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Robust Control for Free-Joint Manipulators

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Abstract

This paper presents a robust control scheme of free-joint manipulators to overcome actuator failures and uncertainties in Cartesian space where tasks are planned. The control scheme has the adaptation law for the upper bound on the norm of uncertainties through the Lyapunov function approach. To solve the dynamic singularity problem in the controller, the singular and nonsingular regions are investigated based on a computer simulation. Then a singularity-free Cartesian trajectory planning is achieved in order to guarantee the availability of the control scheme. To illustrate the validity of the proposed control scheme, simulation results for a three-link planar robot arm with a free joint are shown.

1 Introduction

The dynamics and control of underactuated manipulators with second-order nonholonomic constraints, which is a non-integrable constraint on the acceleration, has drawn a great attention in recent years [1]-[4] [11]. The advantages of using underactuated systems reside in the fact that they weigh less, and consume less energy than their fully-actuated counterparts, thus being useful for applications such as space robotics [4] [10] and redundant robots. The underactuated robot concept is also useful for the reliability or fault-tolerant design [8] [9] of fully-actuated manipulators working with dangerous materials or in remote or hazardous areas such as space, underwater, nuclear power plants, etc. where the repair or replacement of actuators is a very difficult task [4] [8] [9] [11].

Robot tasks are usually planned in Cartesian space or operational space. Even both actuators and brakes of passive joints may fail, or passive joints may have neither actuators nor brakes originally. Therefore, a study on the control in Cartesian space without braking passive joints is needed [2] [11].

When the passive joints are *free-swinging joints* or *free-joints*, that is, these joints move freely under the influence of external forces and gravity, a robust Cartesian control scheme overcoming the uncertainties for free-joint manipulators is proposed in this paper. The form of the proposed controller has basically inverse dynamics form. The controller has the assumption that a nominal decoupling matrix (control input matrix) should be nonsingular. In order to guarantee the availability of the control scheme, a singularity-free trajectory planning overcoming the dynamic singularities is performed within the nonsingular regions shown in Cartesian space via a computer simulation.

Simulation results for a three-link planar manipulator with one free-swinging passive joint are presented to show the feasibility of the proposed control scheme.

2 System Description

The forward kinematics in position level is as follows. $p_e = f(q) \in \Re^m$ where $p_e \in \Re^m$ is the manipulator's end-effector position and orientation vector with respect to the base frame, $q \in \Re^n$ is the joint position vector and $f(q) \in \Re^m$ is the nonlinear sinusoidal function of the joint variable vector.

The Jacobian relationship is obtained as follows: $\dot{p}_e = J(q)\dot{q} \in \Re^m$ where $J(q) = \partial f(q)/\partial q \in \Re^{m\times n}$. The Jacobian matrix $J(q)$ can be partitioned as follows: $J(q) = (J_a(q)\ J_p(q)) \in \Re^{m\times n}$ where $J_a(q) \in \Re^{m\times r}$ is the active part of the Jacobian matrix and $J_p(q) \in \Re^{m\times p}$ is the passive part of it. Here, $n(= r+p)$ is the number of the total joints, r is the number of the actuated or active joints, and p is the number of the unactuated or passive joints.

The dynamic equation of an n-link rigid underactuated manipulator with active joints (r) and free joints (p) can be written in joint space as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B\tau_a + d(t) = \begin{pmatrix} \tau_a + d_a(t) \\ O_p + d_p(t) \end{pmatrix} \tag{1}$$

where $q = (q_a^T\ q_p^T)^T \in \Re^{(n=r+p)}$ is the vector of joint variables, $q_a \in \Re^r$ is the position vector of active joints, $q_p \in \Re^p$ is the position vector of passive(free) joints, $M(q) \in \Re^{n\times n}$ is the symmetric positive definite inertial matrix, $C(q,\dot{q})\dot{q} \in \Re^n$ is the vector of centrifugal and Coriolis torques, $G(q) \in \Re^n$ is the vector of gravitational torques, $\tau_a \in \Re^r$ is the actual control torque vector applied to the active joints, $O_p \in \Re^p$ is the zero torque vector at the passive joints, and $d(t) = (d_a^T\ d_p^T)^T \in \Re^n$ is a norm-bounded external disturbance vector, for which $\|d(t)\| \leq d_{max}$ where d_{max} is an *unknown* positive constant.

