Three-dimensional incompressible viscous solutions based on the unsteady physical curvilinear coordinate system

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ABSTRACT

The development of unsteady three-dimensional incompressible viscous solver based on unsteady physical curvilinear coordinate system is presented. A 12-point finite analytic scheme based on local uniform grid spacing is extended for nonuniform grid spacing. The formulation of a condition is suggested to avoid the oscillation of the series summations produced by the application of the method of separation of variables. SIMPLER and pressure Poisson equation techniques are used for solving a velocity-pressure coupled problem. The matrix is solved using the Generalized Minimal RESidual (GMRES) method to enhance the convergence rate of unsteady flow solver and the Kinematic boundary condition of a free surface flow. It is demonstrated that the numerical solutions of these equations are less mesh sensitive.

1. Introduction

In the physical curvilinear coordinate system, each component of the velocity has the same direction as the direction of each coordinate line and has physical value. The physical curvilinear component form of the divergence of a velocity vector has been widely used to make the pressure equation in velocity-pressure coupled problem because of its simple form. The physical curvilinear components form of the velocity was first introduced to give physical meaning to covariant or contravariant component forms lost by transformation by Truesdell [1]. Demirdzic et al. [2] derived the physical curvilinear components form in nonorthogonal coordinates for Reynolds-averaged Navier-Stokes equations. The equations of the Cartesian forms were transformed directly into physical curvilinear component forms by a two-step procedure; then they were applied to a two-dimensional problem. Until now, practical applications have been limited to two-dimensional problems, because of large storage requirements and numerical error associated with the evaluation of geometric tensor and connection coefficients on the cell face. An approach which is different from Demirdzic et al.'s approach [2], was tried by Lee [3]. In his approach, the partial differential equations with the coordinate-free vector form are transformed into the physical curvilinear coordinate system using general transformation laws.

To derive the governing equations of the unsteady flow, two kinds of approaches have been suggested. One is to use the concept of Lie derivatives (Schouten, [4]). Ogawa and Ishiguro [5] derived the field equations using the contravariant components form in moving coordinates under the concept of Lie derivatives. In the present research, the other approach taken by Warsi [6] is used because of its simplicity in expressing the time derivative.

Numerical methods that have been generally used to discretize partial differential equations are finite volume, finite difference, and finite element methods. Among these methods, the finite volume methods have been widely used in compressible and incompressible flows. The Roe's approximate Riemann solver (Roe [7]) especially has been used to calculate the numerical flux at cell faces primarily used in compressible flow. However, this scheme is based on a one-dimensional problem in which the waves travel normal to the cell face. Higher order schemes for the numerical flux vector was studied by Whitfield et al. [8]. Another method used to discretize partial differential equation is the finite analytic method by Chen and Chen [9]. This method provides an upwinding effect and avoids the truncation error. Several methods based on a one-dimensional, a two-dimensional, and a three-dimensional problem have been studies. In the present research, a 12-point scheme based on a hybrid method that combines a two-dimensional and a one-dimensional problem is selected to discretize unsteady three-dimensional viscous incompresible equations by Patel et al. [10]. The standard form of finite analytic scheme results in nonconservative forms. Therefore, the accuracy can suffer due to the lack of conservation. However, the modeling of three-dimensional equations based on a hybrid method is noted to be more accurate in converting differential equations into discretized equations than that based on a one-dimensional problem of the finite volume method. In the present research, the finite analytic coefficients based on the local uniform grid spacing by Patel et al. [10] are extended for nonuniform grid spacing.

Each analytic coefficient has a series summation. Unfortunately, in many instances, the series summation does not converge and oscillates. The success of a finite analytic scheme depends on the calculation of a series summation produced by using the method of separation of variables in the discretization of the partial differential equation. In this research, this problem is addressed by formulating the new condition to avoid the oscillation. The efforts have been made in the calculation of finite analytic coefficients.

The Kinematic condition has been applied to update the unknown location in the incompressible free surface flow. This condition can be derived using the concept of the material surface and is coupled with the momentum equations. This condition is required as the boundary condition to solve the free surface motion and is a hyperbolic PDE. The method is introduced to solve this condition. No artificial dissipative terms are required and added in this method.

The methodologies developed and derived in this research are validated by computing a three dimensional lid-driven cavity laminar flow and a free surface turbulent flow.

