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DEVELOPMENT OF A NEW MODEL FOR NONLINEAR-DISPERSIVE WAVES OVER ARBITRARY DEPTHS

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1. INTRODUCTION

Wave nonlinearity and dispersivity have mutually counteracting effects on the wave evolution process; i.e., the former makes the wave profile steeper, while the latter milder. Therefore to describe evolution of nonlinear water waves under general condition such as nonlinear random waves over arbitrary depths, both the wave nonlinearity and dispersivity must be properly taken into account in the wave modeling.

Among the previous nonlinear dispersive wave models, Boussinesq equations (Peregrine, 1967) are the most popular one, and recently they have been widely applied to nearshore wave computation and related subjects such as nearshore current simulation. The Boussinesq equations, however, suffer from the inherent disadvantage of being shallow water equations. Therefore to extend their applicable range several efforts have been recently made (Madsen et al. 1991; Madsen & Sørensen 1992; Nwogu 1993; Beji & Nadaoka 1996a; etc.). Among these, Beji & Nadaoka (1996a) presented a very simple and systematic procedure to obtain a more general form of the Boussinesq equations for varying depth and this model manifests itself as perfect energy conservation characteristics when compared with the similar type equations by Madsen & Sørensen (1992) and Nwogu (1993). The improved Boussinesq equations obtained by Beji & Nadaoka (1996a) are written with a scalar parameter β as:

$$\frac{\partial\eta}{\partial t}+\nabla\cdot\left[(h+\eta)\bar{u}\right]=0, \tag{1}$$

$$\frac{\partial\bar{u}}{\partial t}+(\bar{u}\cdot\nabla)\bar{u}+g\nabla\eta=\tfrac{1}{2}(1+\beta)h\left\{\nabla\left[\nabla\cdot\left(h\frac{\partial\bar{u}}{\partial t}\right)\right]-\tfrac{1}{3}h\nabla\left(\nabla\cdot\frac{\partial\bar{u}}{\partial t}\right)\right\}$$
$$+\tfrac{1}{2}\beta gh\left\{\nabla\left[\nabla\cdot(h\nabla\eta)\right]-\tfrac{1}{3}h\nabla\left(\nabla^2\eta\right)\right\}, \tag{2}$$

where η is the free surface displacement, $\bar{u}$ is the depth-averaged horizontal velocity vector, h is the varying water depth as measured from the still water level and g is the gravitational acceleration. The two-dimensional horizontal gradient operator is denoted by ∇ . The new form of the equations is similar to the standard form of the Boussinesq equations, except for the mixed dispersion terms in the momentum equation. Note that when β=0 these equations recovers the original Boussinesq equations by Peregrine (1967).

Despite these efforts to extend the applicable range of the Boussinesq equations, their inherent limitation as shallow water wave equations may not be removed. For instance, these improved Boussinesq equations give the linear shoaling characteristics within 1% error only for h/L<0.3 (Nwogu 1993; Beji & Nadaoka 1996a). Even for a *nearshore* region, however, the numerical simulation sometimes needs to cover deep water waves. Figure 1 illustrates such a case, in which incident shallow water waves are decomposed as they pass over a bar into shorter and hence deep water waves (Byrne 1969; Beji & Battjes 1993; Ohyama & Nadaoka 1994, etc.). To properly simulate this wave field one needs a wave model which can handle both sahllow and deep water waves. Irregular nonlinear waves with a broad- banded spectrum is another example to show the necessity of this type of wave model.

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Recently we have developed a fully-dispersive nonlinear wave model via a new approach named "*multiterm-coupling technique*", in which the velocity field is represented with *a few* vertical-dependence functions. The Galerkin method is invoked to obtain a solvable set of coupled equations for the horizontal velocity components and shown to provide an optimum combination of the prescribed depth-dependence functions to express an arbitrary velocity field under wave motion. The obtained equations can describe nonlinear waves under general conditions, such as nonlinear random waves with a broad-banded spectrum at an arbitrary depth including very shallow and far deep water depths. The single component forms of the new wave equations, one of which is referred to as "*time-dependent nonlinear mild-slope equation*", are shown to produce various existing wave equations such as Boussinesq and mild-slope equations as their degenerate forms.