This equation (1) can be partitioned as follows.

$$\begin{pmatrix} M_{aa} & M_{ap} \\ M_{pa} & M_{pp} \end{pmatrix}\begin{pmatrix} \ddot{q}_a \\ \ddot{q}_p \end{pmatrix}+\begin{pmatrix} F_a \\ F_p \end{pmatrix}=\begin{pmatrix} \tau_a + d_a \\ O_p + d_p \end{pmatrix} \quad (2)$$

where both $M_{aa} \in \Re^{r\times r}$ and $M_{pp} \in \Re^{p\times p}$ are also symmetric positive definite matrices, $M_{ap} = M_{pa}^T \in \Re^{r\times p}$, and $F(q,\dot{q}) = (F_a^T\ F_p^T)^T = C(q,\dot{q})\dot{q} + G(q)$.

Property 1: [5] *There exist positive constants* j_{max}, $j_{d_{max}}$, m_{min}, m_{max}, c_{max}, g_{max}, f_g *and* f_c *such that* $\|J(q)\| \leq j_{max}$, $\|\dot{J}(q,\dot{q})\| \leq j_{d_{max}}\|\dot{q}\|$, $m_{min} \leq \|M(q)\| \leq m_{max}$, $\|C(q,\dot{q})\| \leq c_{max}\|\dot{q}\|$, $\|G(q)\| \leq g_{max}$, $\|F(q,\dot{q})\| \leq f_g + f_c\|\dot{q}\|^2$.

Multiplying (1) by JM^{-1}, we have

$$\begin{aligned} J\ddot{q} + JM^{-1}(C\dot{q} + G) &= JM^{-1}(B\tau_a + d(t)) \\ &= \tilde{J}_a\tilde{M}_{aa}^{-1}\tau_a + JM^{-1}d(t) \quad (3) \end{aligned}$$

where $JM^{-1}B = \tilde{J}_a\tilde{M}_{aa}^{-1}$, and $\tilde{M}_{aa}$ and $\tilde{J}_a$ are called the *effective inertial matrix* and *effective Jacobian matrix* of the robot arm, respectively, and defined as follows, $\tilde{M}_{aa} = M_{aa} - M_{ap}M_{pp}^{-1}M_{pa} \in \Re^{r\times r}$, $\tilde{J}_a = J_a - J_pM_{pp}^{-1}M_{pa} \in \Re^{m\times r}$, and $D_a(q)$ called *decoupling matrix* for the system, is defined by

$$D_a(q) = J(q)M^{-1}(q)B = \tilde{J}_a(q)\tilde{M}_{aa}^{-1}(q) \in \Re^{m\times r}. \quad (4)$$

The relationship to map the joint acceleration to the acceleration of the end-effector is obtained as $\ddot{p}_e = J\ddot{q} + \dot{J}\dot{q}$. Substituting the above equation into (3), we can obtain the following differential equation,

$$\ddot{p}_e - b(q,\dot{q}) = D_a(q)\tau_a + D(q)d(t) \quad \in \Re^m \quad (5)$$

where $D(q) = J(q)M^{-1}(q) \in \Re^{m\times n}$, $b(q,\dot{q}) = \dot{J}(q,\dot{q})\dot{q} - J(q)M^{-1}(q)\left(C(q,\dot{q})\dot{q} + G(q)\right) \in \Re^m$.

Property 2: *By Property 1, there exist positive constants* θ_{d_a}, θ_{b_0}, θ_{b_1} *and* θ_d *such that*

$$\|D_a(q)\| \leq \theta_{d_a},\ \|b(q,\dot{q})\| \leq \theta_{b_0} + \theta_{b_1}\|\dot{q}\|^2,\ \|D(q)\| \leq \theta_d. \quad (6)$$

3 Robust Control Design

3.1 Robust Control and Stability Analysis

A robust controller is made by the following form

$$\tau_a = \hat{D}_a^{\#}(q)(v_r - \hat{b}(q,\dot{q})) \quad (7)$$

where $\hat{D}_a^{\#}(q) \in \Re^{r\times m}$ is a pseudoinverse matrix [7] [11] of $\hat{D}_a(q)$, and $\hat{D}_a$ and $\hat{b}$ are the *nominal model* of D_a and b with the guessed system parameters.

