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VSS 이론을 이용한 SISO 비선형 시스템에 대한 강인성 제어

임 규 만† , 김영수
초당대학교 전자공학과

Robust Control for SISO Nonlinear System using VSS Theory

Kyu-Mann Im† ,Young-Soo Kim
Dept. of Electronic Engineering, Chodang University

Abstract - In this paper, a robust control scheme for a class of SISO nonlinear dynamical system is proposed by using output-feedback linearization method. The presented control scheme is based on the VSS control theory concept. In this control scheme, we assume that the nonlinear dynamical system is minimum phase, i.e., the relative degree of the system is $r < n$ and zero dynamics is stable. We also assume that the states of zero dynamics are not accessible. It is shown that the global asymptotically stability is guaranted under the proposed control scheme. The feasibility of the proposed control scheme is verified through a computer simulation.

1. Introduction

Nonlinear control has emerged as an area of extensive research activity recently.[1]-[2]. An important class of problems in this area concerns the study of disturbance inputs for an analysis and controller synthesis purposes. Feedback stabilization of nonlinear systems at a specified equilibrium is a central topic in control theory and it has been a subject of research by many authors, e.g., see[3]-[5]. The works of Artstein, Sontag-Sussman [6], and Vidyasagar[7] are among the most significant contributions in the study of stabilization using Lyapunov-like techniques. The robust control approach has been developed for the effective control of uncertain linear/nonlinear dynamical systems. The robust control technique does not require the exact functional natures and the accurate parameter values of the system. The robust control scheme including the variable structure control is based on the construction of the control effort overcoming the uncertainty. Therefore, this control scheme needs a priori knowledge of the uncertainty bounds. When dealing with minimum phase nonlinear systems, a stable/unstable decomposition is usually used and the controller must contain only the part with stable inverse. Therefore, the problem of synthesizing control algorithms for plants with unstable zero dynamics is very important. Recently, Hauser et al.[8] proposed a new control design based on an approximate linearized model. For many nonlinear systems, uncertainties are common in control practice. So the design of a robust controller that deals with a nonlinear system with significant uncertainties is an important subject.

The paper is organized as follows. In section II, mathematical tools is presented and output feedback linearization is discussed. In section III, VSS controller is presented and its stability analysis is shown in the Lyapunov sense. In section IV, the feasibility of the proposed control scheme is verified through a computer simulation.

2. Mathematical Tools

Given f, a C^{∞} vector field on R^n, and h, a C^{∞} scalar field on R^n the Lie derivative of h with respect to f is defined as the inner product of the gradient of h with f

$$L_f h = \langle dh, f \rangle = \sum_{i=1}^{n} \frac{\partial h}{\partial x_i} f_i . \tag{2.1}$$

We can see that $L_f h$ is also a C^{∞} scalar field on R^n. Higher order Lie derivatives can be defined inductively as follows:

$$L_f^k h = L_f(L_f^{k-1} h), \quad k = 2, 3, \cdots$$

Given f, g a C^{∞} vector fields on R^n, the Lie bracket $[f, g]$ is a vector field defined by

$$[f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g \tag{2.2}$$

where $\frac{\partial f}{\partial x}$ and $\frac{\partial g}{\partial x}$ are the Jacobians. $[f, g]$ is also a C^{∞} vector field on R^n. One can define iterated Lie brackets $[f, [f, g]]$, $[f, [f, [f, g]]]$ etc. The following notation is standard:

$$\begin{aligned} ad_f^0 g &= g \\ ad_f^1 g &= [f, g] \\ ad_f^2 g &= [f[f, g]] \\ &\vdots \\ ad_f^k g &= [f, ad_f^{k-1} g] \end{aligned}$$

The purpose of this section is to show how single-input single-output nonlinear systems can be locally given, by means of a suitable change of coordinates in the state space, a normal form of special interest, on which several important properties can be elucidated. The point of departure of the whole analysis is the notation of relative degree of the system, which is formally described in the following way.

The single-input single-output nonlinear system

$$\begin{aligned} x(t) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)) \end{aligned} \qquad (2.3)$$

is said to have relative degree r at a point x^o If

(i) $L_g L_f^k h(x) = 0$ for all x in a neighborhood U of x^o and all $k < r-1$

(ii) $L_g L_f^{r-1} h(x) \neq 0$

where the state x is assumed to belong to an open neighborhood U of R^n, $f(x)$ and $g(x)$ are smooth vector fields on R^n, $u \in R$, $y \in R$, and $f(0) = 0$ (i.e., the origin is an equilibrium point).

3. VSS Controller Design

In this section, we propose a control law which guarantees that closed-loop system has the uniformly ultimate bounded stability with a tolerable tracking error. Because the states of zero dynamics are not accessible, we consider these states as bounded disturbances under the assumption that zero dynamics is stable and also we have no priori knowledge concerning the magnitude of these disturbances. The following assumptions are needed for the development of a controller.