In this paper, the outline of the model is presented. For further details of the model, one can refer Nadaoka & Nakagawa (1991), Nadaoka & Nakagawa (1993), Nadaoka, et al. (1994, 1997), Nadaoka (1995), Beji & Nadaoka (1996b, 1997a,c), etc. Among these Beji & Nadaoka (1997c) provides the most detailed explanation of the modeling procedure and the numerical schemes. The subsequent progress of the model such as a spectral-type modeling and the incorporation of breaking effect are found in Beji & Nadaoka (1997b) and Nadaoka & Ono (1998), respectively.

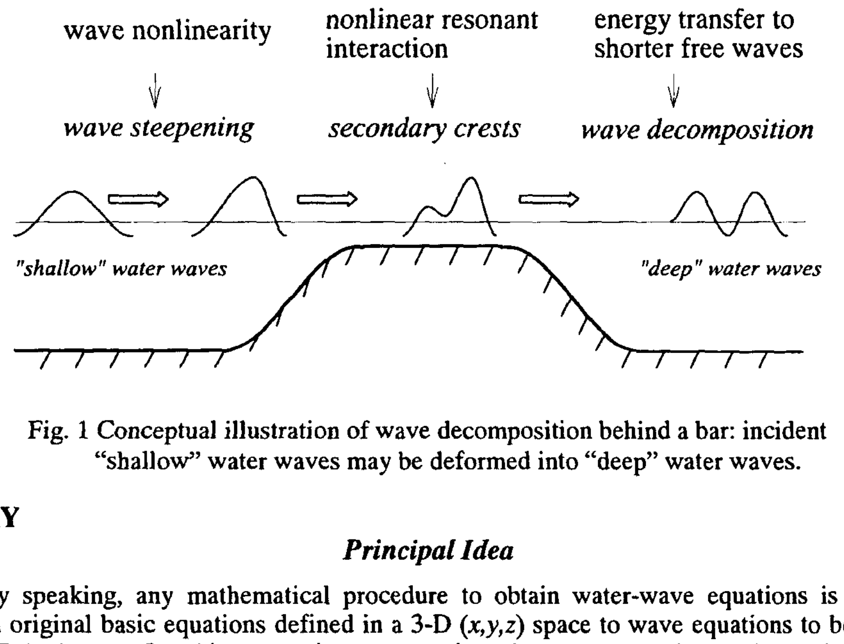

Fig. 1 Conceptual illustration of wave decomposition behind a bar: incident "shallow" water waves may be deformed into "deep" water waves.

2. THEORY

Principal Idea

Generally speaking, any mathematical procedure to obtain water-wave equations is a conversion process from original basic equations defined in a 3-D (x,y,z) space to wave equations to be defined in a horizontal 2-D (x,y) space. For this conversion, we must introduce an assumption on the vertical dependence of the velocity field. For example, the Boussinesq equations are obtained by an asymptotic expansion of the velocity potential around the long wave limit and the vertical dependence of each term is expressed by polynomials (e.g., Mei, 1983). Usually only the first few terms in the expansion including the vertically uniform term as the long wave limit are retained and hence they are not enough to express a velocity field under more general conditions. This in turn suggests that derivation of new wave equations with much wider applicability may be achieved by providing a more reasonable way to express the vertical dependence of a velocity field for more general cases including random waves in deep water.

In our model, the following assumption is introduced for the horizontal velocity vector, $\boldsymbol{u} = (u, v)$:

$$\boldsymbol{u}(x,y,z,t) = \sum_{m=1}^{N} \boldsymbol{U}_m(x,y,t) F_m(z), \tag{3}$$

where

$$F_m(z) = \frac{\cosh k_m (h+z)}{\cosh k_m h}. \tag{4}$$

The choice of cosh functions in the above as the vertical-dependence functions is based on the general 2-D solution of Laplace equation of the velocity potential Φ on the horizontal bottom (e.g., Nadaoka and Hino, 1984),

$$\Phi(x,z,t)=\int_{-\infty}^{\infty}A(k,t)\frac{\cosh k(h+z)}{\cosh kh}\exp(ikx)dk, \tag{5}$$

where k is the wave-number and $A(k,t)$ is a time-varying wave-number spectrum. It should be noted that eq.(5) is valid also for nonlinear waves and hence the use of eq.(4) as the vertical-dependence function $F_m(z)$ is not restricted to linear waves. In the discrete form of eq.(5),