Assumption 1: *It is assumed that* $r \geq m$ *in the design of controller. Then it is selected that* $\hat{D}_a^{\#} = \hat{D}_a^T(\hat{D}_a\hat{D}_a^T)^{-1}$.

Assumption 2: *In the controller (7), it is assumed that* $\hat{D}_a^{\#}(q)$ *exists for all joint configurations of the manipulator during the total control process.*

Remark 1: *Since* $r \geq m$, *the fact that* $\hat{D}_a^{\#}$ *exists is the same that* $\hat{D}_a\hat{D}_a^T \in \Re^{m\times m}$ *is invertible [7] [11].*

With the control law (7), the differential equation (5) can be rewritten as

$$\ddot{p}_e = b(q,\dot{q}) + D_a(q)\tau_a + D(q)\,d(t) = v_r + \eta_1 + \eta_2 \quad (8)$$

where the uncertainty terms η_1 and η_2 are defined by $\eta_1 = (D_a\hat{D}_a^{\#} - I_m)\,v_r + (b - D_a\hat{D}_a^{\#}\hat{b})$, $\eta_2 = D\,d(t)$. The control input v_r is developed by $v_r = v + \Delta v$.

The Cartesian tracking error e and the augmented error s are denoted by $e = p_e - p_{e_d} \in \Re^m$, $s = \dot{e} + \Lambda e \in \Re^m$ where $p_{e_d} \in \Re^m$ is the desired trajectory of the end-effector specified in Cartesian space and Λ is an $m\times m$ positive definite diagonal constant matrix.

The *outer loop* input is defined by $v = \ddot{p}_{e_d} - (K + \Lambda)\dot{e} - K\Lambda e$ where K is an $m\times m$ positive definite diagonal constant matrix. Δv is the control input term overcoming parameter variations and disturbances.

The closed-loop error dynamics for s becomes

$$\dot{s} = -Ks + \Delta v + \eta \quad (9)$$

where $\eta = \eta_1 + \eta_2$ is the *lumped uncertainty* term.

The norm-bound of the lumped uncertainty η is

$$\|\eta\| \leq \|D_a\hat{D}_a^{\#} - I_m\|\|v_r\| + \|b - D_a\hat{D}_a^{\#}\hat{b}\| + \|D\|\|d(t)\|. \quad (10)$$

Assumption 3: *It is assumed that there exists an unknown positive constant* κ_0 *such that*

$$\|D_a\hat{D}_a^{\#} - I_m\| \leq \kappa_0 < 1, \quad (11)$$

Property 3: *By property 2, there exist unknown positive constants* κ_1 *and* κ_2 *such that*

$$\|b - D_a\hat{D}_a^{\#}\hat{b}\| \leq \kappa_1 + \kappa_2\|\dot{q}\|^2. \quad (12)$$

From Assumption 3, Property 3 and the norm-bounded property of disturbances, (10) is calculated as follows: $\|\eta\| \leq \kappa_0\|v_r\| + \kappa_1 + \kappa_2\|\dot{q}\|^2 + \theta_d d_{max}$.