Assumption 1 : Assume that zero dynamics is stable(or nonlinear minimum phase). $a(z, \Psi)$ and $b(z, \Psi)$ can be approximated as follows

$$a(z, \Psi) \cong a(z, \Psi_o) + \frac{\partial a}{\partial \Psi}(z, \Psi_o)\delta\Psi$$

$$b(z, \Psi) \cong a(z, \Psi_o) + \frac{\partial b}{\partial \Psi}(z, \Psi_o)\delta\Psi.$$

where Ψ_o is an equilibrium point of zero dynamics , and without loss of generality, $\Psi_o = 0$.

Assumption 2 : There exists some positive constant vector $\rho_{1\nu}, \rho_{2\nu}$ such that

$$\left\| \frac{\partial a}{\partial \Psi}(z, \Psi_o)\delta\Psi \right\| \le \rho_{1\nu}^T \mu_\nu(t, z)$$

$$\left\| \frac{\partial b}{\partial \Psi}(z, \Psi_o)\delta\Psi \right\| \le \rho_{2\nu}^T \mu_\nu(t, z)$$

where

$$\rho_{1\nu}^T = (\rho_{11}\ \rho_{12}\ \rho_{13}),\ \rho_{2\nu}^T = (\rho_{21}\ \rho_{22}\ \rho_{23})$$

are unknown parameter vectors and $\mu_\nu(t, z) = (1\,\|z\|\,\|z\|^2)^T$.

From the above assumptions, the time derivative of z_r can be rewritten as

$$\dot z_r = a(z, \Psi_o) + b(z, \Psi_o)u + \eta(z, u, t) \qquad (3.1)$$

where

$$\eta(z, u, t) = \frac{\partial a}{\partial \Psi}(z, \Psi_o)\delta\Psi + \frac{\partial b}{\partial \Psi}(z, \Psi_o)\delta\Psi u.$$

Because control input $u(t)$ must be bounded, the norm of $\eta(z, u, t)$ can satisfy the following inequality.

$$\|\eta(z, u, t)\| \le \sigma_\nu^T \varphi_\nu(t, z)$$

where $\sigma_\nu^T = (\gamma_1\ \gamma_2\ \gamma_3)$ is unknown parameter vector and $\varphi_\nu(t, z)$ can be any positive vector function. In this paper, we set $\varphi_\nu(t, z)$ to be the same as $\mu_\nu(t, z)$ as follows

$$\varphi_\nu(t, z) = (1\ \|z\|\ \|z\|^2)^T.$$

Throughout this paper, the norm $\|\cdot\|$ is assumed to be the Euclidean vector norm. Now we utilize the VSS concept to derive a control law. First let us define a sliding surface as follows.

$$s(z_1, \cdots, z_r) = a_1 z_1 + \cdots + a_{r-1} z_{r-1} + z_r \qquad (3.2)$$

where a_i, $i = 1, \cdots, r-1$, are chosen so that the following polynominals $p(s)$ are Hurwitz.

$$p(s) = s^r + a_{r-1}s^{r-1} + \cdots + a_1 \qquad (3.3)$$

Now, we consider a following VSS-like type control law

$$u = u_{eq} + u_\Delta \qquad (3.4)$$

where u_{eq} is the equivalent control input of the nominal system, u_Δ is the control input overcoming the uncertainties (or disturbance which represent the term concerning zero dynamics). We derive u_{eq} from the fact that the derivative of $1/2s^2$ along the trajectory of the closed-loop system should be equal to zero and u_Δ is found such that

$$s[\dot s] < -\beta\|s\|,\ \beta > 0. \qquad (3.5)$$

This inequality implies that the trajectory reaches the sliding surface in a finite time and stays on the sliding surface thereafter. Now we discuss how to derive the u_{eq} and u_Δ which satisfy the above conditions. The time derivative $\dot{s}$ can be expressed as

$$\dot{s}(z_1 \cdots z_r) = \alpha_1 \dot{z}_1 + \alpha_2 \dot{z}_2 + \cdots + \dot{z}_r = 0$$
$$= \alpha_1 z_2 + \cdots + \alpha_{r-1} z_r + a(\mathbf{z}, \Psi_o)$$
$$+ b(\mathbf{z}, \Psi_o) u + \eta(\mathbf{z}, \mathrm{u}, t).$$

If we choose u_{eq} as follows

$$u_{eq} = \frac{1}{b(\mathbf{z}, \Psi_o)}\{-\alpha_1 z_2 - \cdots - \alpha_{r-1} z_r - a(\mathbf{z}, \Psi_o)\} \quad (3.6)$$

then

$$\dot{s}(z_1 \cdots z_r) = b(\mathbf{z}, \Psi_o) u_\Delta + \eta(\mathbf{z}, \mathrm{u}, t). \quad (3.7)$$