$$\Phi(x,z,t)\cong\sum_{i=1}^{i\max}A(k_i,t)\exp(ik_ix)\Delta k\frac{\cosh k_i(h+z)}{\cosh k_ih}, \tag{6}$$

we need a large number of the spectral component $A(k_i,t)$ in case of broad-banded random waves. However this fact does not necessarily mean that N in eq.(3) should be a large number, in spite of the resemblance between eqs.(3) and (6). This is true if each function, $\cosh k_i(h+z)/\cosh k_ih$, in eq.(6) can be expressed by eq.(3) with a *few* prescribed $F_m(z)$. In fact it is shown that an optimum combination of *few* terms in eq.(3) obtained by a Galerkin procedure may provide almost perfect approximation of $\cosh k_i(h+z)/\cosh k_ih$ with arbitrary k_ih including that for very shallow and far deep water depths. This is the most important finding to provide a basis of the new formulation of wave equations described in what follows.

Derivation of Fully-Dispersive Nonlinear Wave Equations

With this basis of formulation, the continuity and irrotational Euler equation in 3-D (x,y,z) space may be converted to give the new wave equations in the following manner. The vertical integration of the continuity equation over the entire depth with the substitution of eqs.(3) and (4) yields

$$\frac{\partial\eta}{\partial t}+\sum_{m=1}^{N}\nabla\cdot\left[\frac{\sinh k_m(h+\eta)}{k_m\cosh k_mh}\boldsymbol{U}_m\right]=0. \tag{7}$$

To obtain the evolution equations of $\boldsymbol{U}_m$ $(m=1,\cdots,N)$, on the other hand, we may apply the Galerkin method to the momentum equation. Namely, after substituting eqs.(3)and (4) into the momentum eqation, the resulting equation is multiplied by the depth dependent function $F_m(z)$ and vertically integrated from $z=-h$ to η. Since the depth-dependence function has N different modes, we obtain a total of N vector equations corresponding to each mode:

$$\sum_{m=1}^{N}a_{nm}\frac{\partial\boldsymbol{U}_m}{\partial t}+b_n\nabla\left[g\eta+\frac{1}{2}\left(\boldsymbol{u}_s.\boldsymbol{u}_s+w_s^2\right)\right]=\sum_{m=1}^{N}\left[c_{nm}\nabla(\nabla\cdot\boldsymbol{U}_m)_t+\boldsymbol{d}_{nm}\cdot(\nabla\cdot\boldsymbol{U}_m)_t\right]\quad(n=1,2,\cdots,N) \tag{8}$$

where $\boldsymbol{u}_s$ and w_s are the velocity components at the free surface $z=\eta$, and

$$a_{nm}=a_{mn}=\frac{1}{2\cosh k_mh\cosh k_nh}\left\{\frac{\sinh(k_m+k_n)(h+\eta)}{k_m+k_n}+\frac{\sinh(k_m-k_n)(h+\eta)}{k_m-k_n}\right\},\quad b_n=\frac{\sinh k_n(h+\eta)}{k_n\cosh k_nh},$$

$$c_{nm}=c_{mn}=\frac{1}{k_m^2\cosh k_mh\cosh k_nh}\left[\frac{\cosh k_m(h+\eta)\sinh k_n(h+\eta)}{k_n}\right.$$

$$\left.-\frac{1}{2}\left\{\frac{\sinh(k_m+k_n)(h+\eta)}{k_m+k_n}+\frac{\sinh(k_m-k_n)(h+\eta)}{k_m-k_n}\right\}\right],\quad w_s=-\sum_{m=1}^{N}\nabla\cdot\left[\frac{\sinh k_m(h+\eta)}{k_m\cosh k_mh}\boldsymbol{U}_m(x,y,t)\right]. \tag{9}$$

The coefficients $\boldsymbol{d}_{nm}$ in eq.(8) have rather complicated mathematical forms, but may be evaluated as being nearly equal to $\boldsymbol{D}_{nm}$ shown in eq.(12) later. In this evaluation the neglected terms are $O(\varepsilon\nabla h)$. Equations (7) and (8) constitute a solvable set of equations for $2N+1$ unknowns, η, $\boldsymbol{U}_m$ $(m=1,\cdots,N)$, and describe their evolution as wave equations. It should be noted that no approximation has been introduced on the nonlinearity and that the full-dispersivity can be attained by taking only a few components; hence eqs.(7) and (8) may be referred to as "*fully-dispersive nonlinear wave equations*".