The control input Δv is defined as the following form. $\Delta v = -\hat{\rho}\frac{\alpha}{h_\alpha(\|\alpha\|)}$, $\alpha = Rs$, $\hat{\rho} = \hat{\theta}^T\,\psi$ where R is an $m\times m$ positive definite diagonal constant matrix, $\theta \in \Re_+^7$ is an unknown positive constant vector and $\hat{\theta} \in \Re_+^7$ is its estimate, and $\psi \in \Re_+^7$ is a known continuous function, and $h_\alpha(\|\xi\|)$ is a positive function to alleviate the chattering of the control input. For example, $h_\alpha(\|\xi\|)$ can be defined as follows. $h_\alpha(\|\xi\|) = \begin{cases} \|\xi\| & \text{if } \|\xi\| > \epsilon \\ \epsilon & \text{if } \|\xi\| \le \epsilon \end{cases}$ or $h_\alpha(\|\xi\|) = \|\xi\| + \epsilon$. An adaptation law with a σ-modification term is also defined as follows. $\dot{\hat{\theta}} = \Gamma(\frac{\psi\|\alpha\|^2}{h_\alpha(\|\alpha\|)} - \sigma\hat{\theta})$ where Γ is a $\gamma \times \gamma$ positive constant adaptation matrix and $\sigma = diag(\sigma_1, \sigma_2, \cdots, \sigma_\gamma)$, $\sigma_i > 0$ for $i = 1, 2, \cdots, \gamma$.

Property 4: *When the initial estimates are selected as a nonnegative value ($\hat{\theta}(0) \ge 0$), there exist unknown positive constants κ_3 and κ_4 such that*

$$\|v_r\| \le \|v\| + \|\Delta v\| \le \|\ddot{p}_{e_d}\| + \kappa_3\|\dot{e}\| + \kappa_4\|e\| + \hat{\rho}. \quad (13)$$

Remark 2: *In the above Property 4, it is assumed that $\|\Delta v\| \le \hat{\rho}$. Since $\dot{\hat{\theta}} + \Gamma\sigma\hat{\theta} = \Gamma\frac{\psi\|\alpha\|^2}{h_\alpha(\|\alpha\|)}$ is a low-pass filter with a positive input, $\hat{\theta}(t) \ge 0\ \forall t$ when $\hat{\theta}(0) \ge 0$. Therefore, $\hat{\rho} = \hat{\theta}^T\psi\ \ge 0\ \forall t$ and so $|\hat{\rho}| = \hat{\rho}$.*

We impose the norm-bound of lumped uncertainty as follows.

$$\|\eta\| \le \bar{\theta}_1 + \bar{\theta}_2\|\dot{q}\|^2 + \bar{\theta}_3(\|\ddot{p}_{e_d}\| + \hat{\rho}) + \bar{\theta}_4\|\dot{e}\| + \bar{\theta}_5\|e\| \quad (14)$$

where $\bar{\theta}_1 = \theta_d d_{max} + \kappa_1$, $\bar{\theta}_2 = \kappa_2$, $\bar{\theta}_3 = \kappa_0$, $\bar{\theta}_4 = \kappa_0\kappa_3$, $\bar{\theta}_5 = \kappa_0\kappa_4$. From Assumption 3, it is assumed that $0 \le \bar{\theta}_3 = \kappa_0 < 1$.

We now summarize the proposed robust continuous controller as follows:

$$\tau_a = \hat{D}_a^{\#}(q)(v_r - \hat{b}(q,\dot{q})) \in \Re^r,\ v_r = v + \Delta v \quad (15)$$

$$v = \ddot{p}_d - (K + \Lambda)\dot{e} - K\Lambda e \in \Re^m \quad (16)$$

$$\Delta v = -\hat{\rho}\frac{\alpha}{h_\alpha(\|\alpha\|)} \in \Re^m, \quad \alpha = Rs \in \Re^m \quad (17)$$

$$\hat{\rho} = \hat{\theta}^T\psi,\ \psi = (1\ \|\dot{q}\|^2\ \|\ddot{p}_{e_d}\|\ \|\dot{e}\|\ \|e\|)^T \quad (18)$$

$$\dot{\hat{\theta}} = \Gamma(\frac{\psi\|\alpha\|^2}{h_\alpha(\|\alpha\|)} - \sigma\hat{\theta})\ \in \Re^5 \quad (19)$$

where $\hat{\theta} \in \Re^5_+$ is the estimate of $\theta \in \Re^5_+$, R is an $m \times m$ positive definite diagonal constant matrix, and Γ is a 5×5 positive constant adaptation matrix and $\sigma = diag(\sigma_1, \sigma_2, \cdots, \sigma_5)$, $\sigma_i > 0$ for $i = 1, 2, \cdots, 5$.

