left(\sigma_+ a - \sigma_- a^\dagger\right) + i\hbar\mathcal{E}\left(\sigma_+ e^{-i\omega_L t} - \sigma_- e^{i\omega_L t}\right). \tag{1}$$

where $a(a^\dagger)$ is the field annihilation(creation) operator; $\sigma_\pm$, the atomic operators such that $[\sigma_+, \sigma_-] = 2\sigma_z$, with ω_L, the driving field frequency; and $\mathcal{E}$, the driving field amplitude; and g, the vacuum Rabi frequency. With the dissipation included, the system will be described by the master equation

$$\dot{\rho} = \frac{1}{i\hbar}[H,\rho] + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \frac{\gamma}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-) \tag{2}$$

where 2κ represent the energy dissipation rate from the sides of cavity and γ, from the atom, . For brevity, however, let us first consider the case of negligible atomic and cavity dissipation. This variant Jaynes-Cummings model[2] reveals a variety of interesting behaviors. One of us has shown that on resonance, i.e., $\omega_A = \omega_C = \omega_L$, the mean photon number $\langle a^\dagger a\rangle$ varies sinusoidally from zero to $4|\alpha|^2$ with $\alpha = \mathcal{E}/g$, thus centered at $2|\alpha|^2$. The oscillation then shows another kind of "collapse-and-revival" type behavior as time proceeds[3]. In case where $\omega_A \neq \omega_C = \omega_L$, the problem is again analytically[5] solvable. The behaviors of $\langle a^\dagger a\rangle$ is shown in Figs. 1 (a) and (b) for the two cases. The scales of the photon number oscillation are almost exactly the same although the time scales are quite different.

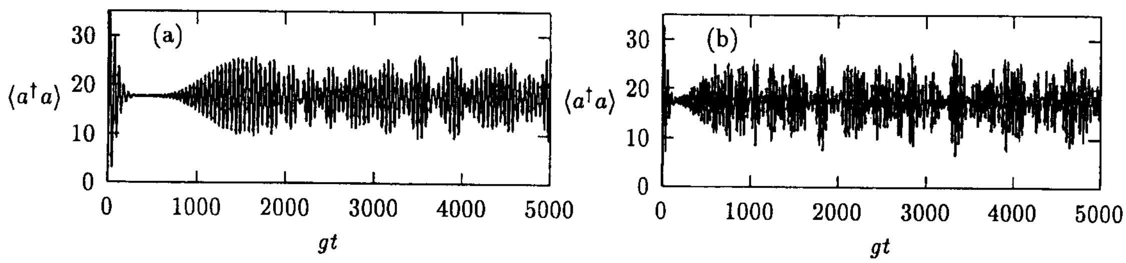

Fig. 1. (a) $\omega_L = \omega_C = \omega_A - 5g$, (b) $\omega_L = \omega_C = \omega_A$. In both cases, $\alpha = 3.0$, $\kappa = \gamma = 0$.

When general detunings are introduced, however, the oscillation scale of $\langle a^\dagger a\rangle$ (as well as the time scale) can be fundamentally different from the foregoing two restricted cases. To show this, we first present the temporal behavior of $\langle a^\dagger a\rangle$ in Fig. 2 for $\alpha = 1$, with $\delta_{AL} = \omega_A - \omega_L = 10g$, $\delta_{AC} = \omega_A - \omega_C = 9.94g$. In our knowledge, such a high excitation of cavity mode for the given driving field amplitude has not been reported previously. In Figs. 3 (a)–(c), we scan the driving laser frequency for the resonant atom–cavity system. In (a), we have the well-known vacuum-Rabi splitting as the driving field is sufficiently small. However, as the driving field intensity increases, it is apparent that the maximum cavity mode excitation occurs *not* at the locations of the Mollow sidebands, but at several "unexpected" locations of the cavity resonance, depending on the other parameters such as $\mathcal{E}$ and δ_{AL}. And in (d), we fix the laser frequency at $\delta_{AL}/g = -1$ and scan the cavity resonance frequency. Indeed, it is this configuration that corresponds to the experiment of Mossberg group. It is, however, quite surprising that the highest cavity mode excitation occurs simply elsewhere other than the Mollow sidebands.