2. Transformation

In the following analysis, x_i are Cartesian coordinates, ξ^i are curvilinear coordinates and $\xi^{(i)}$ are physical curvilinear coordinates. First, consider the general transformation laws under the two change of the coordinates system x_i to ξ^i and ξ^i to $\xi^{(i)}$. The relationship between the coordinates x_i and ξ^i can be expressed as follows.

$$\xi^i = \xi^i(x_j, t) \tag{1}$$

The relationship between the coordinates ξ^i and $\xi^{(i)}$ can be expressed as follows:

$$\xi^{(i)} = \xi^{(i)}(\xi^i) \quad \text{where} \quad \Delta\xi^{(i)} = \sqrt{g_{ii}}\,\Delta\xi^i \tag{2}$$

$\sqrt{g_{ii}}$ are evaluated at ξ^k = constant and $k \neq i$. and $\xi^{(i)}$ resemble the coordinate stretching in each direction of ξ^i. In view of transforming the coordinates from x_i to ξ^i and ξ^i to $\xi^{(i)}$, the vector $d\underline{r}$ can be written as:

$$\begin{aligned} d\underline{r} &= \frac{\partial \underline{r}}{\partial \xi^i} d\xi^i = \underline{a}_i d\xi^i \\ &= \frac{\partial \underline{r}}{\partial \xi^{(i)}} d\xi^{(i)} = \underline{a}_{(i)} d\xi^{(i)} \quad \text{where } \underline{a}_{(i)} = \frac{1}{\sqrt{g_{11}}}\underline{a}_i \end{aligned} \tag{3}$$

The repeated indices imply a sum. $\underline{a}_i$ are covariant base vectors in the curvilinear coordinate system and $\underline{a}_{(i)}$ are covariant base vector in the physical curvilinear coordinate system.

The physical curvilinear components of a velocity vector, $u^{(i)}$, can be defined as the magnitude of the i^{th} component projected onto the i^{th} physical curvilinear coordinate direction, and are expressed as:

$$u^{(i)} = \underline{u} \cdot \underline{a}_{(i)} \tag{5}$$

In the Cartesian coordinate system, $u^{(i)}$ are identical to the physical components of the velocity u(i). To obtain the divergence, gradient and Laplacian operators of a vector in the physical curvilinear coordinate system, one starts from the covariant and the contravariant derivatives of the base vectors. Detailed description of this can be found in Lee[3]

From now, the partial differential equations with the vector forms will be given in terms of the physical components in physical curvilinear coordinates. Now, using the general transformation laws, for a scalar ϕ, the gradient can be written as:

$$\nabla\phi = \frac{\partial \phi}{\partial \xi^{(i)}}\underline{a}^{(i)} = g^{(ik)}\frac{\partial \phi}{\partial \xi^{(k)}}\underline{a}_{(i)} \tag{6}$$

Also, the gradient of a vector $\underline{u}$ can be expressed using equation (5) and the covariant derivatives of the base vectors in the physical curvilinear coordinate system as:

$$\nabla\underline{u} = \frac{\partial \underline{u}}{\partial \xi^{(i)}}\underline{a}^{(i)} = u^{(k)}_{,(i)}\underline{a}_{(k)}\underline{a}^{(i)} = g^{(ij)}u^{(k)}_{,(i)}\underline{a}_{(k)}\underline{a}_{(j)} \tag{7}$$

The quantity $u^{(k)}_{,(i)}$ is called the covariant derivative of the physical curvilinear components of a vector $\underline{u}$. One can easily evaluate the divergence of a vector as:.

$$\begin{aligned} &\nabla\cdot\underline{u} = \frac{\partial \underline{u}}{\partial \xi^{(i)}}\cdot\underline{a}^{(i)} = u^{(i)}_{,(i)} \\ &\text{where } u^{(i)}_{,(i)} = \frac{\sqrt{g_{ii}}}{J}\frac{\partial}{\partial \xi^{(i)}}\left(\frac{J}{\sqrt{g_{ii}}}u^{(i)}\right) \end{aligned} \tag{8}$$

The Laplacian can be evaluated by the divergence of the gradient of a vector $\underline{u}$ as:

$$\nabla^2\underline{u} = g^{(jk)}\left[u^{(m)}_{,(j)}\Gamma^{(i)}_{(mk)} - u^{(i)}_{,(m)}\Gamma^{(m)}_{(jk)} + \frac{\partial u^{(i)}_{,(j)}}{\partial \xi^{(k)}}\right]\underline{a}_{(i)} \tag{9}$$

