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Prediction of Harbour Resonance by the Finite Difference Approach

Jung Lyul Lee* and Chan Sung Park*

INTRODUCTION

When the strong wind or long wave energy is transferred into the water body of a harbour, the harbour exhibits oscillatory resonant motions which often cause significant damage to moored ships and navigation hazards. Therefore, a number of theoretical and numerical investigations of such resonant oscillations have been carried out but most of them were limited to harbours connected with open sea of constant depth. As a first investigation, Lamb (1932) analyzed the free oscillation in closed rectangular and circular basins. His solutions then clarified the natural periods and modes of free surface oscillations related to these special configurations. As the next approach to the practical situation, McNown (1952) studied the forced oscillation in a circular harbour which is connected to the open sea through a narrow mouth. Since the radiation effect was ruled out, his results showed a harbour resonance as it does in a closed basin. Similar research was also carried out on rectangular harbours (Kravtchnenko and McNown, 1955).

Miles and Munk (1961) and Ippen and Goda (1962) realized that the open-sea was important in allowing for the loss of energy radiated from a harbour. As an approach to the arbitrary-shaped harbour but the constant depth region, Hwang and Tuck (1970) and Lee (1971) developed numerical models by using the boundary integral element method. For application to real depth-varying harbours, Mei and Chen (1975) provided a hybrid finite element model, whereas Raichlen and Naheer (1976) developed a finite difference model.

The study of harbour resonance has been extended to take into account the effect of bottom friction(Kostense et al., 1986), wave nonlinearity (Lapelletier and Raichlen, 1987; Mei and Agnon, 1989; Zhou et al., 1991), irregular wave incidence (Goda, 1985; Ouellet and Theriault, 1989), viscous dissipation (Gember, 1986) and porous breakwater (Yu and Chwang, 1994).

The object of this study is to develope a finite difference version for harbour resonance numerical model and to compare the developed model to Raichlen and Naheer (1976)'s finite difference approach, which is used inside the harbour matched at the entrance to a solution for the open-sea based on the Helmholtz equation.

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* Dept. of Civil and Environmental Eng., Sungkyunkwan University, Suwon Science Campus, Suwon 440-746, Korea

BASIC FORMULATION

Madsen and Larsen (1987) presented an implicit approach that can be handled with a relatively small amount of computational effort. The governing equation is given by a set of 1st-order differential equations of hyperbolic type as

$$\frac{\partial S}{\partial t} - i\omega S + \frac{1}{n}\nabla \cdot n\mathbf{R} = 0 , \qquad \frac{\partial \mathbf{R}}{\partial t} - i\omega \mathbf{R} + C^2 \nabla S = 0 \tag{1}$$

where n is given by Cg/C, C is the phase speed and Cg is the group velocity. The differential equations (1) appear to be similar to the system of equations governing nearly horizontal flow in shallow water by introduction of the intermediate complex vector variable **R**. The ADI(Alternating Direction Implicit) algorithm as described by Lee (1994) is invoked on a staggered grid system.

In the present study, the radiating waves are supposed to be absorbed through all the transmissive boundaries except for no flux land boundary. If the wave radiates toward the x -direction, the S can be accounted for by the following Neumann relationship of complex at the transmissive boundary as

$$\frac{\partial S}{\partial x} = -ikS \tag{2}$$

When the $S_{i,j}$ is situated at the center of grid cell adjacent to boundary, the unknowns posed at boundaries can be determined by finite differencing according to the position which they are sided as;

$$P_{i,j} = C_x F_l S_{i,j} , \quad P_{i+1,j} = C_x F_r S_{i,j} \tag{3}$$

$$Q_{i,j} = C_y F_d S_{i,j} , \quad Q_{i,j+1} = C_y F_u S_{i,j} \tag{4}$$

where,

$$F_l = \frac{ik_x + 4/\Delta x}{ik_x - 4/\Delta x}, \quad F_r = \frac{ik_x + 4/\Delta x}{-ik_x + 4/\Delta x}, \quad F_d = \frac{ik_y + 4/\Delta y}{ik_y - 4/\Delta y} \text{ and } F_u = \frac{ik_y + 4/\Delta y}{-ik_y + 4/\Delta y}$$

The more detailed description of the treatment of boundary condition is omitted here.

NUMERICAL RESULTS

The present model was examined on a rectangular harbour with linearly varying depth presented by Raichlen and Naheer (1976) as shown in Fig. 1. The model layout is composed of an inner basin represented by the length l =31.15cm times the width b =6.03cm and an outer basin of size $B \times L$. The width and length of the outer basin were selected as $B=20b$, $L=5b$, respectively, after examined so as to yield the best fitting results when compared to the analytic solution and Raichlen and Naheer (1976)'s results.