Theorem 1: *Under Assumptions 1 ~ 3, if we apply the control law (15) ~ (19) to the underactuated robot manipulator system (5), then the tracking errors are globally uniformly ultimately bounded (GUUB).*

Proof : Let's define a Lyapunov function candidate,

$$V = \frac{1}{2}s^T Rs + \frac{1-\bar{\theta}_3}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \quad (20)$$

where $\tilde{\theta} = \hat{\theta} - \theta \in \Re^5_+$.
Taking the time derivative of V along the solution of the system, and substituting the closed-loop error dynamics for s (9), the norm-bound of lumped uncertainty (14), the control law (15)~(17) and the adaptation law (18)~(19) make the following results:

$$\begin{aligned}\dot{V} &= s^T R(-Ks + \Delta v + \eta) + (1-\bar{\theta}_3)\tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \\ &\le -s^T RKs - (1-\bar{\theta}_3)\tilde{\theta}^T\sigma\hat{\theta} + \bar{w}(\rho,\hat{\rho},\|\alpha\|) \quad (21)\end{aligned}$$

where
$\theta_i = \frac{\bar{\theta}_i}{1-\bar{\theta}_3}$, $i = 1, 2, \cdots, 5$, $\theta = (\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5)^T$, $\psi = (1\ \|\dot{q}\|^2\ \|\ddot{p}_d\|\ \|\dot{e}\|\ \|e\|)^T$, $\rho = \theta^T\psi(\|\dot{q}\|, \|\ddot{p}_d\|, \|\dot{e}\|, \|e\|)$, and $\tilde{\rho} = \hat{\rho} - \rho = (\hat{\theta} - \theta)^T\psi = \tilde{\theta}^T\psi$, and $\bar{w}(\rho,\hat{\rho},\|\alpha\|) = \frac{\|\alpha\|}{h_\alpha(\|\alpha\|)}[h_\alpha(\|\alpha\|) - \|\alpha\|][\hat{\rho}\bar{\theta}_3 + \rho(1-\bar{\theta}_3)]$.
Here, the following relationship holds: $\tilde{\theta}^T\sigma\tilde{\theta} + \tilde{\theta}^T\sigma\theta \ge \frac{1}{2}(\tilde{\theta}^T\sigma\tilde{\theta} - \theta^T\sigma\theta)$. Now, we have the following.

$$\dot{V} \le -\frac{1}{2}\lambda_{min}(Q)\|z\|^2 + w(\rho,\hat{\rho},\|\alpha\|) \quad (22)$$

where $z = (s^T\ \tilde{\theta}^T)^T$, and $Q = \begin{pmatrix} 2RK & 0 \\ 0 & (1-\bar{\theta}_3)\sigma \end{pmatrix}$, and $w(\rho,\hat{\rho},\|\alpha\|) = \frac{1}{2}(1-\bar{\theta}_3)\theta^T\sigma\theta + \bar{w}(\rho,\hat{\rho},\|\alpha\|)$, and $\lambda_{min}(\cdot)$ is the minimum eigenvalue of its argument.
From (20), $V(z) = \frac{1}{2}z^T Pz \le \frac{1}{2}\lambda_{max}(P)\|z\|^2$, where $P = \begin{pmatrix} R & 0 \\ 0 & (1-\bar{\theta}_3)\Gamma^{-1} \end{pmatrix}$ and $\lambda_{max}(\cdot)$ is the maximum eigenvalue of its argument. Thus ,

$$\dot{V} \le -\mu V + w(\rho,\hat{\rho},\|\alpha\|) \quad (23)$$

where $\mu = \frac{\lambda_{min}(Q)}{\lambda_{max}(P)}$, and both Q and P are positive definite matrices.