From now, think about an unsteady coordinate system. The time derivative of a scalar or a vector F is given by Warsi [6].

$$\begin{aligned} \frac{\partial F}{\partial t}\Big|_{x_i} &= \left[\frac{\partial F}{\partial \tau} + \frac{\partial F}{\partial \xi^{(i)}}\frac{\partial \xi^{(i)}}{\partial t}\right]_{\xi^{(i)}} \\ \frac{\partial F}{\partial \tau}\Big|_{\xi^{(i)}} &= \left[\frac{\partial F}{\partial t} + \frac{\partial F}{\partial x_i}\frac{\partial x_i}{\partial \tau}\right]_{x_i} \end{aligned} \tag{10}$$

Here, x_i represent the fixed Cartesian coordinate system, while $\xi^{(i)}$ represent the unsteady physical curvilinear coordinate system. Also, τ represent the time in this coordinate system. The components of the grid speed can be easily evaluated by replacing F with $\xi^{(i)}$ in the second equation of the equation (10).

$$w^{(i)} = \frac{\partial \xi^{(i)}}{\partial t} = -\frac{\partial \xi^{(i)}}{\partial x_l}\frac{\partial x_l}{\partial \tau} \quad (11)$$

The divergence of the grid speed vector $\underline{w}$ is written as:

$$\nabla \cdot \underline{w} = \frac{\sqrt{g_{ii}}}{J}\frac{\partial}{\partial \xi^{(i)}}\left(\frac{J}{\sqrt{g_{ii}}} w^{(i)}\right) = -\frac{1}{J}\frac{\partial J}{\partial \tau} \quad (12)$$

J is the Jacobian of the inverse transformation. The vector form of incompressible Reynolds-averaged Navier-Stokes equations, with the body force in unsteady coordinate system, is given as:

$$\frac{\partial \underline{u}}{\partial \tau} + (\nabla \underline{u}) \cdot \underline{v} = -\nabla P + \nu_E \nabla^2 \underline{u} + [\nabla \underline{u} + (\nabla \underline{u})^T] \cdot \nabla \nu_E$$
$$\text{where } \underline{v} = \underline{u} + \underline{w},\ P = p + \frac{z}{Fn^2} + \frac{2}{3}k \quad (13)$$

Here, Fn is a Froude number, P is a total pressure, p is a static pressure. The procedure for the transformation of the incompressible viscous equations, based on an unsteady physical curvilinear coordinate system, is now introduced using the derivations for the gradient, divergence operator, Laplacian and the time derivative, one can get the equations in an unsteady coordinate system. Each physical covariant base vector $\underline{a}_{(i)}$ is independent; therefore an unsteady physical curvilinear component form of the Reynolds-averaged Navier-Stokes equations can be derived in the scalar form as:

$$R_{eff}\frac{\partial u^{(i)}}{\partial \tau} + R_{eff} v^{(j)} u^{(i)}_{,(j)} = R_{eff}[-g^{(ij)}\frac{\partial P}{\partial \xi^{(j)}}$$
$$+ \frac{\partial \nu_E}{\partial \xi^{(j)}}\{g^{(jk)} u^{(i)}_{,(k)} + g^{(ik)} u^{(j)}_{,(k)}\}] \quad (14)$$
$$+ g^{(jk)}[u^{(m)}_{,(j)}\Gamma^{(i)}_{(mk)} - u^{(i)}_{,(m)}\Gamma^{(m)}_{(jk)} + \frac{\partial u^{(i)}_{,(j)}}{\partial \xi^{(k)}}]$$

3. Discretization of the momentum equation

Reynolds averaged Navier-Stokes and continuity equations in the physical curvilinear coordinate system were derived. Equation (14), which has a nonconservative form, is rearranged into the standard form for the use of the finite analytic method. In the present research, the 12-point scheme by Patel et al. [10] is used, modified and extended The new formulation of a condition is suggested to reduce or avoid the oscillation of the series summation for any cell Reynolds number.