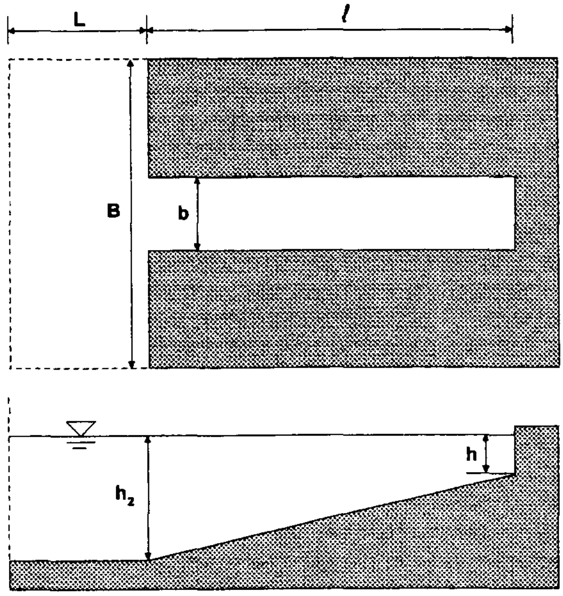

Fig. 1 Definition sketch of a rectangular harbour with linearly varying depth.

The grid spacings in x and y-directions are uniformly given by $\Delta x = l/60$ and $\Delta y = b/5$, respectively. The h_2 is given 7.62cm as the depth of outer basin and h the depth at the backwall.

Fig. 2 shows the harbour resonance determined for a basin of constant depth. The numerical results show a remarkable agreement with analytic solutions. For four ratios of depths, the computed resonance curves were compared to the numerical results obtained by Raichlen and Naheer (1976) as shown in Fig. 3. The agreement is seen to be good. However, the results shown are for the particular cases to provide most satisfied ones. The magnitude of resonance peak appears to vary somewhat according to the outer basin size. The reason is probably due to the imperfect absorption of radiating waves which is caused by use of the finite difference algorithms used instead of a matching approach.

CONCLUSION

The numerical simulation has been accomplished on a rectangular harbour of linear varying depth. In conclusion, the present finite difference model provided the satisfactory results in most of cases but both width and length of outer basin, width in particular, appeared to have influence

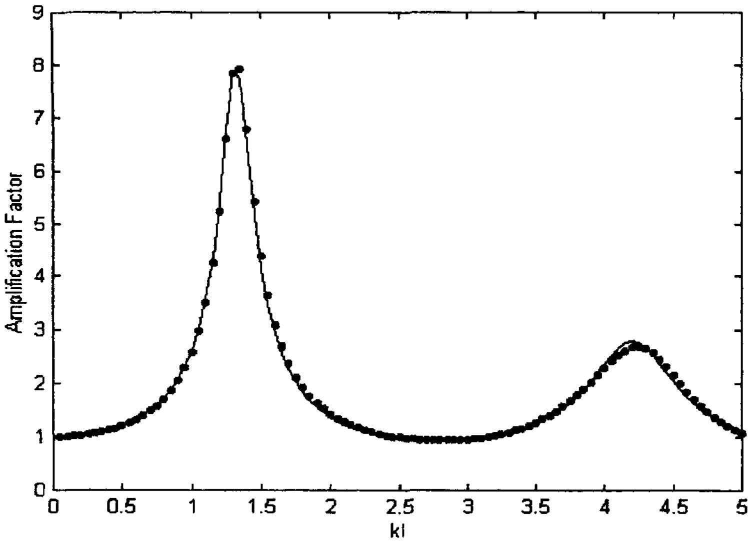

Fig. 2 Resonance curve for a harbour of constant depth.
●: Computed by present model; — : Analytic sol..