The wave-number parameters k_m $(m=1,\cdots,N)$, are to be specified with the linear dispersion relation, $\omega_m{}^2=gk_m\tanh k_m h$, by prescribing the angular frequencies ω_m $(m=1,\cdots,N)$ as a set of input data for the computation to properly cover the wave spectrum concerned. Therefore k_m must be treated as spatially varying quantities, according to the variation in $h(x,y)$.

It should be noted that the different components in the new wave equations are coupled each other even in their linearized form. This is an essential difference as compared with usual spectral method, in which each spectral component evolves independently when waves are linear. For this outstanding feature, the methodology of the formulation described above was named "*multiterm-coupling technique*".

Weakly Nonlinear Version of Fully-Dispersive Wave Equations

A simplified version of eqs.(7) and (8) has been also developed by introducing a weakly-nonlinear formulation. By invoking a Taylor series expansion of $\boldsymbol{u}$ around $z=0$, and keeping only the first-order nonlinear contributions both in the momentum equation and the vertically integrated continuity equation, we obtain finally the weakly nonlinear version of the fully-dispersive wave equations,

$$\frac{\partial \eta}{\partial t}+\sum_{m=1}^{N}\nabla\cdot\left[\left(\frac{\omega_m^2}{gk_m^2}+\eta\right)\boldsymbol{U}_m\right]=0, \tag{10}$$

$$\sum_{m=1}^{N}A_{nm}\frac{\partial \boldsymbol{U}_m}{\partial t}+B_n\nabla\left[g\eta+\eta\frac{\partial w_0}{\partial t}+\frac{1}{2}\left(\boldsymbol{u}_0.\boldsymbol{u}_0+w_0^2\right)\right]=$$
$$\frac{\partial}{\partial t}\sum_{m=1}^{N}\left[C_{nm}\nabla(\nabla\cdot\boldsymbol{U}_m)+\boldsymbol{D}_{nm}(\nabla\cdot\boldsymbol{U}_m)\right]\!,(n=1,2,\cdots,N) \tag{11}$$

where,

$$\omega_m^2=gk_m\tanh k_m h,\quad A_{nm}=\frac{\omega_n^2-\omega_m^2}{k_n^2-k_m^2},\quad A_{nn}=\frac{g\omega_n^2+h\left(g^2k_n^2-\omega_n^4\right)}{2gk_n^2},\quad B_n=\frac{\omega_n^2}{k_n^2},\quad C_{nm}=\frac{B_n-A_{nm}}{k_m^2},$$

$$\boldsymbol{D}_{nn}=\nabla C_{nn},\quad \boldsymbol{D}_{nm}=\frac{2}{k_m^2-k_n^2}\left[\frac{2\nabla k_m}{k_m}\left\{A_{nm}-\left(k_m^2-k_n^2\right)C_{nm}\right\}+\frac{\nabla h\sqrt{\left(g^2k_n^2-\omega_n^4\right)\left(g^2k_m^2-\omega_m^4\right)}}{gk_nk_m}\right]. \tag{12}$$

$\boldsymbol{u}_0$ and w_0 in eq.(11) are the velocities at $z=0$ and may be evaluated as

$$\boldsymbol{u}_0=\sum_{m=1}^{N}\boldsymbol{U}_m,\quad w_0=-\sum_{m=1}^{N}\nabla\cdot\left(\frac{B_m}{g}\boldsymbol{U}_m\right). \tag{13}$$

As shown in (12) the coefficients of the weakly nonlinear version of the equations are considerably simplified as compared with those defined in (9). This is an important advantage in terms of computational efficiency and robustness.

Single-Component (N=1) Forms :
"Narrow-banded nonlinear wave equations"

The linear dispersive characteristics of the fully dispersive wave models described above show almost perfect agreement with the theoretical dispersion curve over wide wave-number domain extending from very shallow to far deep water. Therefore these models may be called also "*broad-banded* nonlinear wave equations", which is applicable to irregular waves with a broad-banded spectrum at an arbitrary depth.