3.1 12-point scheme for the local nonuniform grid spacing

The standard form of the finite agnalytic method is given as:

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 2A\phi_x + 2B\phi_y + 2C\phi_z + S \quad (15)$$

The subscripts x, y, and z indicate derivatives. The stretched coordinates are used to make a standard form of the differential equations in the physical curvilinear coordinate system. The stretched coordinates ξ^{i*} are defined as:

$$\xi^{i*} = \frac{1}{\sqrt{g_{11}}}\xi^i = \frac{1}{\sqrt{g_{ii}}\sqrt{g^{ii}}}\xi^{(i)} \quad (16)$$

Equation (16) shows the relation between each coordinate system. For a local nonuniform grid spacing a different lengths are taken in the formulation. An interpolation formula developed in the Cartesian coordinate system by Chen and Chen [9] is used to consider the effect of the nonuniform length for the local elements in the transformed coordinates ξ^{i*}. The derivation of a 12-point FA coefficients of the three-dimensional differential equations with the nonuniform length in each stetched curvilinear coordinate direction is based on the analytic discretized equation for a one-dimensional local element with nonuniform spacing. Detailed description can be found in [3].

3.2 The series summation

Each analytic coefficient has the series summation which is a function of local cell Reynolds number. Unfortunately, the series summation do not always converge; therefore, it may be said that the accuracy of the solution depends on the calculation of the series summation. Patel et al [10] suggested using the asymptotic expression based on the theory of characteristics, to avoid the calculation of the series summation for only the large cell Reynolds number. For a large positive cell Reynolds number in a one-dimensional problem, the asymptotic expression means that the downstream influence is negligible. Chen et al [11] suggested to use this expression for a cell Reynolds number defined as Ah or Bk ≥ 100. Therefore, an upwind effect is taken into consideration in this range. Here, k and h represent the local spacing in the η and ζ coordinate directions, respectively. The series summation generally does not oscillate for a small cell Reynolds number, while this oscillates for the most part for a large cell Reynolds number. In present work, a vigorous examination has been performed for a wide range of A and B, and the grid spacing, h, k. It is found that the oscillation depends on the sum of an absolute value of each local cell Reynolds number and the asymptotic expression can be used in the range of the cell

Reynolds number, at least Ah or Bk ≥ 30. A basis of the use of the asymptotic expression is suggested as follows:

$$Cri = \sqrt{A^2 + B^2 + (\frac{c_1}{len1})^2 len2} \leq |Ah| + |Bk| \qquad (17)$$

where $len1 = min[h,k]$, $len2 = max[h,k]$, and $c_1 = 28$

The value of the coefficient c_1 took 28 through trial and error. The method by the iteration to calculate the series summation must be used even in large cell Reynolds number if not oscillate. Therefore, one cannot affirm that the asymptotic expression can be used in a specially fixed cell Reynolds number. Three cases are selected to check the condition of equation (17).

[1]. $A = 0.9 \sim 9000000$, $B = 0.09 \sim 900000$, $h = k = 0.01$
[2]. $A = 0.1 \sim 1000000$, $B = 0.01 \sim 100000$, $h = 0.009$, $k = 0.001$
[3]. $A = 0.1 \sim 1000000$, $B = 0.02 \sim 100000$, $h = 0.01$, $k = 0.002$

The variation of the finite analytic coefficients is shown for a large range of the cell Reynolds number in figure 1.

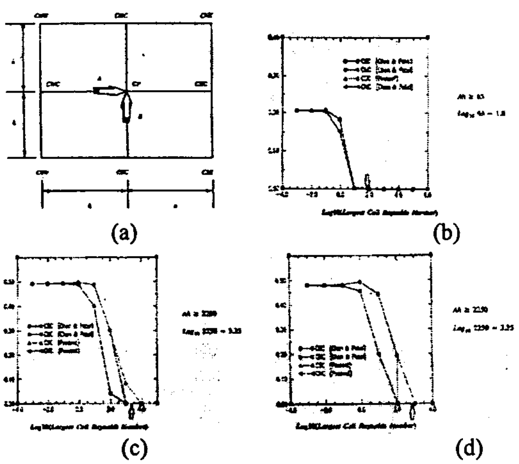


Figure 1. Variation of FA coefficients by changing the cell Reynolds number (b):case 1, (c):case 2, (d):case 3.