The differential inequality (23) has the following solution $V(t,z(t)) \le \frac{w(\rho,\hat{\rho},\|\alpha\|)}{\mu} + [V(t_0,z(t_0)) - \frac{w(\rho,\hat{\rho},\|\alpha\|)}{\mu}]e^{-\mu(t-t_0)}$. Now, since $V(t,z(t)) \ge \frac{1}{2}s^T Rs \ge \frac{1}{2}\lambda_{min}(R)\|s\|^2$ and $V(t,z(t)) \ge \frac{1}{2}(1-\bar{\theta}_3)\lambda_{min}(\Gamma^{-1})\|\tilde{\theta}\|^2$, the boundedness of $s(t)$ and $\tilde{\theta}$ is shown as follows.

$$\|s(t)\| \le [\frac{2V}{\lambda_{min}(R)}]^{\frac{1}{2}},\ \ \|\tilde{\theta}(t)\| \le [\frac{2V}{(1-\bar{\theta}_3)\lambda_{min}(\Gamma^{-1})}]^{\frac{1}{2}}. \quad (24)$$

Consequently, since both s and $\tilde{\theta}$ are globally uniformly ultimately bounded, the stable dynamics $s = \dot{e} + \Lambda e$ guarantees that the tracking errors e and $\dot{e}$ are also globally uniformly ultimately bounded. ■

Remark 3: *Here, we can find the following fact: If $\epsilon \to 0$ and $\sigma \to 0$, then the uniformly ultimately boundedness becomes the asymptotic stability.*

3.2 Cartesian Path Planning Avoiding Dynamic Singularities

The assumption for the nonsingular configurations (Assumption 2) should be satisfied to guarantee the availability of the presented controller. Once the robot manipulator is in the inside of the singular configurations, the above assumption is not guaranteed. Therefore, a path planning avoiding the dynamic singularities is needed [11].

The singularities of the nominal decoupling matrix $\hat{D}_a(q)$ with the guessed nominal dynamic parameters must be shown in joint space. The set of singular points found in joint space can be shown as the regions in Cartesian space via the kinematics. Some regions shown in Cartesian space corresponding to those shown in joint space *may be* or *may not be* the singular regions as known by the inverse kinematics which is one-to-many mapping. These regions are called *"semi-singular regions"*, which means it is doubtful whether those are singular or not. On the other hand, it is guaranteed that the nonsingular regions in Cartesian space are always nonsingular in joint space. Therefore, a path of the end-effector avoiding the dynamic singularities should be formed within the regions in Cartesian space into which the nonsingular regions in joint space are transformed by the kinematics. Then, it is guaranteed that the desired path of the end-effector, which is made within the nonsingular regions in Cartesian space, can avoid the singularities.

We now presents a path planning procedure avoiding the singularities as follows.

1. Obtain the dynamic singularity regions such that $|Det(\hat{D}_a(q)\hat{D}_a^T(q))| \leq \epsilon_d$ for almost all joint configurations in joint space for the given manipulator, where '*Det*' represents the determinant of a matrix and the criterion ϵ_d is a very small positive constant in the neighborhood of zero.

2. Get the semi-singularity regions in Cartesian space corresponding to the singularity regions in joint space by means of the forward kinematics. And find the singularity-free regions in Cartesian space corresponding to the nonsingular regions in joint space.

3. Make a desired path or trajectory within the nonsingular regions in Cartesian space.

4 Simulation Study

The underactuated manipulator to be simulated is a three-link planar robot arm ($n = 3$) with two active joints ($r = 2$) and one passive(free) joint ($p = 1$). The robot's end-effector can control two degrees of freedom ($m = 2$) position in the X-Y plane. In this simulation, it is assumed that the third joint (q_3) is passive.

The simulated underactuated three-link planar manipulator is shown in Fig. 1. It is here assumed that one passive joint is free to swing and has both no actuator and no brake. It is assumed that there are no frictions in the manipulator's joints. It is also assumed that there are no joint limits for the joints and thus the joint angles can vary from 0 (rad) to 2π (rad).

The numerical real and nominal parameters of the simulated manipulator are given in Table I. It is assumed that the lengths of each link are all exactly known. The nominal dynamic parameters used in the proposed controller (15) $\sim$ (19) are set to 70% of the real dynamic parameter values.