An important special case is the single-component (N=1) versions of the wave equations (7) and (8) or (10) and (11), because even with the single component the linear dispersion characteristics of the equations can well approximate the theoretical dispersion curve near the specified wave-number k_p. Therefore these versions may be applied to waves with a narrow-banded spectrum centered at k_p, In this sense they may be called "*narrow-banded* nonlinear wave equations".

The single-component forms of eqs.(10) and (11), for example, may be written as:

$$\frac{\partial \eta}{\partial t} + \nabla \cdot \left[\left(\frac{C_p^2}{g} + \eta \right) \boldsymbol{u}_0 \right] = 0, \tag{14}$$

$$C_p C_g \frac{\partial \boldsymbol{u}_0}{\partial t} + C_p^2 \nabla \left[g\eta + \eta \frac{\partial w_0}{\partial t} + \frac{1}{2} \left(\boldsymbol{u}_0 \cdot \boldsymbol{u}_0 + w_0^2 \right) \right] =$$
$$\frac{\partial}{\partial t} \left\{ \frac{C_p (C_p - C_g)}{k_p^2} \nabla (\nabla \cdot \boldsymbol{u}_0) + \nabla \left[\frac{C_p (C_p - C_g)}{k_p^2} \right] (\nabla \cdot \boldsymbol{u}_0) \right\}, \tag{15}$$

where C_p and C_g denote the phase and group velocities corresponding to k_p as defined by the linear theory.

By specifying C_p and C_g in these equations, we can show that various existing wave equations may be reproduced as the degenerate forms of eqs. (14) and (15). For example, Airy's shallow water equations and Boussinesq equations can be obtained as follows.

(1) Airy's shallow water equations: $C_p = C_g = \sqrt{gh}$

$$\frac{\partial \eta}{\partial t} + \nabla \cdot \left[(h + \eta) \boldsymbol{u}_0 \right] = 0, \qquad \frac{\partial \boldsymbol{u}_0}{\partial t} + \nabla \left(g\eta + \frac{1}{2} \boldsymbol{u}_0 \cdot \boldsymbol{u}_0 \right) = 0. \tag{16}$$

(2) Boussinesq equations: $C_p = \sqrt{gh} \left(1 - k_p^2 h^2 / 6 \right), \; C_g = \sqrt{gh} \left(1 - k_p^2 h^2 / 2 \right)$

$$\frac{\partial \eta}{\partial t} + \nabla \cdot \left[(h + \eta) \boldsymbol{u}_0 \right] + \frac{h^3}{3} \nabla^2 (\nabla \cdot \boldsymbol{u}_0) = 0, \qquad \frac{\partial \boldsymbol{u}_0}{\partial t} + \nabla \left(g\eta + \frac{1}{2} \boldsymbol{u}_0 \cdot \boldsymbol{u}_0 \right) = 0, \tag{17}$$

where all the higher-order terms have been neglected.

Combined Form of the Single-Component Equations: "Time-dependent nonlinear mild-slope equation"

The single-component equations (14) and (15) may be combined, with the introduction of the mild-slope assumption, to give the following equation of η:

$$C_g \eta_{tt} - C_p^3 \nabla^2 \eta - \frac{(C_p - C_g)}{k_p^2} \nabla^2 \eta_{tt} - C_p \nabla (C_p C_g) \cdot (\nabla \eta) - \frac{3}{2} g C_p \left(3 - 2\frac{C_g}{C_p} - \frac{k_p^2 C_p^4}{g^2} \right) \nabla^2 (\eta^2) = 0. \tag{18}$$

By further manipulations, the linearized equation of (18) can be found to lead to the time-dependent (or "*narrow-banded*") mild-slope equation proposed by Smith and Sprinks (1975),

$$\eta_{tt} + \omega_p^2 \left(\frac{C_p - C_g}{C_p} \right) \eta - \nabla (C_p C_g \nabla \eta) = 0, \quad (\omega_p = C_p k_p) \tag{19}$$

and also to Berkhoff's (1972) elliptic equation as an original steady form of the mild-slope equation,

$$k_p^2 C_p C_g Z + \nabla \cdot (C_p C_g \nabla Z) = 0, \tag{20}$$

in which Z denotes a spatially varying wave amplitude. Therefore, eq.(18) can be regarded as an extension of the mild-slope equations to nonlinear waves. In this sense, eq.(18) may be called "*time-dependent nonlinear mild-slope equation*". However its linear dispersion characteristics are not the same as those of the time-dependent mild-slope equation (19) and the similar equations by Kirby et al.(1992) and Kubo et al. (1992), since the latter equation approximates more limited region around ω_p in the dispersion curve. This means that even in the linear version of eq.(18) the new mild-slope equation has an advantage as compared with these previous equations. It is also shown that KdV equation can be obtained as a degenerate form of the unidirectional form of eq.(18).