In the first case, the aspect ratio of the grid spacing in a two-dimensional problem is taken one and the ratio of A and B is taken ten. The summation is not convergent and oscillates even though the cell Reynolds number is less than 100. However, the difference of the value calculated between two approaches is negligible because the hight of the oscillation is small. Generally, the method suggested by Chen et al.[11] and the present method are in good agreement in the first case. In the present suggestion, the asymptotic expression was used in the range, $|Ah|+|Bk| \geq 63$. For small cell Reynolds number less than 1, the value of FA coefficients looks nearly constant and the values of CSC, CNC, CEC and CWC are dominant Relative importance of the summation of the values of CSW, CSE, CNW, and CNE is less than 20 percent. However, as the positive cell Reynolds numbers increase, the priority of CSC, CNC and CEC decrease to zero. Namely, the influence of south, north and east is negligible at large cell Reynolds number. On the other hand, the priority of CWC increase up to 90 percent. In the second case, the aspect ration of the grid spacing, and the ratio of A and B are 9 and 10, respectively. In the present method, the asymptotic expression was used even at very large cell Reynolds number, $|Ah|$ or $|Bk| \geq 2250$ and max[Ah, Bk] > 108. Figure 1c shows the two approaches are in good agreement in case of max[Ah, Bk] < 100 or max[Ah, Bk] > 10000. However, there are a big difference around max[Ah, Bk] = 1000. The series summation must be used if the series summation does not oscillate. The bigger deviation between two approaches is shown in the third case. In the third case, the aspect ration of the grid spacing and the ration of A and B are 5 and 10, respectively. In the present method, the asymptotic expression was used in the range, Ah or Bk ≥ 714. Figure 1d shows a jump of the values at max[Ah,Bk] = 100 in the Chen et al. [11].

For a small cell Reynolds number, max[Ah, Bk] $\leq$ 30, the series summation is generally used to calculate FA coefficients, except for special cases. The asymptotic expression is generally used at very high Reynolds numbers. Therefore, there are no difficulties to compute the flow field at the low and the very high Reynolds numbers. The difficulty is in the flow computation at the intermediate Reynolds number. Equation (20) suggested in the present research gives the criteria for the use of the asymptotic expression in the range of from low to high cell Reynolds number.

4. Kinematic Boundary condition

The free surface Kinematic boundary condition needs to update the unknown free surface location. The free surface is unknown a priori and the Kinematic condition needs to be solved with the momentum equations in a coupled manner. In the following the Kinematic condition will be derived

4.1 Derivation of the kinematic condition

The concept of the material surface is used for the free surface. A material surface moves with the flow and deforms in shape as the flow progresses. There are no mass fluxes in or out of the surface, so that its boundary is always composed of the same fluid particle. If the equation of the surface is represented by F(x,y,z,t) = 0, then the equation of the surface at the time, $t+\delta t$ is expressed as:

$$F(\underline{r}+\delta\underline{r}, t+\delta t) = 0 \tag{18}$$

Using the Taylor's series expansion, the Kinematic boundary condition is derived as:

$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + \nabla F \cdot \underline{u} = 0 \tag{19}$$

If the equation of the surface F(x,y,z,t) is decomposed into z-coordinate and G(x,y,t), the Kinematic boundary condition to be solved for G(x,y,t) can be obtained as:

$$\begin{aligned} \frac{DF}{Dt} &= \frac{\partial}{\partial t}(z-G) + u_i \frac{\partial}{\partial x_i}(z-G) \\ &= -\frac{\partial G}{\partial t} - u\frac{\partial G}{\partial x} - v\frac{\partial G}{\partial y} + w \end{aligned} \tag{20}$$

The time derivative of G in unsteady coordinate system can be expressed using the equation (10) as:

$$\frac{\partial G}{\partial \tau}\Big|_{\xi^i} = \frac{\partial G}{\partial t}\Big|_{x_i} + \frac{\partial G}{\partial x_i}\frac{\partial x_i}{\partial \tau}\Big|x_i \tag{21}$$

It is assumed that the particle on the upper boundary moves up and down along the z-coordinate. Finally, the Kinematic boundary condition in unsteady coordinate system is obtained as:

$$\frac{\partial G}{\partial \tau} + u\frac{\partial G}{\partial x} + \left(v - \frac{\partial y}{\partial \tau}\right)\frac{\partial G}{\partial y} = w \tag{22}$$

To make the standard form of FA scheme, the second derivative terms are added in both sides of equation (22). And then, a modified equation is solved using a 9-point FA scheme.