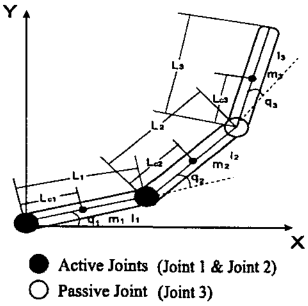


Fig. 1. A three-link planar robot arm with a passive joint: ($q_1 \& q_2$: active joints; q_3: passive joint)

The external disturbance to be inserted into each joint is the random noise whose each magnitude is bounded by 0.5, that is, $|d_i(t)| \leq 0.5$ for $i = 1, 2, 3$.

Table I. Numerical parameter values of the manipulator: $[(L_1, m_1, I_1, L_{c1}) = (L_2, m_2, I_2, L_{c2}) = (L_3, m_3, I_3, L_{c3})]$

Parameters	Values	Link i (i=1,2,3)
Length	Real	0.5
$[L_i(m)]$	Nominal	
Mass	Real	1
$[m_i(kg)]$	Nominal	0.7
Moment of inertia	Real	0.1
$[I_i(kgm^2)]$	Nominal	0.07
COM position	Real	0.25
$[L_{ci}(m)]$	Nominal	0.175

* 'COM' : Center Of Mass

The simulations are performed for two cases according to the motion direction of the planar arm to the gorund as follows: 1. Case 1: Horizontal motion ($G(q) = 0$); 2. Case 2: Vertical motion ($G(q) \neq 0$).

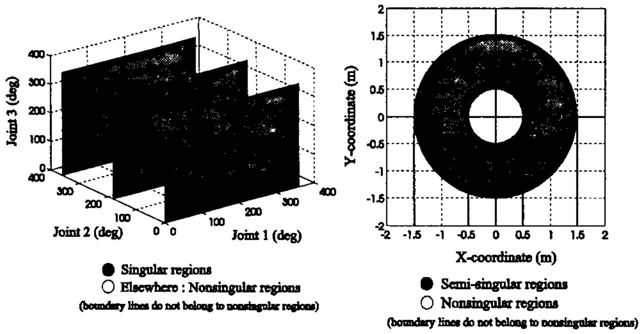
The simulations include the singularity-free Cartesian trajectory planning and the robust control tracking the planned trajectory.

In this simulation, a desired path of the end-effector in Cartesian space is a circle. The used Cartesian task is the circle tracking task that the robot end-effector circulates one time along the specified circle. The type of the trajectory tracking the specified circle is a quintic polynomial with all zero initial and final velocities and accelerations. The total execution time of the circle tracking task (t_f) is 5.0 (sec).

Now, the singularity-free regions are shown by the simulation. A small positive criterion constant ϵ_d is selected as $\epsilon_d = 10^{-7}$.

Because the singularity problem is unaffected by the effect of gravitational torques, the singularity-free regions for both cases are identical. Thus, the singularity-free path planning is performed in the same manner for both cases.

Fig. 2-(a) and Fig. 2-(b) shows the singularity-free regions in joint space and Cartesian space.



(a) Singular/nonsingular regions in joint space (b) Semi-singular/nonsingular regions in Cartesian space

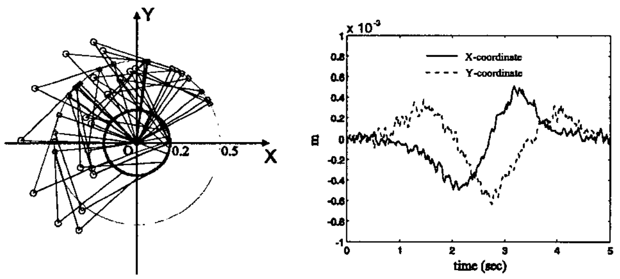
Fig. 2. Singular/semi-singular regions and nonsingular regions in joint space and Cartesian space.

The center point of the desired circle is $(x_{e_{dc}}, y_{e_{dc}}) = (0,0)$. The radius of the circle is $R_e = 0.2$. Therefore, the desired path $x_{e_d}^2(t) + y_{e_d}^2(t) = R_e^2$ is used in the control simulation. The initial and final position of the desired trajectory is $(x_{e_{di}}, y_{e_{di}}) = (x_{e_{df}}, y_{e_{df}}) = (0.2, 0)$.