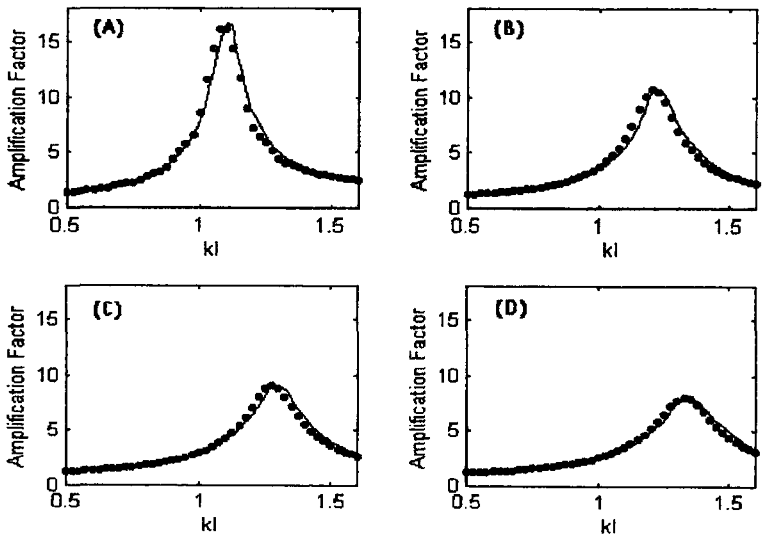

Fig. 3 Resonance curves at the backwall for four depth ratios.
●: Computed by present model; — : Numerical solution by Raichlen and Naheer(1976). ((A) h/h_2=0, (B) h/h_2=0.33 (C) h/h_2=0.67, and (D) h/h_2=1).

on the magnitude of resonance peaks. As the further study, therefore, such sensitivity problem resulted from the size of outer basin is under study. Since the major advantage of present model is that the radiating waves can be handled without introduction of matching condition at the entrance, the present model, which has been developed by use of finite difference scheme, is expected to be easily used even to the complicated harbour without any limitation in application.

REFERENCES

Gember, M., 1986. Modeling dissipation in harbor resonance, *Coastal Engrg.*, 10: 211-252.

Goda, Y., 1985. Random seas and design of maritime structure, University of Tokyo press, Tokyo, Japan.

Hwang, L.-S., and Tuck, E.O., 1970. On the oscillation of harbors of arbitrary shape, *J. Fluid Mech*, 42:447-464.

Ippen, A.T., and Goda. Y., 1963. Wave induced oscillations in harbors: The solution of a rectangular harbor connected to the open sea, Report No. 59, Hydrodynamics Lab., MIT, Cambridge, Mass.

Kostense, J.K., Meijer, K.L., Dinemans, M.W., Mynett, A.E., and van den Bosch, P., 1986. Wave energy dissipation in arbitrarily shaped harbors of variable depth, 20th *ICCE*, 436-437.

Kravtchnenko, J., and McNown, J.S., 1955. Seiche in rectangular ports, Quart. *Appl. Math*, 13:19-26.

Lamb, H., 1932. Hydrodynamics, Dover Publications, New York, N.Y.

Lapelletier, T.G., and Raichlen, F., 1987. Harbor oscillations induced by nonlinear transient long waves, *J. Wtrway., Port, Coast., and Ocean Engrg.*, 113(4):381-400.

Lee, J.L., 1994. Quasi-3D wave-induced circulation model, *J. the Korean Society of Coastal and Ocean Engineers*, 6(4):459-471.

Lee, J.J., 1971. Wave-induced oscillations in harbors of arbitrary shape, *J. Fluid Mech*, 45:375-394.

Madsen, P.A., and Larsen, J., 1987. An efficient finite-difference approach to the mild-slope equation, *Coastal Engrg*, 11:329-351.

McNown, J.S., 1952. Waves transmission through porous structures, *J. Wtrway., Harbor., and Coastal. Engrg. Div.*, ASCE, 100(3):169-188.

Mei, C.C., and Agnon, Y., 1989. Long period oscillations in a harbor induced by incident short waves, *J. Fluid Mech*, 208:595-608.

Mei, C.C., and Chen, H.S., 1975. Hybrid element method for water waves, *2nd Annual Symp. of Waterways, harbor, and Coastal Engrg. Div.*, ASCE, 1:63-81.

Miles, J.W., and Munk, W.H., 1961. Harbor paradox, *J. Wtrway., and Harb. Div.*, ASCE, 87(3):111-130.

Ouellet, Y., and Theriault, T., 1989. Wave grouping effect in irregular wave agitation in harbor, *J. Wtrway., Port, Coast., and Ocean Engrg.*, 115(3):363-383.

Raichlen, F., and Naheer, E., 1976. Wave induced oscillations of harbors with variable depth, *Proc. 15th ICCE.*, ASCE, 3536-3556.

Yu, X., and Chwang, A.T., 1994. Wave-Induced oscillation in harbor with porous breakwaters, *J. Wtrway., Port, Coast., and Ocean Engrg.*, 120(2):125-144.

Zhou, C.P., Cheung. Y.K., and Lee, J.H.W., 1991. Response in harbor due to incidence of second order low-frequency waves, *Wave Motion*, 13:167-184.