Now u_Δ is chosen by

$$u_\Delta = -\frac{1}{b(\mathbf{z}, \Psi_o)}\{\hat{\sigma}_\nu^T \varphi_\nu(t, \mathbf{z}) + k\} \cdot sgn(s) \quad (3.8)$$

where $\hat{\sigma}_\nu$ is estimate of σ_ν. We can summarized the controller structure as follows

$$u = u_{eq} + \frac{1}{b(\mathbf{z}, \Psi_o)}\{-k - \hat{\sigma}_\nu^T \varphi_\nu(t, \mathbf{z})\} sgn(s) \quad (3.9)$$

Now the objective of control is to drive the parameter update law which guarantee that $\mathbf{z}(t)$ converge to zero vector as time goes to infinity. Therefore, we suggest the parameter update law as follows

$$\dot{\hat{\gamma}}_1 = s \cdot sgn(s)$$
$$\dot{\hat{\gamma}}_2 = s\|z\| sgn(s)$$
$$\dot{\hat{\gamma}}_3 = s\|z\|^2 sgn(s) \quad (3.10)$$

where

$$\tilde{\gamma}_1 = \hat{\gamma}_1 - \gamma_1$$
$$\tilde{\gamma}_2 = \hat{\gamma}_2 - \gamma_2$$
$$\tilde{\gamma}_3 = \hat{\gamma}_3 - \gamma_3$$

The stability of the proposed control law is analyzed by the following theorem.

Theorem 3.3 : Under the assumption [1]-[2], the uncertain dynamical system (3.1) with a robust control law (3.9) and parameter update law (3.10), is globally uniformly ultimately bounded.

Proof: The proof is based on the Lyapunov-like function

$$V = \frac{1}{2}(\tilde{\gamma}_1^2 + \tilde{\gamma}_2^2 + \tilde{\gamma}_3^2 + s^2). \quad (3.11)$$

Taking the time derivative of V along the trajectory of (3.7) yields

$$\dot{V} \le \tilde{\gamma}_1 \dot{\hat{\gamma}}_1 + \tilde{\gamma}_2 \dot{\hat{\gamma}}_2 + \tilde{\gamma}_3 \dot{\hat{\gamma}}_3 + s\, b(\mathbf{z}, \Psi_o)\, u_\Delta$$
$$+ |s| \sigma_\nu^T \varphi_\nu(t, \mathbf{z})$$
$$\le \tilde{\gamma}_1 \dot{\hat{\gamma}}_1 + \tilde{\gamma}_2 \dot{\hat{\gamma}}_2 + \tilde{\gamma}_3 \dot{\hat{\gamma}}_3$$
$$- s(\hat{\sigma}_\nu^T \varphi_\nu(t, \mathbf{z}) + k) \cdot sgn(s)$$
$$+ |s| \sigma_\nu^T \varphi_\nu(t, \mathbf{z})$$
$$\le \tilde{\gamma}_1 \dot{\hat{\gamma}}_1 + \tilde{\gamma}_2 \dot{\hat{\gamma}}_2 + \tilde{\gamma}_3 \dot{\hat{\gamma}}_3$$
$$- s\, \tilde{\sigma}_\nu^T \varphi_\nu(t, \mathbf{z})\, sgn(s) - s\, k\, sgn(s). \ \blacksquare$$

From (3.11), $\dot{V}$ can be expressed as

$$\dot{V} \le -s\, k\, sgn(s) < 0.$$

Therefore $s \to 0$ and $\mathbf{z} \to 0$ as $t \to \infty$.

4. Conclusions

In this paper, a robust control scheme for a class a uncertain nonlinear dynamical systems has been proposed. The presented control scheme is based on the VSS robust control structure with an parameter adaptation law for the uncertainty bounds. The uniformly ultimate boundness of the control scheme is guaranted and has been demonstrated by a simulation.

References

[1] Isidori, A., Nonlinear Control Systems : An Introduction, Springer-Verlag, Berlin Heidelberg, 1989.

[2] Nijmeijer, H. and A.J.van der Schaft, "The Disturbance Decoupling Problem for Nonlinear Control Systems," IEEE Trans. Automat. Contr., AC-28 , pp.621-631, 1983.

[3] C. I.Byrnes and A. Isidori, "Local stabilization of minimum phase nonlinear systems, "Syst.Contr. Lett., vol.11, pp.9-17, 1988.

[4] H. Hermes, "On the synthesis of a stabilizing feedback control via Lie algebaic Methods, "SIAM J. Contr. Opt. ,vol.18, pp. 352-361, 1980.

[5] J.Tsinias,"Stabilization of affine in control nonlinear systems," "Non. Anal.", TMA, vol.12, pp. 1283-1296, 1988.

[6] E. D. Sontag,"A Lyapunov-like characterization of asymptotic controllability," SIAM J. Contr. Opt., vol.21, pp.462-471, 1983.

[7] M.Vidyasagar,"Decomposition techniques for arge-scale systems with nonadditive interactions : Stability and Stabilizability,"IEEE Trans. Automat. Contr.,vol.25, pp.773-779, 1980.

[8] Hauser, J., Sastry, S. and Meyer, G.,"Nonlinear control design for slightly nonminimum phase systems : Application to V/STOL aircraft, "Automatica),vol.28, pp.665-679, 1992.