3. NUMERICAL PERFORMANCE

The high performance of the present model has been confirmed by numerical simulations for various cases including a solitary wave, cnoidal and Stokes wave trains, linear random waves, nonlinear irregular waves over a bar, nonlinear directional wave convergence over a focusing lens topography. As an example, Fig.2 shows the numerical result for nonlinear irregular waves over a submerged bar calculated by eqs. (10) and (11) with two components, which is compared with the experimental result by Beji & Battjes (1993).

4. CONCLUDING REMARKS

The present wave model can describe waves under general conditions, such as nonlinear random waves with a broad-banded spectrum at an arbitrary depth including very shallow and far deep water depths. Following this success of the development of the new wave model, several similar works based on the multiterm-coupling technique have been reported. For example, Isobe (1994) employed a variational principle instead of the Galerkin method to derive a set of coupled equations. Nochino (1994) applied the Galerkin formulation to the basic equation in terms of pressure. More recently Kennedy & Fenton (1996) proposed a numerical method in which a Galerkin-type formulation with the multiterm-coupling technique is applied to the Laplace equation of the velocity potential as the basic equation. In these works, polynomials with different order are used for the vertical dependence functions, so that for deep water waves quite large number of components with higher order polynomials must be introduced to obtain precise approximation of the velocity field and hence of the dispersion characteristics. To the contrary, in our model, few components, or only one component for narrow-banded waves, are enough even for deep water waves. This is an important advantage in practical applications.

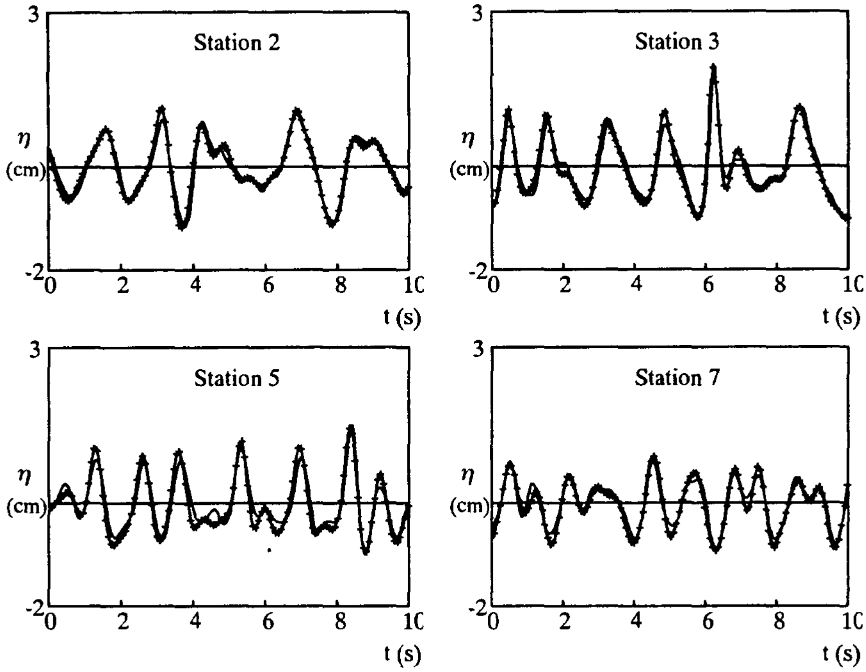

Fig.2 Comparisons of the experimental measurements of nonlinear random wave propagation over a submerged bar (—) with the numerical simulations (+) using eqs.(10) and (11) with two components; $k_1=k_p$ and $k_2=\pi k_p$, k_p being the wave-number corresponding to the peak period T_p=2.0s (Nadaoka et al., 1997). Station 2: upslope, station 3: horizontal bottom of the bar top, station 5: downslope, station 7: horizontal bottom behind the bar.

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