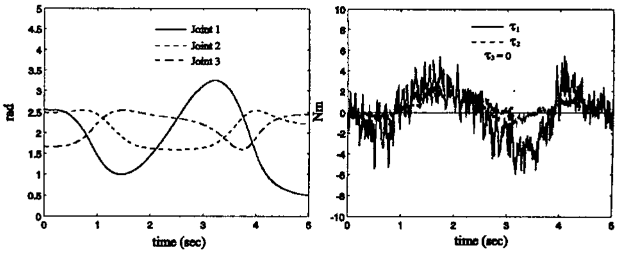
In the control simulation, a singularity-free desired trajectory is used as that defined in the above trajectory planning simulation. The desired trajectory is the circular motion with a quintic polynomial. The task is that the robot end-effector circulates one time along the specified circle in the X-Y plane. The actual initial position of the end-effector in the X-Y plane is the same as the desired initial position.

In the controller, a positive continuous function $h_\alpha(\|\alpha\|)$ is chosen as $h_\alpha(\|\alpha\|) = \|\alpha\| + \epsilon$. Here, it is selected $\epsilon = 0.1.$. The several values used in the simulation are given as follows: $\hat{\theta}(0) = (0\ 0\ 0\ 0\ 0)^T$, $\Gamma = \text{diag}(0.01, 0.01, 0.01, 0.01, 0.01)$, $\sigma = \text{diag}(0.1, 0.1, 0.1, 0.1, 0.1)$, $K = \text{diag}(100, 100)$, $\Lambda = \text{diag}(50, 50)$, $R = \text{diag}(2, 2)$.

For Case 1 (horizontal motion), the control results are shown in Fig. 3. For Case 2 (vertical motion), the results are shown Fig. 4.

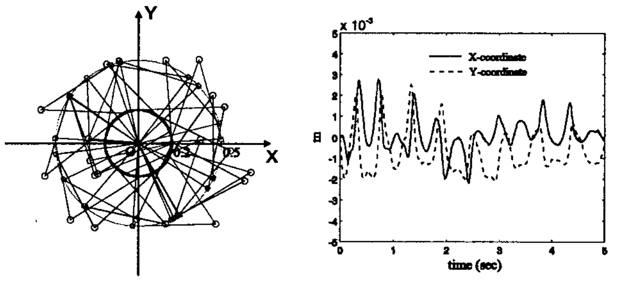


(a) Snapshot of robot motion (b) Position tracking error

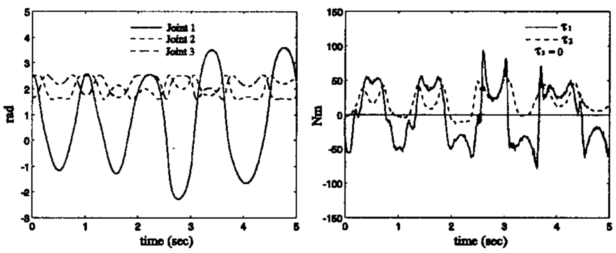


(c) Joint angles (q) (d) Control torque input (τ_a)

Fig. 3. Control results for Case 1 (horizontal motion).



(a) Snapshot of robot motion (b) Position tracking error



(c) Joint angles (q) (d) Control torque input (τ_a)

Fig. 4. Control results for Case 2 (vertical motion).

From the simulation results, it has been shown that the proposed control scheme is feasible and useful.

5 Conclusions

In this work, a robust control scheme for free-joint manipulators overcoming the uncertainties has been proposed in Cartesian space where robot tasks are usually planned. In such an underactuated manipulator system with free joints, both joint actuators and brakes at passive joints may fail due to a hardware or software fault, or passive joints may have neither actuators nor brakes in the original design of the manipulator. The proposed control scheme does not need *a priori* knowledge of the accurate dynamic parameters and the exact uncertainty bounds. To overcome the dynamic singularity problem in the controller, a singularity-free Cartesian path planning has been achieved via a computer simulation. It has been found that the proposed robust control scheme is valid and feasible through simulation results.

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