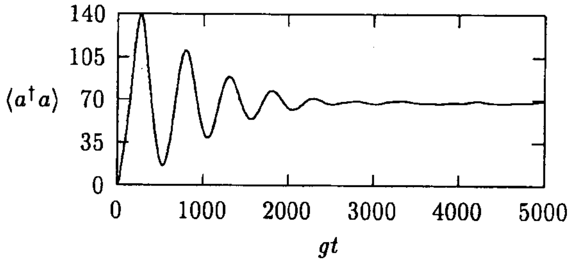

Fig. 2. $\alpha = 1.0$. $\delta_{AL}/g = 10$, $\omega_{AC}/g = 9.94$.

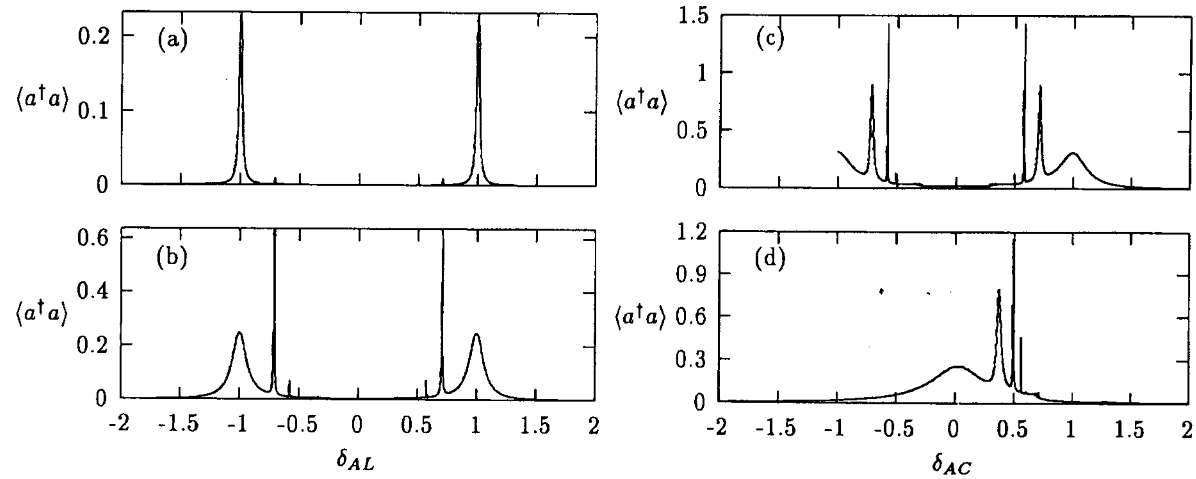

Fig. 3. (a)–(c) The maximum time-averaged cavity photon number vs. δ_{AL} with $\delta_{AC} = 0$. (a) $\alpha = 0.01$, (b) 0.05, and (c) 0.1. (d) The maximum time-averaged cavity photon number vs. δ_{AC} with $\delta_{AL}/g = -1$ and $\alpha = 0.01$. Note the unfamiliar sharp peaks at the unexpected locations.

In the talk, we will discuss why we have these interesting behaviors of the system in terms of a semiclassical language.

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1. A. Lezama, Yifu Zhu, Manoj Kanskar, and T. W. Mossberg, Phys. Rev. A **41**, 1576 (1990).

2. E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963).

3. Y. T. Chough and H. J. Carmichael, Phys. Rev. A **54** 1709 (1996).

4. D. Kleppner, Phys. Rev. Lett. **47**, 233 (1981).

5. Albeit the solutions are given in the form of infinite series.