5. Results and Discussions

Incompressible viscous equations in the unsteady physical curvilinear coordinate system were derived. The 12-point FA discretization scheme based on local nonuniform grid spacing (Lee [3]) was used. For incompressible flow, the density is constant and the pressure is a primitive variables. The real difficulty in the simulation of the incompressible flow field lies in predicting an unknown pressure field. Several methods currently are being used for the solution of incompressible viscous equations. In all these methods, the momentum equations are used to compute the velocity field, while different equations are employed to compute the pressure field. The present procedure is based on the SIMPLER algorithm and the pressure Poisson equation method. The pressure Poisson equation was written in the physical curvilinear coordinate system and solved by using a 12-point FA scheme based on local nonuniform grid spacing. Detailed description can be found in [3]. The matrix, that consists of the coefficients resulting from the finite analytic method, is solved using the Generalized Minimal RESidual (GMRES) method to enhance the convergence rate of the flow solver. These methodologies are validated by simulating a three-dimensional lid-driven cavity laminar flow. and a three-dimensional free surface turbulent flow.

5.1 Lid-driven three-dimensional cavity laminar flow

The velocity components on the wall are zero, except on the moving wall with a velocity 1. The computations are performed on a grid consisting of 16*16*16 grid points. To obtain the solution for the steady state, only one iteration per each time step is used. All computations are performed using a relaxation factor of 1. Figure 2 shows that the rate of the convergence depends on the size of the dimensionless time step in the range from 0.01 to 2. The computations leads to a fully converged solution with fewer that 200 iterations.

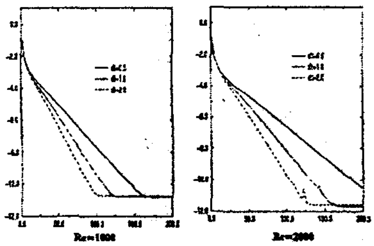


Figure 2 Convergence histories for different time steps and different Reynolds numbers

Peric [12] mentions, if the angle between the two coordinate lines in the two-dimensional problem is greater than 135° or less than 45°, the pressure correction equation does not converge at all, or the convergence rate is too slow. Cho and Chung [13] used a new treatment method for nonorthogonal terms in the pressure correction equation in order to enlarge the ranges for convergence and found that the smaller the angle, the narrower the region of relaxation factor. In the present research, the computations are performed at several inclined angles, 90, 60, 45, 30, 15, 10, and 5 degrees to check the rate of the convergence on the grid skewness. It is found that the solutions always converged, even for very small inclined angles but more iterations were required for convergence for small inclined angles. Figure 3 shows the convergence histories in various inclined angles from 90 to 5 degrees. It was shown that the present code is less mesh sensitive and converges well, even at the large skewness.

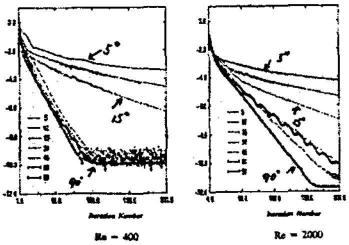
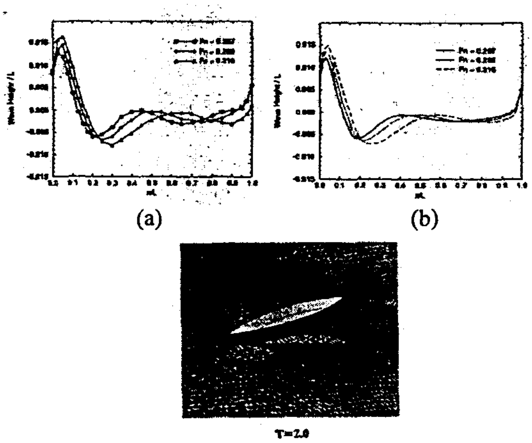


Figure 3 Convergence histories in various inclined angles (degree)

5.2 a three-dimensional free surface turbulent flow

As the next test, three-dimensional free surface turbulent flow was computed. The upper boundaries moves arbitrary with the flow, and the grid in the computational domain is generated every time step until the solution of the steady state is obtained. The Baldwin-Lomax turbulence model [14] was used to calculate the eddy viscosity in the turbulent flow. The computations were performed with three different Froude number under the same conditions. The Froude numbers and the Reynolds number used in the experiment ([15][16]) are 0.267, 0.289, 0.316, and $3.3*10^6$, respectively. A comparison of the wave elevations on the Wigley hull surface for three different Froude numbers and a perspective view of the wave elevation is shown in Figure 4. The residue remains around 10^{-4} after t=0.5. These computations took 400 time steps on a 125*36*35 grid.



(a) (b)

(c)

Figure 4 The wave elevations on the hull surface, (a) : experiment, (b) : computation, and a perspective view (c)

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